2. Literature Review

Some 100 research papers and articles on the current state of mathematics education, both local and worldwide (see the bibliography for details), were reviewed and analyzed. The summary is presented in seven sections below.

2.1. Students’ Perception of Mathematics and Mathematics Learning

2.1.1. Affects and Mathematics Achievement

It is well reported in the literature that affective variables have strong influence on mathematics learning and achievement. Such variables include belief on self, belief on mathematics and belief on mathematics learning (McLeod, 1992). In Hong Kong, mathematics achievement was found to be closely related to affective variables, especially attitude towards mathematics and self concept, which in turn were related to self and parental expectation. Amount of homework time, schooling of the parents and residential size had nothing to do with attitude, learning habit and academic achievement (Cheng & Wong, 1991a; Wong, N.Y., 1992; Wong & Cheng, 1991).

2.1.2. Attitude towards Mathematics

In general, mathematics was highly regarded by students in Hong Kong. Most of them thought it interesting and took it as a favourite subject. They showed a general interest in mathematics lessons but seldom participated in mathematics extracurricular activities. When met with difficulty, most of them would discuss with their classmates, and only a minority would consult their teachers. They hoped that others would teach them how to do it rather than copying the solution from others. Most students thought that mathematics was a subject that requires thinking. Most of them found difficulty in understanding the mathematics lessons and in tackling mathematical problems. More found difficulty in word problems than in numerical problems (Cheng & Wong, 1991a; Wong & Cheng, 1991).

Students’ self-concept of learning mathematics dropped as they advanced through the grade levels. The mean score for “longing for a mathematics lesson” dropped from 2.68 in P.4 to 2.41 in P.6 (in a scale of 1 to 5). Similar results were found in items such as “I don’t want to learn mathematics” (Wong, H.W., 1996). Degree of fondness of mathematics also dropped as students moved up the grade levels, and many students found mathematics lessons boring. Though most students liked mathematics, regarded it as an important subject and were willing to pay effort to learn mathematics outside school, many of them did not have confidence in learning.
mathematics. As found in many other studies, hard-working was perceived as the most important factor contributing to success in mathematics, and the most important motive for doing well in mathematics was for getting into a favourite school or getting a desirable job (Wong & Leung, 1998).

Significant gender differences were found in attitudes towards mathematics. More boys than girls read mathematics “outside readers”, took part in mathematics extracurricular activities and consulted reference books when encountering difficulty in mathematics. More girls hoped that the teacher would tell them everything and there was no need to do too much thinking (Cheng & Wong, 1991b).

2.1.3 Belief in Mathematics Learning and Understanding

Most of the students showed a positive attitude towards mathematics learning. For instance, they agreed that one should read the textbook on top of listening to the teacher in class. They also thought that understanding teacher’s explanation and the mathematics concept behind were important. They believed that just coping with the examination was not enough and that mathematics is a subject that requires thinking. Many students opted for teachers assigning exercises outside the textbook and did not wish to have less homework. They believed that those good in mathematics took a greater initiative in learning and did not confine their practices to homework (Cheng & Wong, 1991b).

In another study by open-ended questions, it was found that “getting the correct answer”, “getting the answer quickly” and “understanding the method concerned” were the three major indicators that students put down for judgement of understanding. Yet in another study by the use of episode writing, “ability to solve mathematics problems”, “having the correct answers”, “accurate and fast solution”, “ability to apply to daily life situations”, “knowing the underlying principle”, “understanding the procedure and strategies”, “ability to clarify concepts”, “knowing the relationships among concepts” and “ability to explain to others” were some of the indicators of understanding as perceived by the students (Wong, 1993b, 1995b).

2.1.4 The Classroom Environment and Home Background

It is well known that the congruence of the perceived and preferred classroom environment is closely related to learning (Fraser, 1988). The same was found in a local project. It is especially true for those who are sensitive to their environments (“high self-monitors”). It was also found that the most crucial factor leading to a good mathematics classroom, as perceived by the students, is the teacher. A good mathematics teacher is one who explains clearly, shows concern towards the students, treats them as friends, ensures that they understand, teaches in a lively way, conscientious, well-prepared and answers students’ queries (in that order). S/he
should provide more exercises and should generate a lively atmosphere but keep good order and a good learning environment, one which is not boring, quiet, with classmates engaged in learning, where order is observed and discussion with classmates after lesson is possible (in that order). The students also indicated that they are more inclined to make an effort if they have had previous experience of success and perceive that they have the ability, and if they can understand the lesson in class. Have someone to ask after school is also helpful (Wong, N.Y., 1993a, 1995a, 1995b, 1996a; Wong & Watkins, 1998). It was also found that most of the secondary school students could not obtain guidance from others (Wong, N.Y., 1992).

