Atomic Model
and
Wave-Particle Duality

NSS Enriching Knowledge Series of the Physics Curriculum
(2008)

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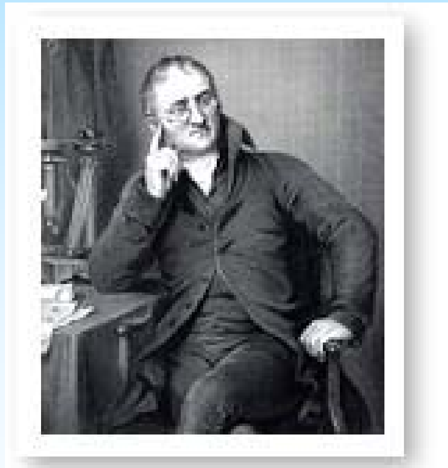
Atomic Theories (400 BC – 1900 AD)

Democritus of Abdera

* Democritus of Abdera (460-370 B.C.)
  - Basic substances: Elements
  - His universe consisted of empty space and an infinite number of atoms (*a-tomos*, the "uncuttable").
  - These atoms were eternal and indivisible, and moved in the void of space.
  - There were no experimental data.
  - ~ 90 elements were discovered by 1900.



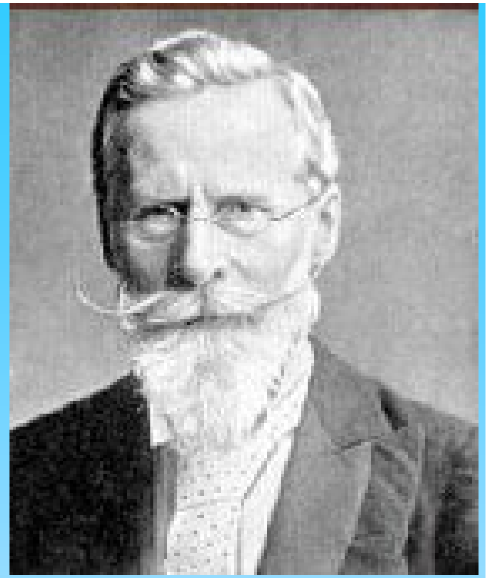
John Dalton

* John Dalton (1808)
  - Law of Definite Proportions:

    (3g of X) + (4g of Y) produces 7g of XY

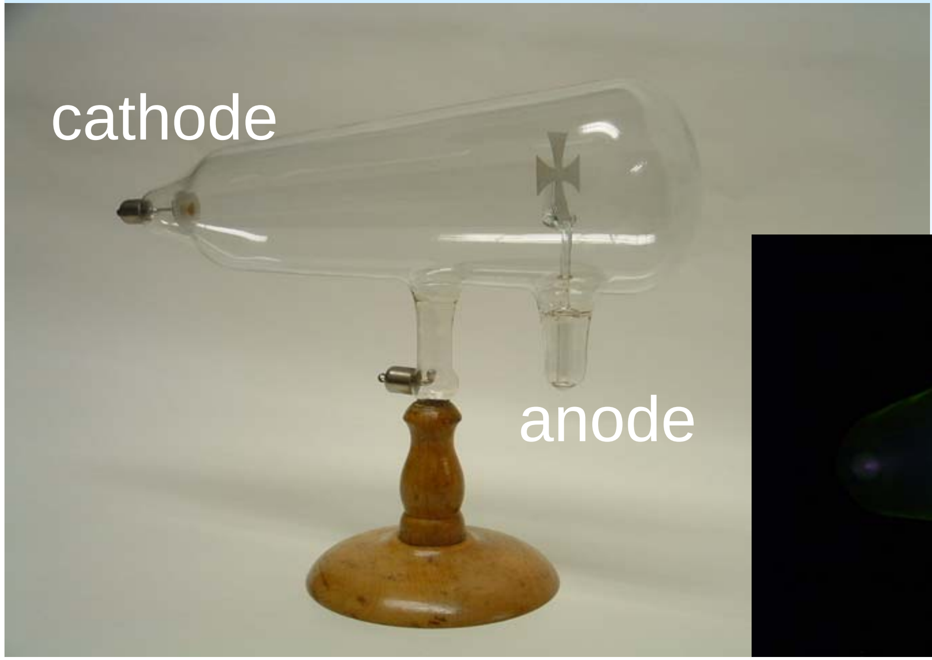
    Dalton suggested – but did not prove - the 'atomic' nature of matter.

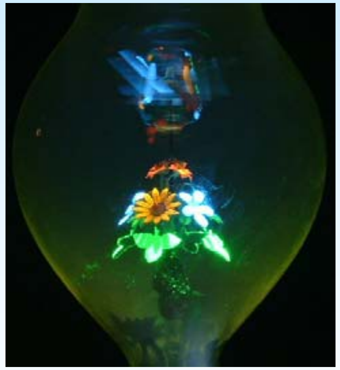
Mysterious Rays using Crookes Tubes



(1832-1919)

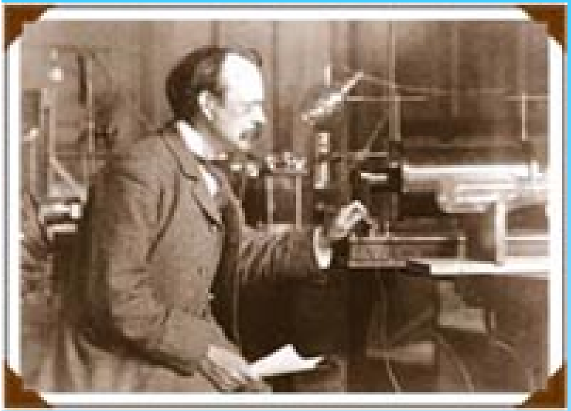
- Crookes tubes are simply evacuated glass tubes with electrodes to which a voltage can be applied.
- Sir William Crookes saw (in 1879) emission from the cathode of such a tube and showed that this emission could be blocked by an object.
- He named this ‘cathode ray’ and believed that these were a stream of particles of some sort.



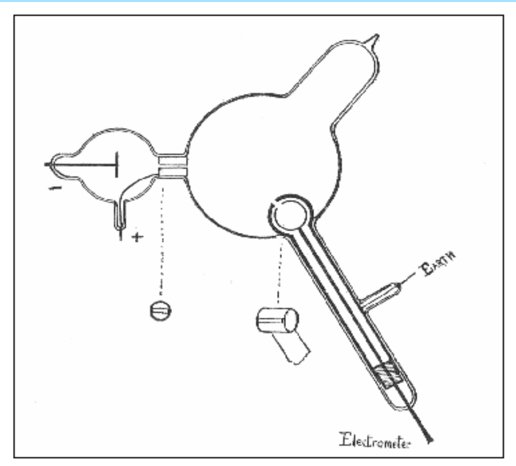


Crookes Flower Tube

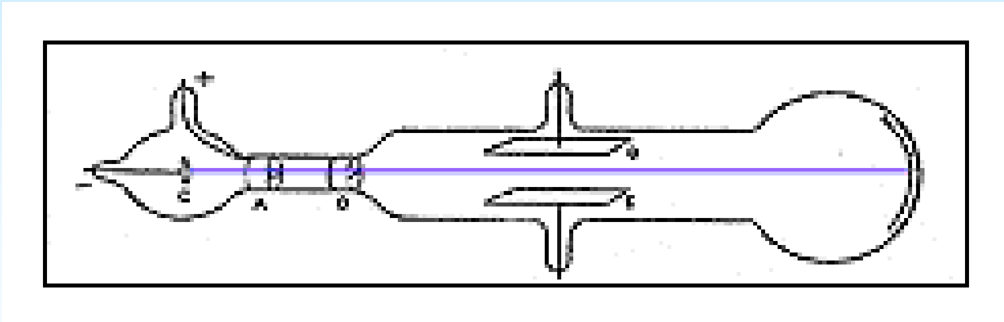
J.J. Thomson and the electron (1897)



- First: Magnetic field was used to bend the 'cathode ray' into the electrometer (which detected the charge.) This showed that the charge could not be separated from the 'ray'.



