Exemplar 8:

Rotational Symmetry about a Point

Objective: To understand rotational symmetry of polygons

Key Stage: 3

Learning Unit: Transformation and Symmetry

Materials Required: (1) Large logos and polygons made from coloured paper
(2) Computer software to demonstrate the rotation of polygons and logos
(3) Dynamic geometry software *Geometer's Sketchpad* (later referred as *Sketchpad*) and the files in 2D_Sym
(4) Worksheet on alphabets and polygons.

Prerequisite Knowledge: (1) The idea of line symmetry (or reflectional symmetry used at Key Stage 3)
(2) The meaning of regular, equilateral and equiangular polygons.

Description of the Activity:

1. The teacher sticks some daily life pictures such as "wheel" or a letter "S" on the blackboard (Fig. 1). Students are asked whether the logo is symmetrical or not. The teacher recalls the idea of line symmetry learnt in primary schools or reflectional symmetry used in Key Stage 3 and guides students to see that the logo is not reflectional symmetrical but is a rotational symmetrical shape.

Fig. 1

2. The teacher further explains the idea of rotational symmetry by illustrating the movement of figures in the files 2D_Symmetry.exe (Fig. 2). By clicking the "Rotate" box, the figure will rotate and stop when the image overlaps with the original figure. Students can count the number of overlapping by counting the number in clicking the "Rotate" box.

Fig. 2

3. The teacher puts a square on the blackboard and discusses with students the number of fold for the square. The figure is rotated to check against students' hypotheses. It is summarized that "A square has 4-fold rotational symmetry".

4. A **regular** pentagon is put on the blackboard. The teacher repeats the discussion and activity as (3). Through questioning and demonstration, the teacher guides students to come to the conclusion that "A regular pentagon has 5-fold rotational symmetry".

5. The teacher raises the conjecture that "A polygon of n sides always has n-fold rotational symmetry about its centre". The teacher distributes the polygons and Worksheet 1 to students and asks them to work on the polygons to check the validity of this conjecture.

6. The teacher invites some students to demonstrate the answers of some polygons. After that, the teacher can use the files 2D_Symmetry.exe to illustrate the rotation of other polygons (Fig. 3) and guides students to summarize the answers to the worksheet.

Fig. 3

7. Students are asked to suggest conditions for the validity of the conjecture in point 5. The teacher invites students to show counter-examples for the above conjecture and asks them to modify the conjecture in (5) to "A **regular** polygon of n sides has n-fold rotational symmetry about its centre." Discussion on why it is true should be made.

8. The teacher further asks students to check whether the conjecture "An equilateral polygon of n sides has rotational symmetry" is true.

9. Students are then asked to summarize their findings. For those students who are very interested in the topic, the teacher can distribute Worksheet 2 to them as an enrichment activity.

Worksheet 1: Rotational Symmetry of Polygons

Count the number of fold of rotational symmetry for the following polygons and put them into appropriate places in the table below:

Right-angled triangles	Equilateral triangles	Isosceles triangles	Scalene triangles
Parallelograms in general		Quadrilaterals in general	
Squares	Rectangles	Rhombuses	Kites
Isosceles Trapeziums		Equilateral hexagons*	*Equiangular hexagons
Regular octagons	Regular heptagons	Equilateral octagons*	*Equiangular octagons
Regular pentagons	Regular hexagons	Equilateral pentagons*	*Equiangular pentagons

Rotational Symmetry	Polygons
No rotational symmetry	
2-fold rotational symmetry	
3-fold rotational symmetry	
4-fold rotational symmetry	
5-fold rotational symmetry	
6-fold rotational symmetry	

*More challenging problems.

Worksheet 2: Rotational Symmetry of Alphabets/Symbols

Count the number of fold of rotational symmetry for the symbols or alphabets provided and record in the following table:

*	÷	$	×	○	%	+	N
T	D	E	Z	S	A	B	H

Rotational Symmetry	Symbols/Alphabets
No rotational symmetry	
2-fold rotational symmetry	
4-fold rotational symmetry	
6-fold rotational symmetry	

Notes for Teachers:

1. All students in their primary schools have learnt line symmetry of 2-D figures. The teacher should explain the reason of using reflectional symmetry instead of line symmetry for both 2-D and 3-D figures at Key Stage 3 as line of symmetry in 3-D relates to the idea of rotational symmetry. To be consistent in the meaning of terms and the movement involved in understanding the symmetrical shapes, the term reflectional symmetry is used instead.

2. For line symmetry, it should be noted that some countries use the terms axial symmetry, reflectional symmetry, reflective symmetry or bilateral symmetry.

3. Although rotational symmetry is included as an enrichment topic in primary schools, the teacher should be aware that some students might have come across it. The teacher may then skip points 1 and 2 of the activity and go directly to the discussion of rotational symmetry of polygons. For other students, it is necessary to introduce the idea of rotational symmetry including the following terms: centre of rotation, 2-fold rotational symmetry, 3-fold rotational symmetry and so on.