In a qualitative study through interviews with 11 students, it was found that the students perceived the “lecture-exercise-lecture-exercise” stereotype as boring. A tight schedule gave them no time to think. Huge class size and boring classes hampered learning too. Being able to understand, previous experiences of success, a teacher who can make full use of activities and teaching aids to create a lively class atmosphere, who is patient, who knows the need of the students, makes abstract concepts tangible and makes clear explanations were factors conducive to learning (Lit, 1998).

It was found in yet another study that the teacher unintentionally funneled the students towards the solution the teachers had in mind and lacked the skill in handling students’ responses. Teachers were most concerned about having to help students get good marks, keeping in pace with the pre-set teaching schedule, and keeping good order in class. Qualitative differences among the students were attributed to their capacity in the eyes of the teachers. Students felt ashamed of asking questions and their perceived obligation was to imitate the teacher’s problem solving processes (Ho, M.F., 1996).

2.1.5 Conception of Mathematics

Students did not think mathematics is an art, and many (around half of the sample) thought that those good in mathematics were cleverer (Cheng & Wong, 1991b). In another study, it was found that students perceived mathematics as the solving of problems by simple methods, as a school subject having definite answers, as a subject of computations, as a set of rules, as a subject that requires thinking, and as a useful subject yet with some parts not easily applicable to daily life (Wong, N.Y., 1995b).

Yet in another study, students tended to identify mathematics by its terminology and subject content. Mathematics was perceived as a subject of “calculables”, that involves thinking and should be something useful in daily life. As a consequence, geometric concepts were underdeveloped (Wong, Lam, Wong, 1998).

In another study with 11 secondary school students, mathematics was perceived by the students as something that does not take a rigid form, challenging, hard, involves deep thinking and yet practical (Lit, 1998)
2.1.6 Learning Habits and Homework

A majority of students used hand-held calculators, possessed a habit of copying from the blackboard and hoped that the textbook could have more figures; and more girls, in contrast with boys, used hand-held calculators and hoped that there could be more figures in the textbook (Cheng & Wong, 1991b).

In a study in 1991 on secondary school students, it was found that on average, students used 38% of their homework time in doing mathematics homework, which was 13.79 hours per week. Girls used more time in mathematics homework than boys (Cheng & Wong, 1991b). In another study conducted in 1992, students used 33% of their homework time in their mathematics homework (Wong, N.Y., 1992).

2.2 Students’ Cognitive Style and Performance

2.2.1 Cognitive Style

It was found that “trying to understand”, “going for revision” and “asking someone for help” were the three major student approaches in tackling mathematics problems (Wong, N.Y., 1995b). Children’s proficiency in mathematics deteriorated as they advanced through the grade levels, and mathematics, just next to English, was the subject which children found most difficult. Students started to encounter greater difficulty in learning mathematics as early as P.3, and at that grade level, the individual differences began to widen (Wong, H.W., 1996).

The major weaknesses in algebra among school students were found to be misinterpretation of algebraic symbols and transition problems from arithmetic to algebra. Students found difficulty in setting up equations too. Common errors included those that arose from inability to handle letters, ignoring the hidden multiplication, inability to handle brackets properly, and the lack of adequate comprehensive abilities. These were attributed to rote-learning without understanding (Law, 1994; Mok, 1994, 1996; Wong, M.P.H., 1996). Transition from arithmetic to algebra, tackling problems with more than one variable, manipulation of symbols, comprehension of finite sequences, making shapes, spatial concepts and the use of graphs were identified as some of students’ major difficulties in learning mathematics (Wong, 1994).

Investigation on mathematics achievement was conducted for local primary school students. Expert problem solvers were contrasted with novice ones. In general, the former could manage complex and higher order rules better, had a better recall, possessed a good understanding of concepts, and were able to pick out commonalities and differences among similar formulas.
They had a better foresight of what they were heading towards before they started, and were able to pick up relevant information and make inferences. Virtually experts saw a problem in a rather different way than novices (Leung, H.P. 1994).