- Second: He showed that the ray could be deflected by electric field – but only after the tube had been very well evacuated by pumps.



He finally combined the E and B fields from which the charge to mass ratio can be deduced.

Thomson's Conclusions

1. Cathode rays are charged particles (which he called "corpuscles", and we now call electrons).
2. These electrons are constituents of the atom. He tried different gases in the tube and different cathode materials, but obtained the same e/m ratio: there is only one kind of electron in all atoms
3. The e/m ratio for ions have been known (from electrolysis) and this electron e/m is 2000 times more than that for hydrogen ions, and Thomson reasoned (correctly) that this must mean that the electron has small mass.

- He got the Nobel Prize in Physics 1906.
- He also proposed the 'Plum Pudding' or 'raisin cake' atomic model. (Electrons are the raisin in the positively charge cake). More on that later.

Thomson 1934

http://www.aip.org/history/electron/jjsound.mp3

Millikan Oil Drop Experiment (1909)

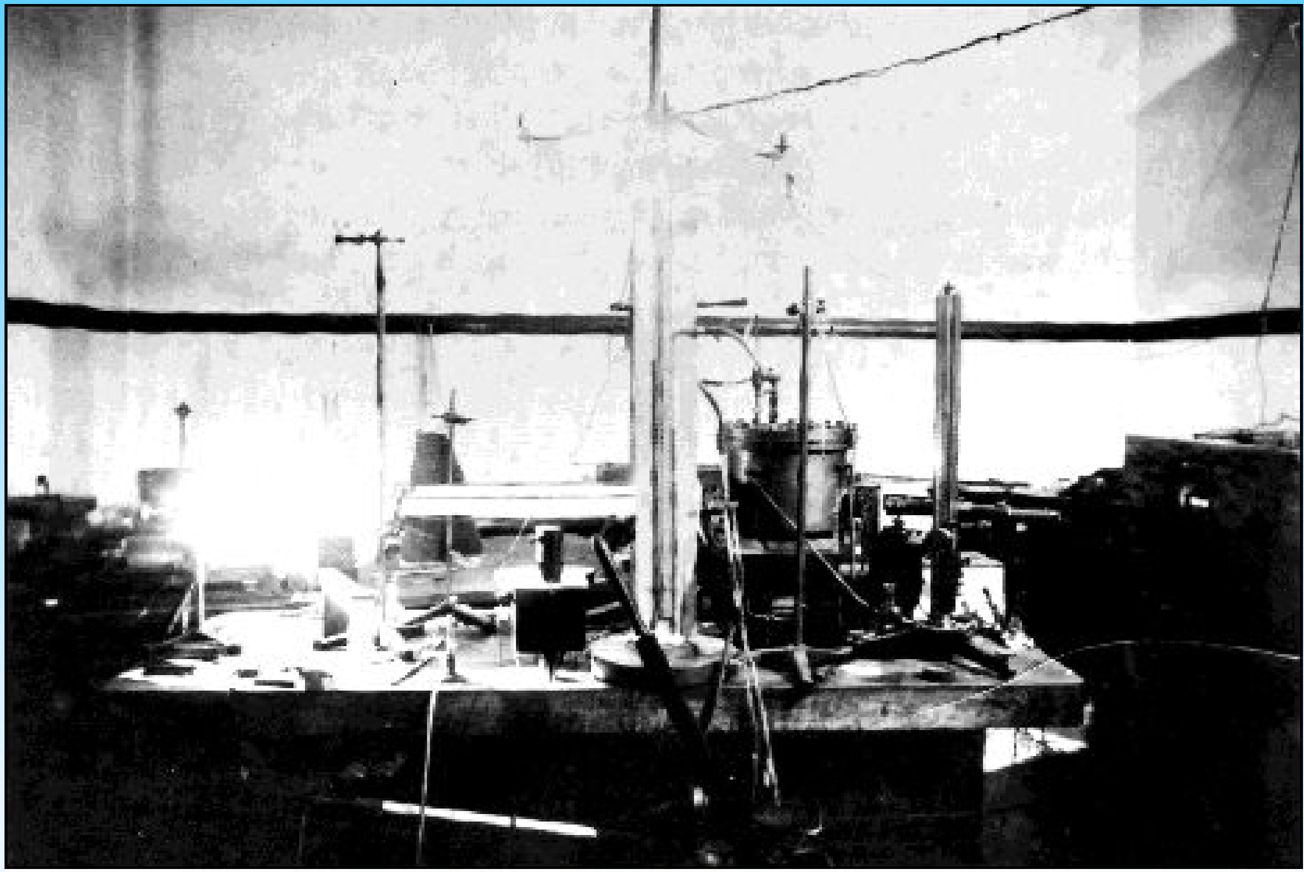
- J.J. Thomson had determined the e/m ratio – which does not depend on materials used. But this did not prove the existence of the electron: There could be a range of different sizes of electrons and still have the same e/m ratio.

- To determine the charge, they experimented with measuring the motion of water droplets 'charged' or ionized by X-rays in an electric field – but unable to get good results due to difficulties such as evaporation of the droplets.

- Robert Millikan's experiment overcame many of those difficulties. The key advance was the use of oil instead of water - the idea occurred to him on a train trip realizing that lubrication oil does not evaporate very fast.

- Millikan was then able to watch the motion of single oil droplets for hours, putting on and taking away charges by X-rays, and measured the change in the velocity.

Robert Millikan



Millikan's oil drop experiment

Millikan's 1911 paper

THE

PHYSICAL REVIEW

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
November 28, 1910.

THE ISOLATION OF AN ION, A PRECISION MEASUREMENT OF ITS CHARGE, AND THE CORRECTION OF STOKES'S LAW.[1]

BY R. A. MILLIKAN.

§ 1. INTRODUCTION.

IN a preceding paper[2] a method of measuring the elementary electrical charge was presented which differed essentially from methods which had been used by earlier observers only in that all of the measurements from which the charge was deduced were made upon one individual charged carrier. This modification eliminated the chief sources of uncertainty which inhered in preceding determinations by similar methods such as those made by Sir Joseph Thomson,[3] H. A. Wilson,[4] Ehrenhaft[5] and Broglie,[6] all of whom had deduced the elementary charge from the average behavior

TABLE I.

