



Exemplar 7 :

Constructing Data Sets from Given Mean, Median and Mode

Objectives : To construct data sets from given mean, median and mode

Learning Unit : Measures of Central Tendency

Key Stage : 3

Materials Required : Nil

Prerequisite Knowledge : Meaning of mean, median and mode

Description of the Activity :

1. The teacher revises with students the meanings of mean, median and mode before the activity if necessary.
2. Students are divided into pairs for discussion and working on the worksheet.
3. The worksheet is divided into 3 parts.
Part I: Constructing Data Set from a Given Mean
Part II: Constructing Data Set from Two Given Averages
Part III: Constructing Data Set from Three Given Averages
4. In each part, students are asked to construct data sets from fewer conditions to more conditions. The teacher should give hints or guidance to students if they have difficulties. The teacher should avoid giving explicit directions. Students are encouraged to find the answers by discovering appropriate strategies.
5. At the end of each part, the teacher invites some students to present their findings and strategies to the whole class for comments.
6. As a consolidation activity, the teacher gives other values of mean, median and mode for students to construct more data sets.

Worksheet: Constructing Data Sets from Given Mean, Median and Mode**Part I: Constructing Data Sets from a Given Mean**

In this part, x , y and z are positive integers.

1. If the mean of **3**, **5** and x is equal to **6**, what is the value of x ?

2. If the mean of **3**, x and y is equal to **6**, give one set of possible values for x and y ?

3. Comparing your answer in Question 2 with your classmates' answers, what do you observe? How many pairs of possible solutions are there for x and y ?

4. Construct a set of x , y and z such that their mean is **6**.

5. List five other sets of x , y and z with the condition that the mean of each data set is equal to **6**. Briefly describe your strategy in finding these values.

Part II: Constructing Data Sets from Two Given Averages

In this part, p , q , r and s are positive integers.

1. Construct a set of p , q and r such that their median is **4**.

2. Modify your data set in Question 1 so that both the median and the mode are **4**. Compare your answer with your classmates'.

3. Can you modify your data set in Question 1 so that its median is **4** and its mode is **5**? Briefly explain your conclusion.

4. By including one more datum s into the data set, i.e. the data set now consists of p , q , r and s , can you construct a data set with median **4** and mode **5**? Briefly explain your answer.

5. Compare the results in Questions 2 and 3. Explain why not all the data sets can be constructed from given medians and given modes.

Part III: Constructing Data Sets from Three Given Averages

In this part, the data constructed should be non-negative integers.

From Question 1 to Question 4, after constructing the data sets, compare your answers with the answers of other groups to see if the answers are unique.

1. Construct a set of three data with mean 3, median 4 and mode 4.

2. Construct a set of four data with mean 3, median 4 and mode 4.

3. Construct a set of five data with mean 3, median 4 and mode 4.

4. Construct a set of six data with mean 3, median 4 and mode 4.

5. Briefly explain your strategy in finding the data set.

6. Which of the answers from Question 1 to Question 4 are not unique? Briefly explain why they are not.

7. Is it always possible to construct a data set from any three given averages? Give an example to support your view.

Notes for Teachers:

1. It is most ideal for students to do this exemplar after they are taught the meanings of mean, median and mode. Otherwise, a revision on the meanings of these terms should be given to the students if necessary.
2. In **Part I** of the worksheet, students are asked to construct data sets with a given mean. For less able students, the teacher can start with a few questions of finding the mean, e.g. (i) find the mean of 5, 6 and 7, (ii) find the mean of 2, 4 and 12, etc. The means of these two data sets are 6, the same mean as in Question 1. Questions of similar types could be set for **Part II** and **Part III**, if necessary, as warm-up exercises. As an alternative, a sample data set could be given as a hint for students to start with.
3. Suggested answers to **Part I** of the worksheet:
(x , y and z are positive integers)
 - Question 1 $x = 10$
 - Question 2 Accept any answer if $x + y = 15$
 - Question 3 There are many different pairs of solutions for x and y .
 - Question 4 Accept any answer if $x + y + z = 18$
 - Question 5 Accept any reasonable explanation
4. Suggested answers to **Part II** of the worksheet:
(p , q and r are positive integers)
 - Question 1 Accept any answer when the median is 4, e.g. 1, 4, 5; 2, 4, 7; etc.
 - Question 2 After modifying the answer in Question 1, the data set should either be in the form $p, 4, 4$ or $4, 4, r$. In either case, the median and the mode are both 4.
 - Question 3 It is not possible to have a set of three data with median 4 and mode 5. (In order to have a mode of 5, two or three of the data must be 5, i.e. $p, 5, 5$ or $5, 5, 5$ or $5, 5, r$. The condition of mode 5 makes it impossible to have a median 4 in the data set.)
 - Question 4 Accept any answer in the form $p, 3, 5, 5$.
In order to satisfy the condition of mode 5, two or more of the data must be 5. With the additional condition that the median is 4, only a maximum of two data can be 5. Considering these two conditions, two of the data must be 5. The data set is in the form $p, q, 5, 5$. Finally, for the median to be 4, the value of q must be 3. Therefore, the data set is in the form $p, 3, 5, 5$.

Question 5 In constructing the data set, if it is not possible to fulfill both the median and the mode, the data set does not exist. Question 3 is an example of the non-existence of a data set for a given median and mode.

5. Suggested answers to **Part III** of the worksheet.

The data should be non-negative integers.

Question 1 1, 4, 4

(see Point 6 below for a suggested strategy in finding the answer)

Question 2 0, 4, 4, 4

Question 3 0, 3, 4, 4, 4 or 1, 2, 4, 4, 4

Question 4 0, 0, 4, 4, 4, 6 or 0, 1, 4, 4, 4, 5 or
0, 2, 4, 4, 4, 4 or 1, 1, 4, 4, 4, 4

Question 5 Accept any reasonable explanation.

A suggested explanation:

The median is considered first as there is a restriction on its position in the data set. The mode is considered next as there is a requirement for it to appear most frequently. Finally, the mean is looked at because it provides more flexibilities than the median and the mode on constructing a data set.

Question 6 Answers to Questions 3 & 4 are not unique.

Accept any reasonable explanation.

Question 7 It is not always possible to construct a data set for any three given averages.

Accept any correct example.

Here are some examples:

- A set of three data with mean 9, median 4 and mode 4
- A set of four data with mean 3, median 4 and mode 1
- A set of five data with mean 3, median 2 and mode 4
- A set of six data with mean 1, median 4 and mode 4

6. A suggested strategy for answering Question 1 of **Part III**:

To construct a set of three data with mean 3, median 4 and mode 4

- Start with the data set 3, 3, 3 for satisfying the condition of mean '3' first.
- Change some of the data but keep '3' as the mean and bring in '4' as the median. For example, change the data set 3, 3, 3 to 2, 4, 3.
- Adjust the values of the data so that their mean remains as '3', median as '4' and bring in '4' as the mode. In order to keep '3' as the mean, the sum of the

three data must be 9. So, in the process of adjusting the values of the data in the data set, students should bear in mind that the sum of the three data is 9. For the sum to be 9 and mode to be 4, it should not be too difficult to see that two out of the three data must be 4. Therefore, the remaining datum is $9 - 4 - 4 = 1$. The data set is 1, 4, 4.

7. If students have difficulties in giving explanations, the teacher could provide them with guiding questions, e.g. Which datum do you fix first in the data set? How is this datum related to one of averages? Which of the averages should be considered first in deciding the data set?
8. Instead of listing the answers in the space provided, the teacher can design tables for students to fill in. Some of the data could also be given in the table for the students to use as a starting point.
9. In the case more than one set of data could be constructed, all the data sets are accepted as answers because there is no 'best' answer.