2.2.2 Performance in Public Examinations

In analyzing the examiner’s reports for CE Mathematics, 1993-1997 (see Appendix), it was found that the major weaknesses of the candidates lie in carelessness, mixing up information, making wrong assumptions, missing information given by the question, inability in understanding the question fully, lack of basic knowledge and skills (including those learned in junior forms), mixing up with units, poor presentation, lack of clearly-explained answers and inability to use mathematical arguments to support explanations. Statistics (especially cumulative frequency polygon), probability, A.P./G.P., ratio of similar figures, geometric proofs were some of the content areas that students were weak in. Some weaknesses appeared in the choice and use of mathematical techniques. In general, candidates were weak in 3-dimensional visualization, answering open-ended questions, giving explanations, and geometric proofs. Initial observations have it that most of the above concern attitudes and approaches to problems rather than knowledge and techniques. The major problem is not knowing what were given and what was asked, and then making tacit assumptions. Students are generally weak in tackling non-routine questions.

The examiner’s reports of CE Additional Mathematics, 1993-97 (Appendix) were also analyzed. Careless mistakes remain serious. Recalling formulas incorrectly, taking cubes and cube roots of numbers, missing out the testing of turning points and missing integration constant are some of the common careless mistakes. Though the failure to comprehend the questions and the inability to handle information remain as problems, inadequate understanding of knowledge is far more serious in CE Addition Mathematics than in CE Mathematics. It could be due to the demand on technicality of the subject. Particular weaknesses were found in vectors, trigonometry (like compound angle formula and general solution), complex numbers, turning points and inequalities involving absolute values.

As for AS Mathematics and Statistics (1994-97) (Appendix), many candidates were unable to make use of the information (“hints”) to solve problems. Calculus is a weak area for students. Some even cannot distinguish between definite and indefinite integrals. The concept of probability distributions, and the use and interpretation of graphs are problematic too. They cannot apply their knowledge to solve “non-typical” problems. Students’ weaknesses in AS Applied Mathematics fall mainly in the knowledge and techniques categories. Geometric and practical interpretations are especially weak areas for students. Overlooking the given conditions and/or instructions is common. For AL Pure Mathematics (1995-97), limits, complex numbers, matrices, polar coordinates, continuity, differentiability and 3-dimensional problems are weak areas for students. Many of them make use of formulas without checking
the conditions of the formulas or even if they are using the correct version of the formulas. Logical reasoning and handling problems involving a variety of topics are weak (see Appendix for details).

### 2.3 The Asian Learner

Students under the Confucian Heritage Culture (CHC) outperformed their Western counterparts in a number of international comparisons, especially in the area of mathematics. This was the case despite the fact that their class size was larger, their curriculum was examination driven, and there was a stress on memorization (Leung, F.K.S., 1995c; Wong, N.Y., 1998b). Their examinations addressed low level cognitive goals, were highly competitive, and exerted excessive pressure on teachers and students (Biggs, 1994; Wong, N.Y., 1998b). Disapproval was frequently used to control social behaviour in the classroom (Winter, 1990). Modern Chinese parents placed great emphasis on the academic achievements of their children (Ho, D.Y.F., 1986), while their children studied hard to meet the expectations of their parents and they often attributed their academic success and failure to the efforts they put into their work (Hau & Salili, 1991, 1996).

In the Third International Mathematics and Science Study (TIMSS), Hong Kong again outperformed the Western countries in mathematics. One possible trade-off of Hong Kong students’ superior achievement in mathematics was their relatively low confidence in mathematics. When compared to East Asian countries, Hong Kong pupil’s mathematics achievements were less impressive. Also, there were still a lot of simple and essential concepts and skills that many Hong Kong students failed to master (Leung & Wong, 1997a, 1997b).

In a comparison of the mathematics classes in Beijing, Hong Kong and London, it was found that Hong Kong and Chinese students were more disciplined. A typical Beijing and Hong Kong lesson would be more or less like “revision - introducing the topic - explanation - examples - classwork - student working on the board - summary - homework”, whereas a typical London lesson would be something like “dealing with classroom routines - student working on their own with teacher going around helping - recording of the grades of student performance”. Conformity in the format of the solution was emphasized by teachers in Beijing and Hong Kong. The use of rigorous mathematical language was stressed in Beijing mathematics classes. In Hong Kong, skills in solving mathematics problems were emphasized, whereas in London, care was given to individual difference (Leung, F.K.S., 1995c).

The “mentor/mentee relationship” was used to explain the “the teacher as the authority in the classroom” phenomenon, which was often regarded as a hampering effect, and a mixture of authoritarianism and student-centredness in the CHC classroom was identified (Biggs, 1994).
The TIMSS video study further revealed that more time was spent on applying mathematics concepts and in thinking in the Japanese classrooms when compared with the German and American ones. Concepts were developed rather than simply stated. It was also pointed out that the key point of the reform in Japan (which took place in classroom teaching rather than mere revision of curriculum documents) was teachers’ participation in “lesson study groups”. In Japan, it was not assumed that teaching would change when surrounding elements changed, but the assumption was rather to let teachers share the goal of student learning and let professional knowledge grow when the teachers were given opportunities to improve teaching (Stigler & Hiebert, 1997).