Negative Drop.

Distance between cross-hairs = 1.010 *cm.*
Distance between plates = 1.600 *cm.*
Temperature = 24.6° *C.*
Density of oil at 25° C. = .8960.
Viscosity of air at 25.2° C. = .0001836.

	G sec.	*F* sec.	*n*	$e_n \times 10^{10}$	$e_1 \times 10^{10}$
	22.8	29.0	7	34.47	4.923
	22.0	21.8	8	39.45	4.931
	22.3	17.2			
$G=$ 22.28, $V=7950$	22.4	—	9	44.42	4.936
	22.0	17.3			
	22.0	17.3			
	22.0	14.2	10	49.41	4.941
$F=$ 8.65	22.8	8.6	14	68.65	4.904
	23.1	8.7			
	23.2	9.8	13	63.68	4.900
	—	9.8			
$F=$ 10.63	23.5	10.7	12	59.12	4.927
	23.4	10.6			

Electric charge is quantized!

$e = 4.891 \times 10^{-10}$ E.S.U. the smallest quantity of electricity capable of separate existence.

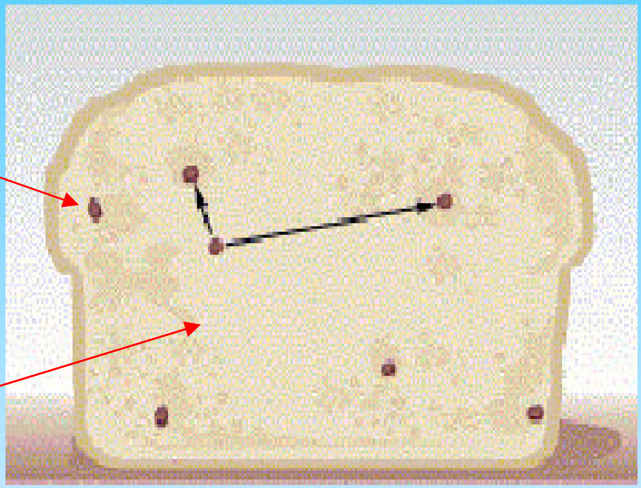
Thomson and Millikan: electrons have definite charge and mass

- Atoms contain negatively charged electrons.
- Electrons have mass about 2000 times less than hydrogen atom, but the (negative) electron charge is equal to the (positive) charge of the ion in magnitude.
- Atoms are neutral: there must be positive charge in them. How is the positive charge distributed within the atom? And what about the distribution of the mass?

Thomson's 'Plum Pudding' or 'Raisin Cake' Model of the atom

* Raisins are the electrons.

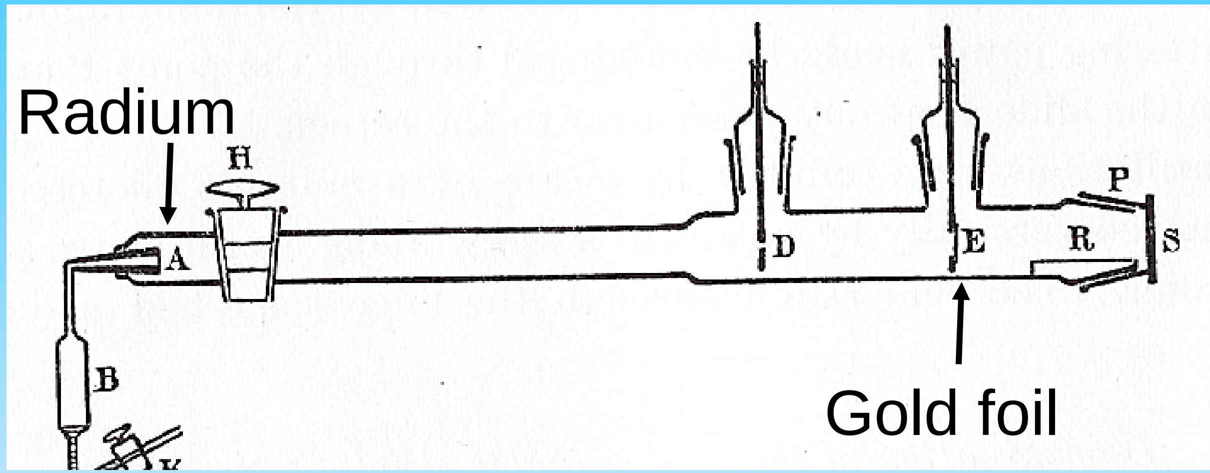
  Positive charge and mass distributed uniformly about the atom ('the bread') and the size of the 'bread' is about 10^{-10}m (atomic size).



Rutherford asked his student to use particles from radium as projectiles to probe this 'raisin cake'.

The sample used was a gold foil (which can be very thin).

Rutherford's expectation and the surprise!



* The positively charged α particles have 7.7 MeV of energy, and 8000 times more massive than the electrons ('the raisins') – they will not be deflected by the electrons or the uniform positive background.
* Surprise! Surprise! They found a small number of those 7.7 MeV a particles deflected by very large angles – even 180 degrees!

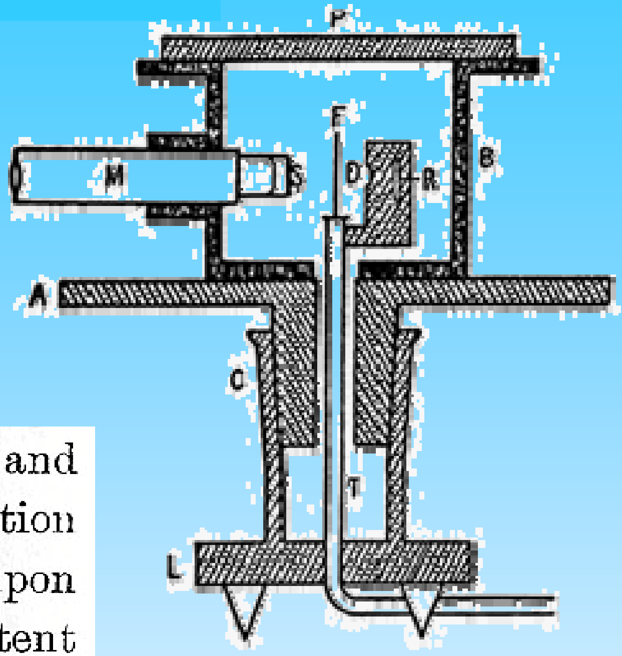
Scattering of α Particles by Matter

The Scattering of the α-Particles by Matter.

By H. Geiger, Ph.D.

(Communicated by Prof. E. Rutherford, F.R.S. Received February 1,—Read February 17, 1910.)