4. They can be described as follows:
   A 2-dimensional figure has rotational symmetry if it can be rotated about a central point (called the **centre of rotation**) in such a way that the figure after rotation appears to be in exactly the same position as the original figure. If this happens **n times** when the figure is rotated one complete turn, then the figure has **n-fold** rotational symmetry (or rotational symmetry of **order n)** (Fig. 4). However, if n = 1, the figure has no rotational symmetry.

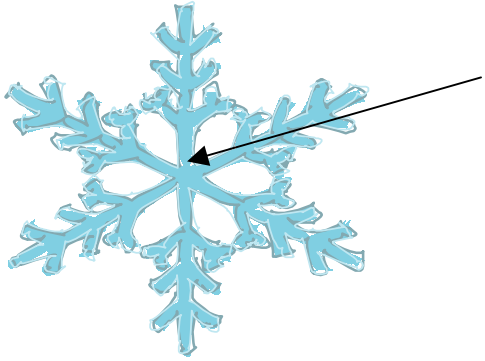

Fig. 4

5. The teacher can use Appendix B to prepare the polygons or use the *Sketchpad* files eqlater5.gsp, eqlater6.gsp, eqlater8.gsp to produce the corresponding

equilateral polygons or eqang5.gsp, eqang6.gsp and eqang8.gsp to produce the equiangular polygons.

6. Suggested answer to Worksheet 1:

Rotational Symmetry	Polygons
No rotational symmetry	Right-angled triangles, isosceles triangles, scalene triangles, isosceles trapeziums, kites, quadrilaterals in general, equilateral pentagons, equiangular pentagons, equilateral hexagons, equiangular hexagons, equilateral octagons, equiangular octagons
2-fold rotational symmetry	Parallelograms in general, rhombuses, rectangles
3-fold rotational symmetry	Equilateral triangles
4-fold rotational symmetry	Squares
5-fold rotational symmetry	Regular pentagons
6-fold rotational symmetry	Regular hexagons
7-fold rotational symmetry	Regular heptagons
8-fold rotational symmetry	Regular octagons

7. The teacher can get the template for the letters or symbols from the annex. Suggested answers to Worksheet 2:
   (a) No rotational symmetry: A, B, D, E, T
   (b) 2-fold rotational symmetry: S, H, %, Z, ÷
   (c) 4-fold rotational symmetry: +, ×
   (d) 6-fold rotational symmetry: *

8. The letter "O" has an infinite number of lines of symmetry. It has an "infinite fold rotational symmetry". A set of concentric circles also has such property.

9. The teacher should guide students to arrive at the conclusion that a n-sided regular polygon has n-fold rotational symmetry.

10. For the discussion of points 7 and 8, students should be able to recognize that "regularity" is crucial in determining rotational symmetry. Irregular polygons may have no rotational symmetry even they may be equilateral. Equilateral triangles and quadrilaterals will have rotational symmetry (3-fold and 2-fold respectively) while other equilateral polygons in general have no rotational symmetry (Fig. 5).

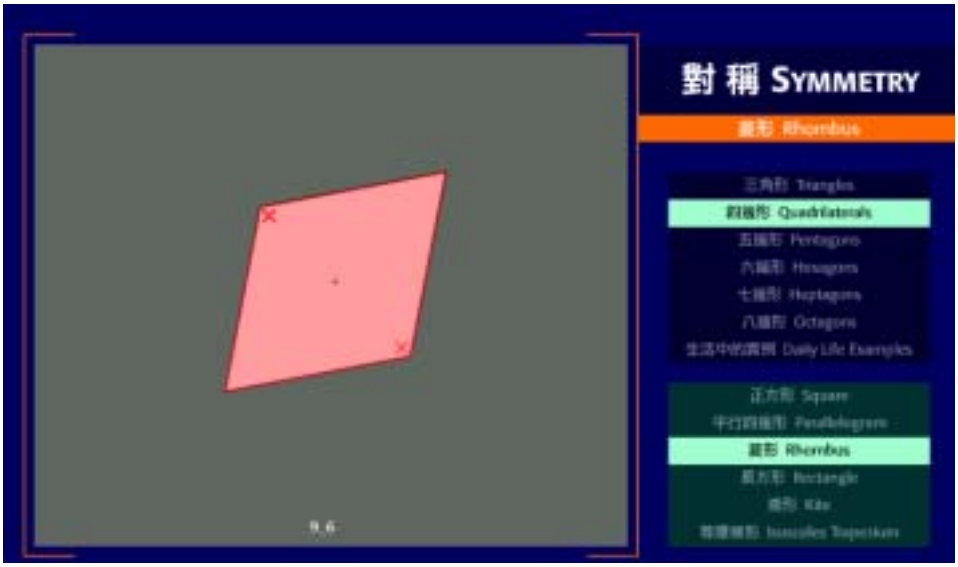

Fig. 5

11. For more able students, the teacher can ask them to explore further the number of folds for equiangular polygons.

Annex

Template of symbols or alphabets

Cut off the following symbols and find the number of folds for each of them by rotation:

*	÷	$	×
○	%	+	N
T	D	E	Z
S	A	B	H