2.3.1 The Position of Practices and Examinations

It was repeatedly found that CHC students have strong preference of surface approaches to learning. It was pointed out that the excellent academic performance of CHC learners was due to the synthesis of memorizing and understanding (Watkins & Biggs, 1996), and that CHC learners possessed cultural potential for the deepening of understanding by means of repetitive learning (Marton, 1997), which is different from rote learning (Biggs, 1994). Fung (1996) pointed out that “…‘mathematics is learnt through intensive drilling and practice’ … While nobody will object to the necessity of practice (especially in the subject mathematics), it is all too often that overemphasis on practice has jeopardized the development of understanding because it is possible for pupils to reproduce certain procedures correctly without understanding what they actually are”. Though rote learning should be avoided, continuous practice with increasing variations could deepen understanding (Marton, 1997; Watkins & Biggs, 1996; see also Fung & Wong, 1997).

Confucian education is often hampered by examinations (Lee, Zhang, & Zheng, 1997; Leung, F.K.S., 1995a; Zhang, 1993; Zhang & Lee, 1991) but it was pointed out that Confucianism does not necessarily imply rote learning, nor can it be equated with an examination culture. There are long traditions in Confucianism and other Chinese cultures such as Buddhism on the orientation of understanding and enlightenment (Wong, N.Y., 1998b).

2.3.2. “Product” and “Process”

There has been dispute in the literature on the process-based curriculum (Howson & Wilson, 1986), and this can be seen as the major difference between the British and US mathematics curricula (Wong & Lam, 1997; Wong & Wong, 1997). In the “California Math War” (Jackson, 1997a, b), the relationship between “product” (content) and “process” was discussed. It was pointed out that conceptual understanding and problem solving ability on the one hand, and basic skills on the other, were not separated. Understanding is gained through techniques and
there is no need to segregate learning dimensions for process ability learning targets (Fung & Wong, 1997; Wong, 1995c; Wu, 1998). Fung (1996) also pointed out that “content and process are interactive elements of the curriculum. A learner’s process abilities grow with his or her extent of exposure to subject content” (see also Wong, K.M., 1994). Mere introduction of “pseudo-realistic tasks” may hamper mathematical understanding (Wong, K.M., 1997a, 1997b, 1998). Trimming down the curriculum is necessary in introducing flexibility and high order abilities (Wong, 1996d).

2.4 World Trend of the Mathematics Curriculum

On analyzing the mathematics curriculum standards of U.K., U.S.A, Taiwan, Mainland China, Hong Kong, Japan, Singapore, Australia, New Zealand and Germany (Baden-Wurttemberg), it was found that all of them are going for an expanding goal at the turn of the century. Attitude, confidence and appreciation are receiving more emphasis (Wong & Wong, 1997). Quality education as versus coping-with-exam-education is stressed. Its audience is the general public (universal) but at the same time individual differences should be addressed (Ma, 1996b, Ma & Lam, 1996). Upon analyzing the mathematics curriculum in Mainland China, it was found that mathematical awareness and mathematisation were stressed. Application of mathematics in realistic problems, the attitude of using mathematics in daily life problems, confidence in using mathematics, and communicating with mathematics were seen as a realization of this educational goal in the mathematics curricula in various countries. Sense making in the teaching process was further taken as an example of such a realization and this should be taken into consideration in the revision of the curriculum, curriculum materials (textbooks) and classroom teaching (Ma, 1996a; see also Ngan & Leung, 1996). However, Wong, K.M. (1997a, 1997b) raised the precaution of advocating simplistically the introduction of real-world situations into mathematical problems as a means to promote the learning of mathematics. Most “real-life” mathematics problems are just artificially created. Mathematics should be learned in its own right.

It was repeatedly pointed out that the heart of contemporary mathematics curriculum reform lies in the approach rather than content (Lim, 1995; Lin, 1998). In the Taiwan experience, an implementation plan was set up including experimental teaching, sharing sessions, and publication of parents’ guidebooks and teacher’s newsletters (Lin, 1998). A 10 year plan of implementation was also laid down in the Shanghai mathematics curriculum reform (Kong & Wong, 1998).