It is also of interest to refer here to experiments made by E. Marsden and myself (see 'Roy. Soc. Proc.,' A, vol. 82, p. 495, 1909) on the diffuse reflection of the α-particles. It was found that some of the α-particles falling upon a metal plate appear to be reflected, *i.e.* they are scattered to such an extent that they emerge again on the side of incidence. It was shown that from gold 1 in about 8000 of the incident α-particles suffers reflection, and that this



Geiger and Marsden's apparatus

"It was quite the most incredible event that ever happened to me in my life. It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you." Rutherford

The Scattering of α and β Particles by Matter and the Structure of the Atom

E. Rutherford, F.R.S.*

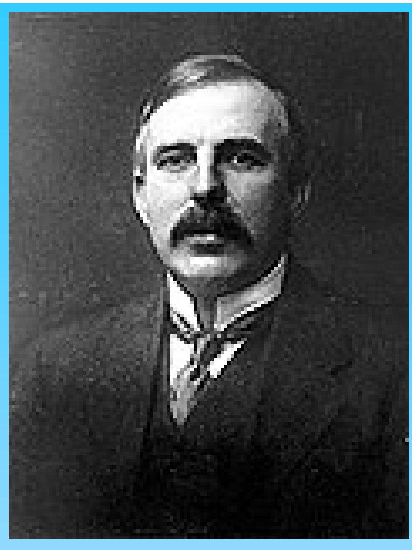
Philosophical Magazine (1911)

* Rutherford realized that such large deflection could not possibly be resulted from a single scattering in the Thomson's model.
* However, he was able to show that the probability of multiple scatterings was far too small to explain the observations.
* Since electric field is proportional to $1/r^2$, such high field requires concentration of charge to a very small r.
* The positive charge CANNOT be the 'bread' surrounding the electrons – all the charge must be concentrated to a very small NUCLEUS. How small?

Cambridge Physics - Discovery of the Nucleus

A Rutherford scattering applet

Structure of Nucleon



Rutherford

- In Rutherford's scattering experiment, he kept seeing that the atomic number Z (number of protons in the nucleus, equivalent to the positive charge of the atom) was less than the atomic mass A (average mass of the atom) implying something besides the protons in the nucleus were adding to the mass.
- He put out the idea that there could be a particle with mass but no charge. He called it a neutron.

Chadwick

- Rutherford's former student James Chadwick, using a new refined particle detection, was able to determine that the neutron did exist and that its mass was about 0.1 percent more than the proton's. In 1935 he received the Nobel Prize for his discovery.

Summary

Atoms: Electrons + Nucleus (Protons + Neutrons)

Atomic Radius ~ 10^{-10} m

Nucleus ~ < 30 fm

Hydrogen Atom 1.6 x 10^{-27} kg or 940 MeV/c^2

Electrons:

Charge -1.6 x 10^{-19} C

Mass 9.1 x10^{-31} kg or 0.5 MeV/c^2

		#e	#p	#n	Z	A
^{1}H	Hydrogen	1	1	0	1	1
^{238}U	Uranium	92	92	146	92	238

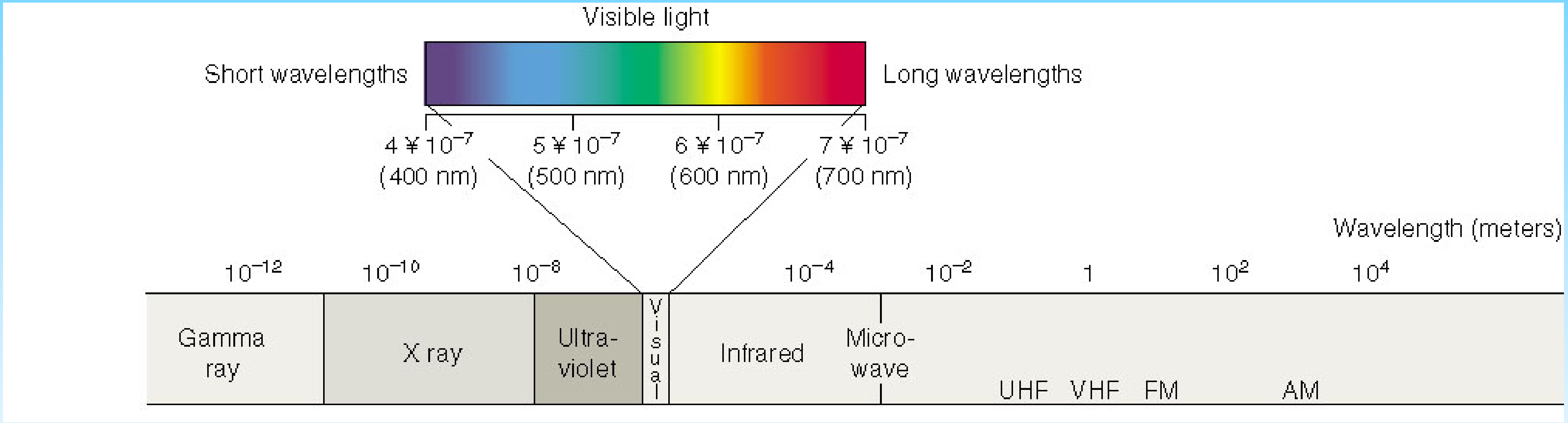
Matter, Energy, Heat and Light

- We long know matter, energy, heat and light are closely related.
- Need fuel (wood or coal) to keep a fire burning, giving out heat, and keep a train running.
- When there is heat, there is light (do not think of light as the visible part only!)
  – and you can tell the temperature by the colour of the light.

Everyone can see the connection between heat and light!

Electromagnetic Radiation

- Light is an electromagnetic radiation, which includes also gamma rays, X-rays down to microwaves and radio waves.



- Each kind of wave has its own frequency and wavelength.
- Visible light has wavelengths between 400 nm and 700 nm, where nm (nanometer) is 1 nm $=10^{-9}$ m.

Maxwell Equations for Electromagnetic Radiation

$$\nabla \cdot D = 4\pi\rho \qquad \nabla \times H = \frac{4\pi}{c} j + \frac{1}{c}\frac{\partial D}{\partial t}$$

$$\nabla \cdot B = 0 \qquad \nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0$$

The Maxwell equations require additional **equation of continuity** for charge density and current density.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$$

In empty space, Maxwell equations have a wave solution of

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

E ~ sine or cosine functions

EM Waves: Power

Average over one period, we have

$$< P >= \frac{1}{T}\int_0^T P(t)dt = \frac{1}{2\mu_0 c} E_0^2 A \qquad (<\sin^2\phi> = 1/2)$$