Continuation of the mathematics curriculum between primary and secondary levels was emphasized in both Malaysia and Shanghai (Kong, 1997; Lim, 1995). In the overall educational aim in the Shanghai mathematics curriculum, mathematical calibre was mentioned. This may be likened to mathematical literacy advocated in the West. Modes of thinking and high-order abilities, in particular problem solving abilities, were also promoted. Relationships with real life situation were also stressed (Kong, 1996, 1997).
Differentiated curriculum and electives were introduced in various Asian educational systems in their mathematics curriculum reform (Kong, 1997; Lin, 1998; Ma & Lam, 1996; Park, 1997; Su, 1996). However, it was pointed out that Western curricula should not be transplanted directly for local use since the latter has a long tradition of a rigid and canonical curriculum. We should allow more flexibility (Wong & Wong, 1997).

2.5 Goals of Mathematics Education

It was recommended by a task group of the CDC/EA Joint Working Party that “students be brought up in such a (mathematics) classroom culture and environment that they can: (a) acquire active and effective learning habits so that they are able to read and know how to access knowledge; able to write and to speak clearly in order to express their views and to communicate with others; willing to think, to query, to challenge and to probe; (b) have first-hand mathematical experience so that they realize the dual natures of mathematics as an exact science as well as an imaginative endeavour, as an abstract intellectual pursuit as well as a concrete subject with real-life applications; appreciate the beauty, the import, the power as well as the limitation of mathematics”. Furthermore, the task group suggested that “in the course of achieving these aims the subject content will necessarily come in so that students will learn basic mathematical concepts and skills, and to learn how to apply them to solve problems in everyday life or in their future pursuit, be it academic or vocational. Thus, mathematics should be treated not merely as a technical tool, which it certainly is, but more importantly as an intellectual endeavour and a mode of thinking. This will help students to form their own conception of the discipline, and convince them that mathematics is an intellectually rewarding discipline which plays a central role in human culture in a more general context”.

“Practical aims include (a) developing the ability to apply mathematics to daily life situations (mathematical literacy); (b) equipping students with the mathematics needed in a majority of professions; and (c) providing a mathematical foundation for further study in science and other related disciplines. Disciplinary aims include (a) enabling students to manipulate numbers, symbols and other mathematical objects; (b) developing number sense, symbol sense, spatial sense and a sense of measurement as well as the realization of structure and pattern; (c) facilitating reasoning, deduction and logical thinking; (d) facilitating the ability to utilize mathematics to conceptualize, formulate and solve problems; and (e) enabling students to express ideas using the language of mathematics. Cultural aims include (a) developing an appreciation of the aesthetic nature of mathematics; and (b) developing an awareness of the role of mathematics in various cultures from ancient to modern times, and its relation to other disciplines.”

The focus of the mathematics curriculum for the various levels should be as follows: primary: numbers, shapes and measurements (mostly inductive reasoning and heuristics); junior secondary: operations, patterns, functions and their graphs, algebraic concepts, geometric
concepts and statistical concepts (deductive reasoning); senior secondary: inverse operations, 3-dimensional spatial sense, probabilistic concepts (generalization and abstraction) (Wong, Wong, Lam, 1995).

2.6 Delineation of the Local Mathematics Curriculum and Teaching

The current mathematics curriculum is a product of the late 70s, and holistic reform of the curriculum was urged to meet the expanding goals of school mathematics under universal education (Leung, K.T., 1997; Wong, N.Y., 1995c, 1996d, 1997, 1998c). Situational analysis should be the first step in the reform (Leung, F.K.S., 1995b).

Leung, K.T. (1995) commented that the current curriculum document was too bulky and that it should not place too much bindings on teaching. Fung (1996) further commented that “(a) the development and planning of the mathematics curriculum lack a coherent direction, and liaison between primary and secondary practitioners is in sufficient; (b) the role of mathematics teaching in secondary education is identified with serving public examinations; and (c) curriculum planners at that time (the 80s) did not even bother to establish a holistic viewpoint in response to questions such as why and how mathematics has to be studied, why those materials were selected and how they are to be communicated to pupils.”

The impact of hi-tech, the enhancement of ability rather than skill, and individual differences were identified as the three major issues in universal mathematics education. It was doubtful whether streaming, with its labeling effect and the inability of converting from a lower to a higher track, and under the examination culture in Hong Kong, was able to tackle the problem of individual differences. Likewise, the implementation of standard-based curricula such as TOC was also queried. There is a likelihood of fragmenting the curriculum. The shift from standards (as feedback for further learning) to standardization (a common standard), the difference between quality of learning and checklist of learning outcome and curriculum control were discussed. In comparing the curriculum reforms in Shanghai and Hong Kong, it was concluded that, on top of the above, there is also the dilemma between the goals of schooling for education and for selection (Kong & Wong, 1998).