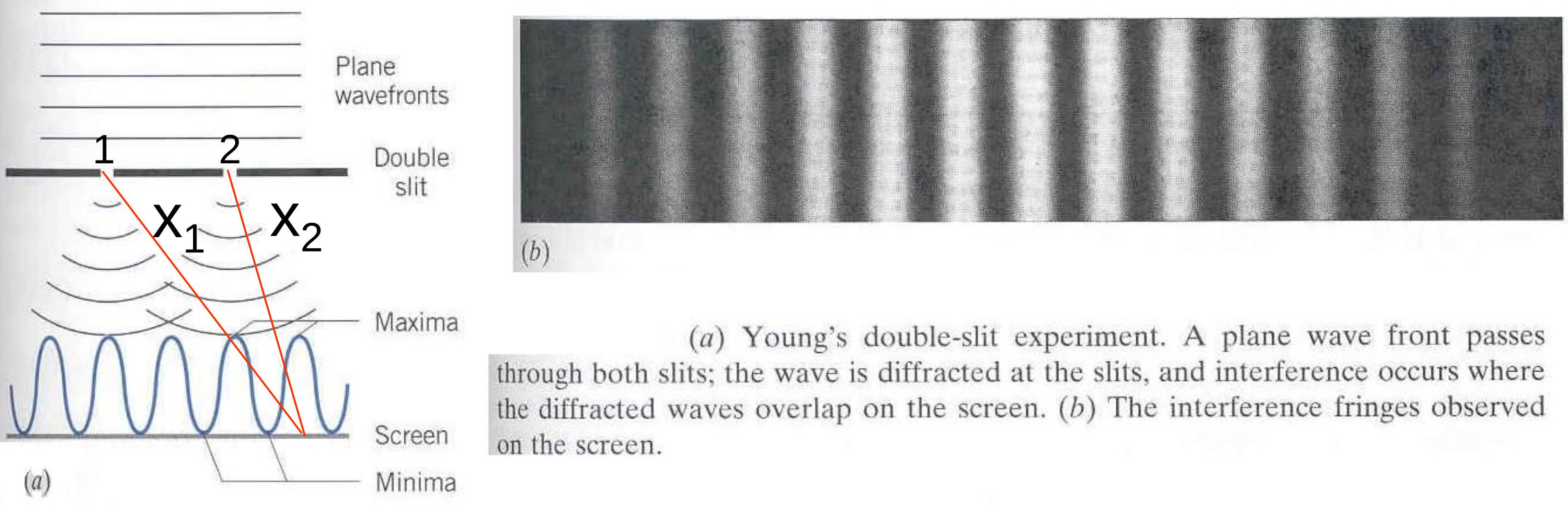
Notice that $<P> \sim E_0^2$

Intensity of the wave ~ energy carried by the wave per unit area per unit time ~ **square of the wave amplitude**

i.e. $I \sim E_0^2$

Interference of Waves

(Young's Double Slit Experiment)



(*a*) Young's double-slit experiment. A plane wave front passes through both slits; the wave is diffracted at the slits, and interference occurs where the diffracted waves overlap on the screen. (*b*) The interference fringes observed on the screen.

Maximum (constructive interference, in phase) when $|x_1 - x_2| = n\lambda$.

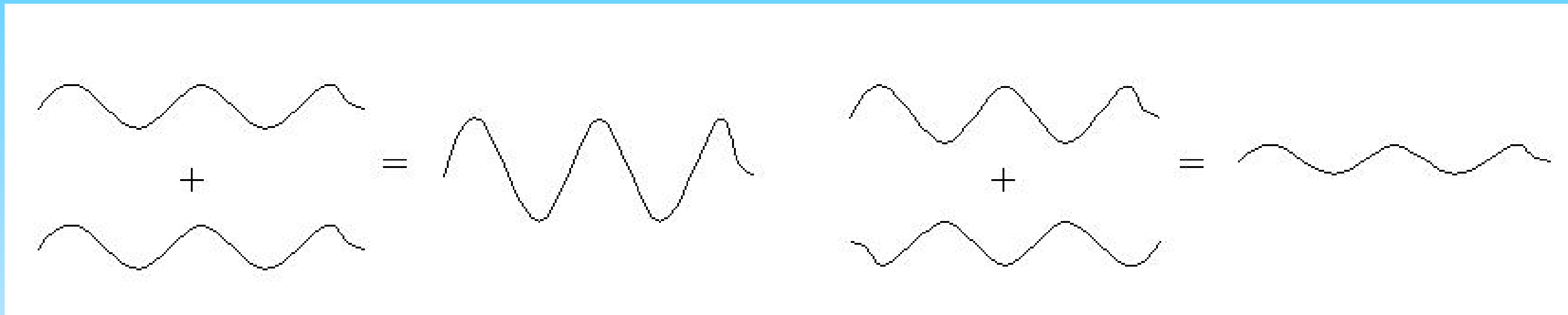
Minimum (destructive interference), out of phase when $|x_1 - x_2| = (n+1/2)\lambda$.

For screen distance (*L*) >> slits separation (*d*), we have

$|x_1 - x_2| \sim d \sin\theta = n\lambda$ *for constructive interference*. ($\tan\theta = x/L$)

Superposition of EM Waves

EM waves obey the **principle of superposition**.

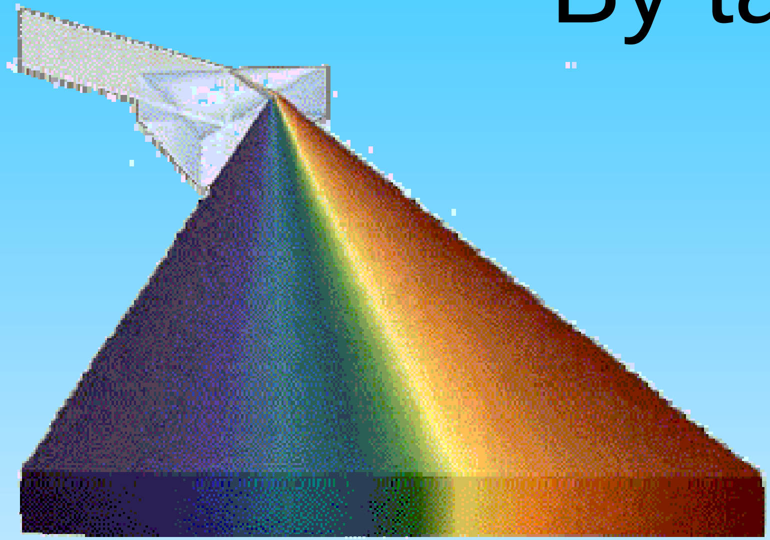


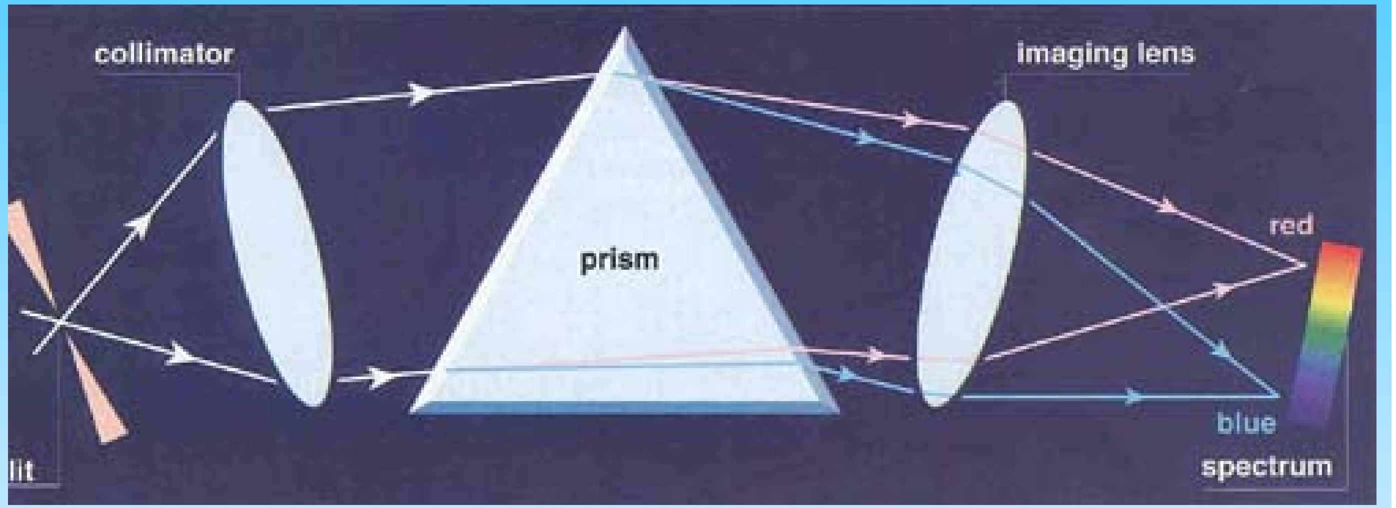
Two (light) wave trains add up and become a *new* wave.

However the principle of superposition does not apply to particles or solid objects. For example, adding two electrons does not give you a new electron.

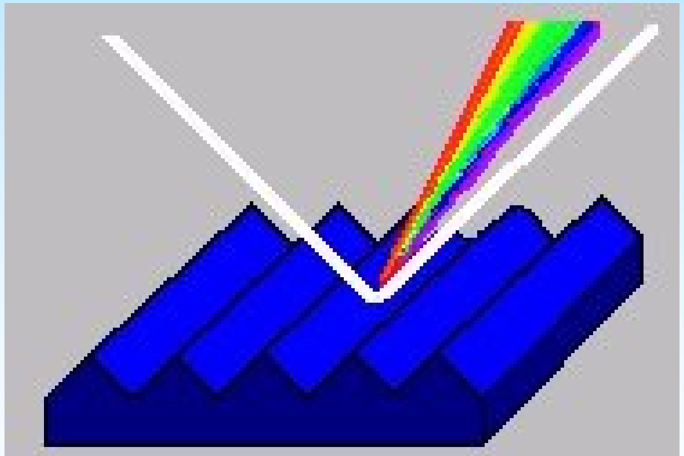
Waves and particles are fundamentally different!
Classical picture only!!!

How do you study light?
By taking a spectrum.





- Newton was the first to use a prism to spread out the light.
- Nowadays, diffraction gratings are used.



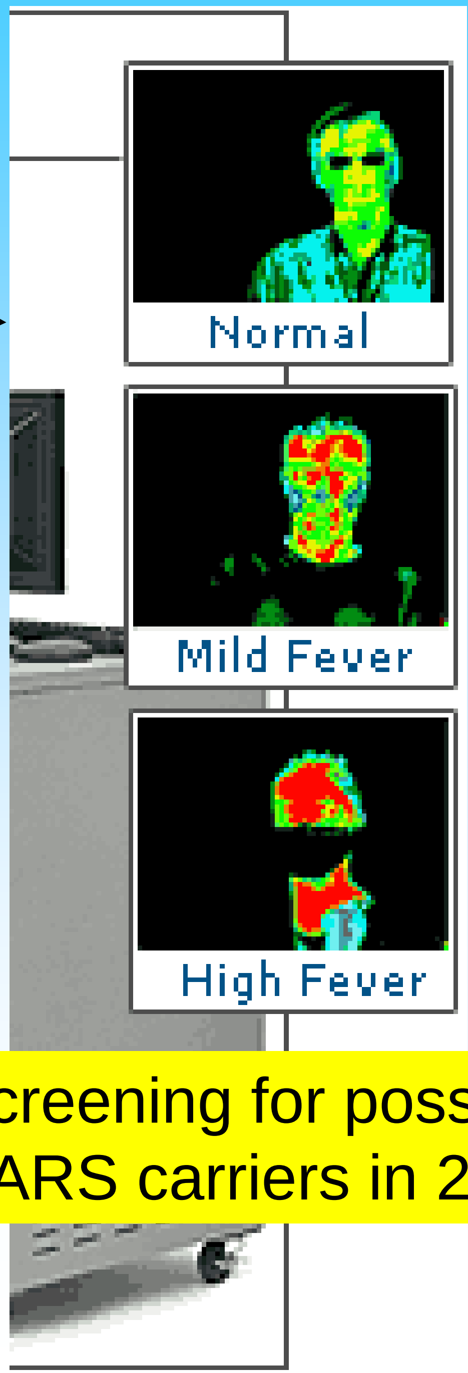
Diffraction grating

Thermal (Black Body) Radiation: Kirchhoff (1859)

- Kirchhoff recognized heat radiation (energy) can be emitted and absorbed by matters (bodies).
- Idealized '**Black Body**' absorbs radiations of all colours and its temperature is determined by the balance between emission and absorption.
- Two black bodies put side to side must come to the same temperature independent of what these bodies are made of.
- Thus, the radiation can only be of the form $F(\lambda, T)$, where F is some universal function of wavelength and temperature only.

Examples of Black Body (Thermal) Radiation

- ANY and ALL things above absolute zero temperature (-273°C, or 0°K) emit thermal radiations.
  - Human (36°C) emits in the infrared.
    - You don't know who he/she is.
    - But you know his/her temperature!
  - Charcoal (~ 700°C) emits in the red.
  - Ordinary light bulb (2800°K)
  - Quartz halogen bulb (3300°K)
  - The sun (5500°K)
  - And even the universe (2.7°K microwave radiation from the Big Bang ~13.5 billion Years ago!)



Screening for possible SARS carriers in 2003.

Stefan-Boltzmann Law (1884)

- Maxwell established his Theory of Electricity and Magnetism (1864)
- Boltzmann's statistical mechanical formulation of Second Law of Thermodynamics appeared in 1877
- Boltzmann then showed that statistical mechanics plus Maxwell's Equation gives:
  - Radiation pressure $p = u/3$ where u is density of the radiation energy
  - $u = \sigma T^4$ (Stefan-Boltzmann Law for Thermal Radiation)