The authors concluded that (a) Asian countries should not simply import curricula from Western countries since they have different cultural backgrounds, (b) de-emphasis of grades and downplay of examination culture is essential, (c) we should make room for the professional development of teachers for better mathematics teaching; mere change in curriculum document is not enough, (d) we should not take for granted that “Confucian” education = rote learning = examination driven-ness (Kong & Wong, 1998).
2.6.1 Primary Level

A teacher survey in the late 70s on the primary mathematics syllabus revealed that the major difficulties in teaching primary mathematics were the varying standards (mixed-ability) of the students, the curriculum being too tight, and the lack of suitable teaching aids and high quality textbooks. Difficult topics included application of fractions, application of percentages, mixed mode calculations, word problems, conversion of units which are not denary, H.C.F. & L.C.M., ratio, capacity and volume (PTU, 1980).

In a focus group interview, the current state of primary mathematics was delineated. Articulation between the kindergarten, primary and secondary curricula, the reliance on textbooks, the inability to cope with individual differences, and the aptitude test driven curriculum were seen as the major problems in primary mathematics education (Wong, Lam & Wong, 1995). Another teacher survey was conducted in 1997 and it was found that primary school students’ attitudes towards mathematics, as perceived by the teachers, were generally positive, much better than the attitudes of those in secondary schools when comparing with a similar survey conducted in 1996. Most of the teachers took textbooks for reference in preparing their lessons, and they hesitated to perform curriculum tailoring. On average, P.4, P.5 and P.6 teachers used 0.81, 2.3 and 2.5 supplementary books on aptitude test exercises respectively. Over 40% reflected that the aptitude test brought about negative effect on teaching. The major concern in teaching was examination scores. Next was application in real life situations. Less than 20% of the teachers considered appreciation of mathematics in their teaching. Lecturing and exercises were most frequently used, a bit more than the case for secondary schools. Most of the teachers only used the short recess time to give guidance to those in need, as a means to cater for individual differences. In general, the teachers thought that the existing curriculum was too heavy, too difficult and not interesting. But as in the case for secondary schools, most of them only anticipated a minor change (Wong & Cheung, 1997).

In analyzing the Shape and Space dimension of two local primary textbooks, it was found that the development of spatial sense was dominated by stereotyped techniques, most of which were out of the reach of the students at that developmental stage. Diversity to cater for individual differences was lacking, and it was suggested that more effort to deal with students’ misconception should be given (Kan, Ma, So, & Wong, 1995, 1996).

The primary mathematics syllabuses of Hong Kong and Mainland China were compared. It was found that the scope of the latter was more focused and the former stressed the relationship with daily life application (Ma, 1995). Ngan (1997) presented the proposed primary mathematics syllabus for the year 2001 in which the development of abilities was stressed. A study group reviewed the syllabus and suggested that more examples and guidelines should be given along side with the curriculum. Contents should be treated more deeply, and the scope could be narrowed down. Individual differences should be addressed. Development of abstract concepts should not be shadowed by the urge of applying mathematics in real life situations (HKAME Study Group, 1997).
2.6.2 Secondary Level

In a focus group interview, the following problems of the secondary school mathematics curriculum were identified: (1) low motivation to learn to think mathematically, (2) doubts about the usefulness and relevance of learning mathematics, (3) teaching approaches stripped of mathematical thinking, (4) examination question types dictating classroom teaching, (5) unified textbook format reinforcing the status quo, and (6) poor linkage between curricula in different grades and topics (Wong, Wong & Lam, 1995).

In another teacher survey, it was found that the major student learning problems were the inability to memorize formulas, reluctance to think, and a weak foundation in mathematics. The major concern of teachers was that students could have good examination results. Emphasis was laid on lecturing, homework, classwork and discussions in class rather than group activities, games and projects. Most of the teachers reflected that they were not competent in leading games and activities. Other areas that teachers reflected incapability included cultivation of discussion and the enhancement of thinking skills. Most teachers would consider cutting away those topics that students could not comprehend or those that were not going to be examined. Most of the teachers thought that the existing curriculum was not interesting enough, and a majority of them welcomed a minor change instead of a drastic one. They would like to strengthen the application of mathematics in real life situations (Wong, P.H., 1996).

For the proposed 2001 Secondary School Mathematics Syllabus published recently, Prof. Zhang Dian Zhou of East China Normal University pointed out that a more far-reaching vision was expected. More should be said on what the current reform is heading for. The goals should be gradually projected into the targets and then finally realized in the subject matter (Zhang, 1998b).

2.6.3 Sixth Form

Leung, K.T. (1995) commented that those candidates who sat for the AL Pure Mathematics examination lacked holistic consideration, and this was attributed to the lack of training in plane geometry. The real number system should also be emphasized, and matrices and linear algebra should be deleted.

There have been comments on the AS Mathematics and Statistics syllabus that not enough guideline was given to teachers in the implementation, including suggested depth of treatment. The curriculum was examination oriented and too much emphasis was laid on the application (tool) rather than logical reasoning (way of thought) and theoretical considerations. There was inconsistency among topics, and the “target audience” was unclear. Those who took Additional Mathematics found it too easy and Arts students found it too difficult. This resulted in very
diversified suggestions on the addition and deletion of topics among teachers (Lai, 1994; Suen, 1996).

The course structure of “CE Math/CE Add Math, S6 Statistics/S6 Math/S6 Further Math” was proposed with Add Math not linking with (i.e. not a prerequisite) to any of the 6th form mathematics curriculum. S6 Math is a prerequisite to S6 Further Math. It was further proposed that S6 Math should comprise AS Mathematics, AL Mathematics & Statistics, and AL Pure Math modules; S6 Statistics should comprise AS Statistics and AL Math & Statistics; and Further Math should be equal to AL Pure Math (Figure 1). Subject contents of different modules were also given in details (Cheung, 1994).

![Curriculum Model of Sixth Form Mathematics Proposed by P.H. Cheung](image)

**Fig.1** Curriculum Model of Sixth Form Mathematics Proposed by P.H. Cheung

### 2.6.4 The Target Oriented Curriculum and Assessment

There has been a fear of fragmenting the curriculum with TOC. It was suggested that not making cells deliberately for process abilities was appropriate since process abilities could only be enhanced with the teaching of subject content. Precision in TOC assessment might hamper learning and reinforce comparison. There was a worry that the unclear relation between TOC assessment and public examinations might lead to turning TOC assessment into a selection process and making the curriculum even more examination driven (Wong & Lam, 1997; Wong & Tso, 1997). It was further suggested that the bands of performance should be simplified. TOC assessments should be downplayed to checking of prerequisite knowledge (Wong, N.Y., 1997c). Chow (1997) argued that the assessment of high order thinking skills was possible.
A close resemblance was found with the curriculum approaches between the new Shanghai curriculum and TOC (Kong & Wong, 1998), with cognitive, affective and psycho-motor objectives considered (Kong, 1997). There has been comments that such a standard-based curriculum was fragmented (Zhang, 1998a).

2.7 The Anticipated Change

The role of the curriculum document in the curriculum was critically reviewed (Wong, N.Y., 1996d). Leung, K.T. (1995) advocated a concise syllabus supplemented by teaching capsule developed by teachers which should include core, elective and self-study activities. He further elaborated that the future curriculum should consist of the following components: (1) a concise curriculum document showing a list of topics, (2) a commentary or chart which depicts the inter-relationship among topics, (3) examination syllabus, (4) sample paper. Ways of teaching should not be prescribed. The curriculum should not be taken as a binding document (Leung, K.T., 1997). It was also mentioned that, in the California Math War, the intention of the curriculum standards eschewing prescriptions on how to teach was admirable (Wu, 1998). A high quality teacher is the key person to get curriculum standards implemented in the classroom (Wong, 1996b; Wu, 1998). Teacher professionalism should go hand in hand with curriculum reform (Chow, 1996).

Suen (1998) suggested that the curriculum should be developed by full-time experts to secure a high quality of the curriculum document. Supporting materials and teacher training courses were essential in the implementation, and alternative assessment was suggested to counteract the examination orientation.

A clear goal, catering for individual differences, continuation and a detailed implementation plan were essential (Lam, 1995). A curriculum structure for school mathematics, from primary to senior secondary, was proposed from an epistemological point of view. The entire curriculum rested in a framework of the four stems of number and symbol manipulation, shape and space, measurement, and handling information (Fung & Wong, 1997).