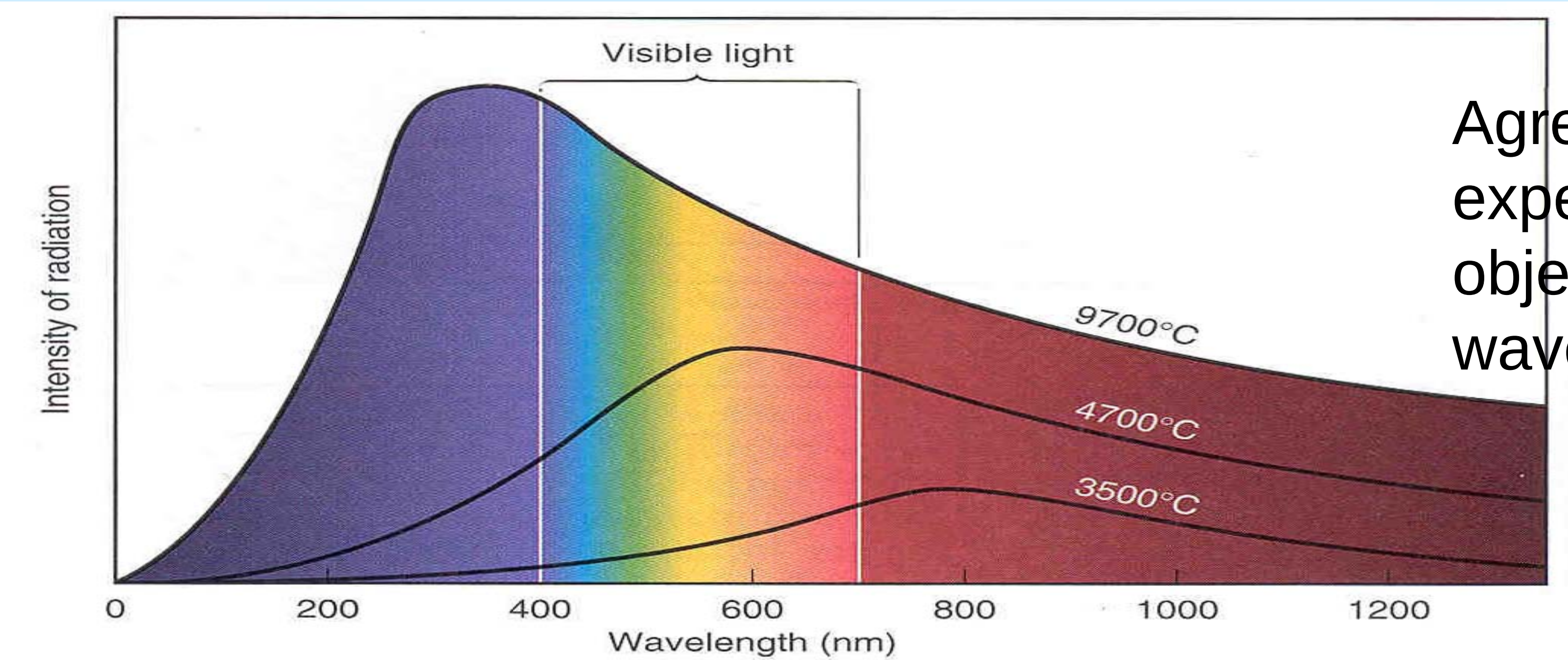
W. Wien's Displacement Law (1893)

Using Statistical Mechanics and Maxwell's Equation, W. Wien established that for the function *F*,

$$T\lambda = constant.$$

In particular, the peak of the function *F* (peak emission) wavelength is

$$\lambda_{max}\,(\text{in nm}) = \frac{2{,}900{,}000}{T}$$



Agreement with experiment. 'Hotter object gives shorter wavelength light.'

Summaries for Emission by Black Bodies (Thermal Radiation)

1. Hotter objects emit more total radiation per unit surface area.
   - Stefan-Boltzmann's Law

$$E(T) \equiv V \int \rho(\nu, T) d\nu = a V T^4 .$$

2. Hotter objects emit *bluer* photons (with a higher frequency ν and average energy.)
   - Wien's Displacement Law:
   - $\lambda_{max} = 2.9 \times 10^6 / T(K)$ [nm]

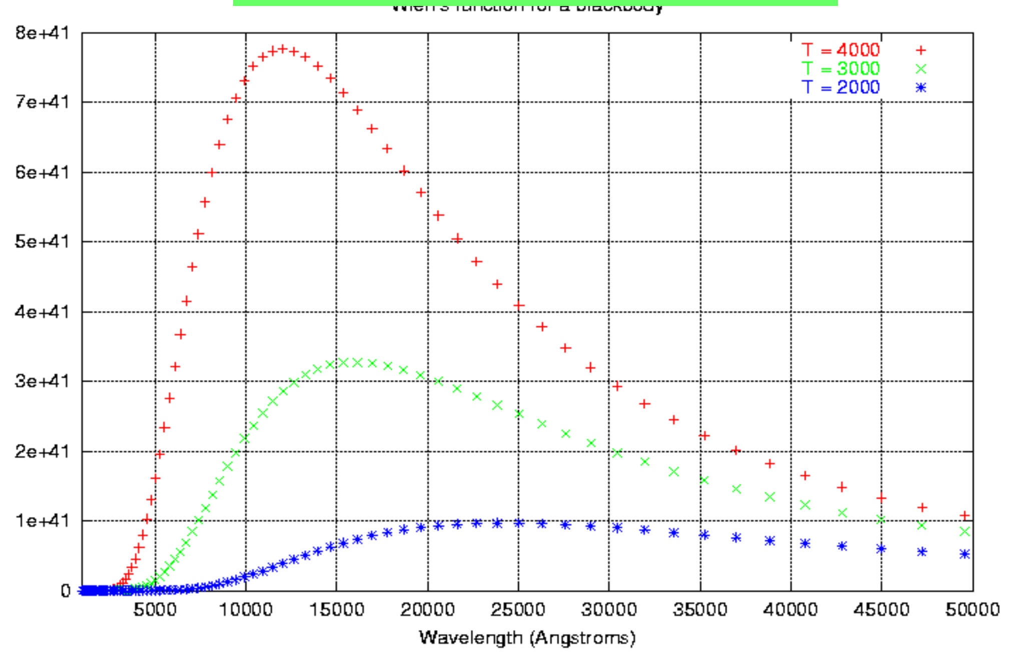
$$\rho(\nu, T) = \nu^3 f(\nu/T)$$

No need to know what the object is, just its temperature T.

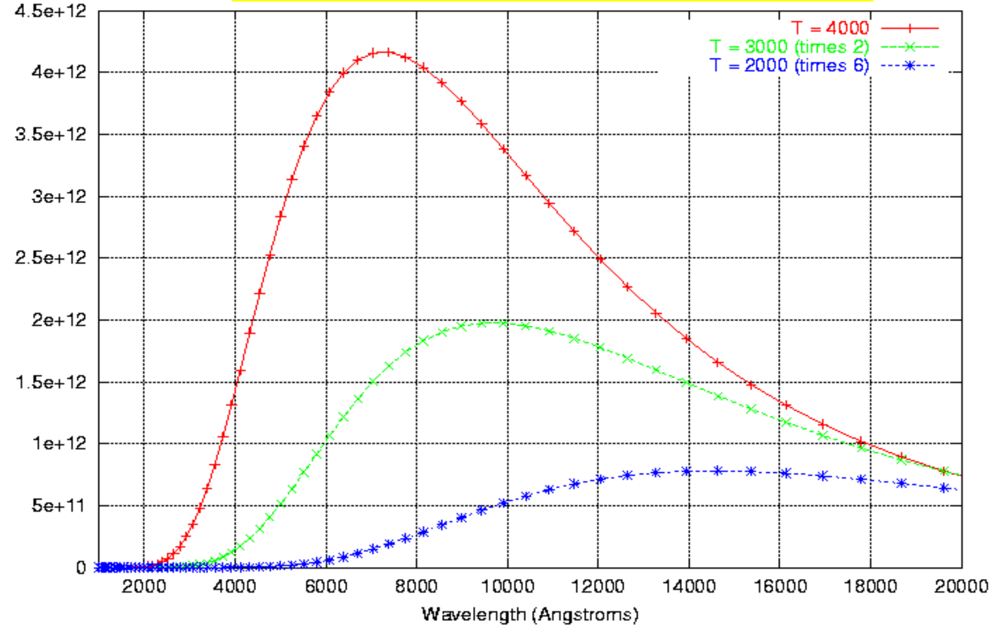
But what is the exact form of the function $f(\nu/T)$?