2.7.1 The Position of Hi-Tech

Wong, N.Y. (1991a, 1991b, 1996c, 1998d) argued that the major impact of hi-tech lied in the de-emphasis of skill, leaving more room for development of concepts, rather than the use of the technology concerned. Hi-tech also changed the mode of getting information and learning. More details were found in Fung & Wong (1997).
2.7.2 Individual Differences

In a focus group interview, it was found that the tailored syllabus was well received by “Band 5 schools”, though they expected more elaboration from the Education Department. The message of curriculum tailoring and the spirit of the tailored syllabus were not promoted well enough. However, teachers suggested that the tailored syllabus should only be taken as a guideline for setters of public examinations and should not be regarded as a teaching guideline. Teachers indicated that how the syllabus was implemented, which required the professional judgement of the teacher, was far more important than how the curriculum contents were tailored. They also recommended that the spirit (approach) of the curriculum should be sufficiently promoted in future curriculum initiatives and measures should be taken to assure that the textbooks carried the spirit of the curriculum so that the message of the initiative could be conveyed to classroom teaching (Wong & Suen, 1998).

As for catering of individual differences, Leung, F.K.S. (1994) proposed a frustum model in which students with learning difficulties narrowed down their scope of learning gradually and finally confining to a “core curriculum” (Figure 3). It was also pointed out that the “core” curriculum should not be constructed simply by deleting those topics which students found difficult (Siu, 1995; Wong, 1995d). However, Wong, N.Y. (1997b, 1998b) did not support streaming or tracking. The society’s wish to maintain a unified public examination should be dealt with with good care (Wong, 1998a). Flexibility (e.g. moving up a track) and downplaying the labeling effect were the major concerns. It was suggested that options of enrichment/remedial teaching should be introduced to cater for individual differences. An individualized learning model with such a flavour was proposed by Prof. Leou Shian of Taiwan (Leou, 1998) (Figure 2). The proposal was to work with depth of treatment rather than the number of topics. Trimming down of the existing syllabus was a prerequisite.

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**Fig. 2** Individualized Learning Model of Leou Shian (simplified)
2.8 Summary

Students in Hong Kong have a high regard for mathematics as it was often perceived as a major subject. Success in mathematics was seen as a ticket for the future. Influenced by the achievement orientation of the Chinese culture, students made effort in their studies and believed that diligence could help them learn mathematics. Intrinsic motivations were dominated by extrinsic ones. Rigid and conformed modes of learning added extra hardship, and individual differences in student inclinations were not catered for. The examination orientation reinforced the status quo. A feeling of alienation led to taking the mathematics class as boring, and paying great effort without arriving at desired outcomes generated frustrations. Attitude towards mathematics began to deteriorate and a lack of confidence finally led to giving up the subject. What was left when the students left school was a nightmare called mathematics. In fact, most of the problems revealed in the examiner’s report on the Certificate Examination concerned the way candidates approached the problems and the way of thought rather than knowledge and technique.
Basic skills and computations were some of the strong points of mathematics education in the Chinese society. Practice was valued. This could serve as a firm foundation for the enhancement of high level abilities as long as practice and memorization go hand in hand with understanding, which could be deepened with practices with variations. Conceptual understanding and fluency in skills were not separated; only that one should safeguard from rote learning and meaningless over-drilling. Students in Hong Kong preferred exposition rather than more “lively” ways of learning such as the use of games or group discussions, but this does not mean that they were conservative, passive or did not opt for deep learning. They wished that the teachers could explain the way of thought right away.

Much has been said on teaching and learning rather than the curriculum. In fact one should not expect that a curriculum document could solve all the learning problems. The teacher is still the key person in classroom teaching. Having said thus, a favourable curriculum would offer a far-reaching vision and also room for teachers to exercise their teaching at their best. The essence of a curriculum lied in the approach rather than the arrangement of contents. Students constructing their mathematical knowledge and possessing an expanding goal was the world trend in mathematics curriculum reform. Affective domains such as attitude and confidence, high order thinking abilities were taken care of. Standardization was popular but its feasibility in the local situation was doubted, and flexibility to cater for individual difference was emphasized. Trimming down of the bulky curriculum was badly needed to leave room for deep understanding and enhancement of higher order thinking abilities but we should safeguard going for a watered-down curriculum. We should go for depth of treatment, with fewer contents, and not complexity. In fact, skill needed to be de-emphasized with the impact of high-technology. In the local situation where comparison by public examinations was so much stressed, differentiation could be subtly done by the choice of teaching/learning activities and depth of treatment in individual topics rather than the addition and deletion of topics. Labeling effect would thus be downplayed.

Progression through the grade levels was another issue of major concern. From an epistemological perspective, a curriculum framework that is conducive to learning could be developed. Nevertheless, teachers’ ownership of the curriculum, sharing of educational goals and professionalism should be the prerequisites for curriculum change, and the Japanese experience of “lesson study groups” could be useful.
2.9 Bibliography