Wien's Formula: Fitting the curve

$$\rho = \alpha \nu^3 e^{-\beta \nu / T}$$



Experimental data



- Good *approximation*
- No good justifications for that particular formula
- Agree with experiment very well at short wavelengths (high frequency)

Lord Rayleigh's Attempt: UV Catastrophe

Based on sound principles:

- Classical EM Wave Theory of Maxwell gives radiation mode density

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} d\lambda$$

- Boltzmann statistical mechanics gives average energy per mode to be *kT* (Equal Partition Theorem)
- Combining of the two gives energy density *u*(λ)

$$u(\lambda) = \frac{8\pi}{\lambda^4} kT$$

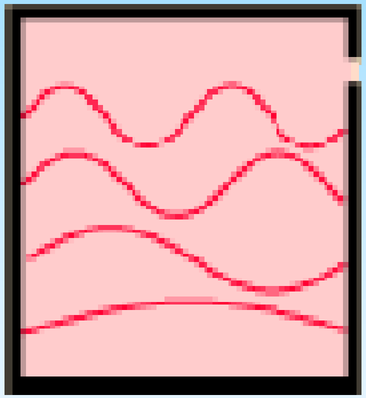
This works well at long wavelengths (small ν), but *u*(λ) goes to infinity as λ goes to zero (ν gets large): UV catastrophe!

Source of Thermal Radiation: Collection of Radiating Dipoles

- Planck, following Kirchhoff (whom he succeeded in 1889 at Berlin), studied the problem using a collection of radiators of all frequencies.
- Lorentz had already by then fully developed the theory of optics using these ‘linear oscillators’ (‘Hertzian radiators’ or resonators).
- Planck sought to provide a theoretical basis for Wien’s formula, and from 1895 on published a series of papers showing how Wien’s formula might be obtained in his model by making various assumptions.
- But improved data finally convinced Planck that Wien’s formula was not correct – and hence the ‘despair’ he sensed.

Planck’s ‘Act of Despair’

Radiation modes in a hot cavity provide a test of quantum theory



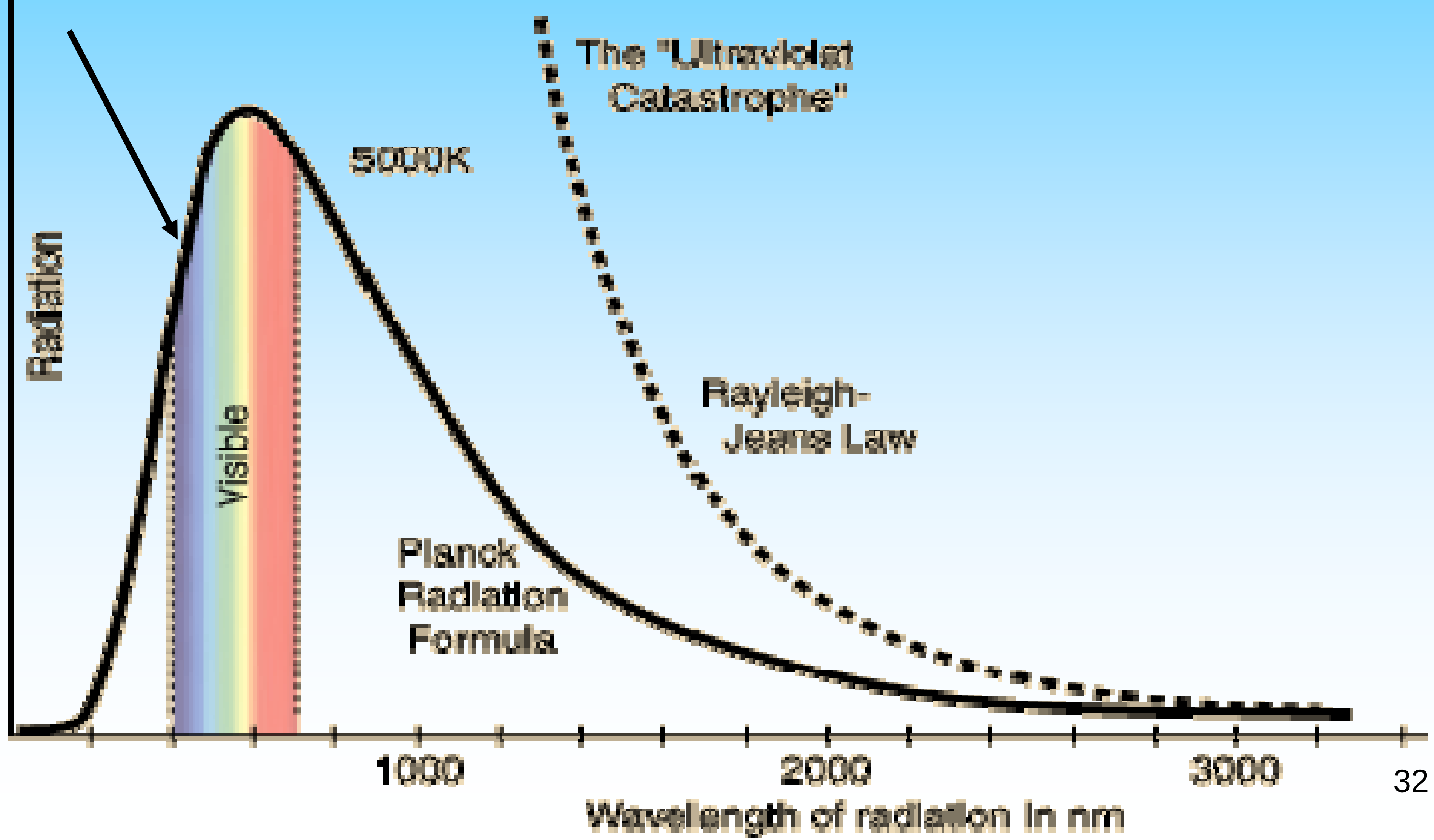
	#Modes per unit frequency per unit volume	Probability of occupying modes	Average energy per mode
CLASSICAL	$\frac{8\pi\nu^2}{c^3}$	Equal for all modes	kT
QUANTUM	$\frac{8\pi\nu^2}{c^3}$	Quantized modes: require $h\nu$ energy to excite upper modes, less probable	$\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$

energy of photon = $h\nu$

Boltzmann’s statistics

It worked beyond expectation!

$$R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\left(\frac{hc}{\lambda}\right)\frac{1}{e^{hc/\lambda kT} - 1}\right]$$



Planck's Quantum of Light

* Energy emission or absorption for an radiator at frequency ν can only be in units, or quanta, of $h\nu$, $2h\nu$, $3h\nu$ .. and not values in between.

* Classically, there is no restriction on the amplitude of the EM waves and energy exchange can be in any amount, for any ν.

* The functional form of the Planck distribution makes it increasingly difficult to emit or absorb light at high frequency because of the very large quanta.

Photo-electric Effect

An experiment which studies electrons emitted from metal surface upon the illumination of light.
(First performed by Hertz, 1887)

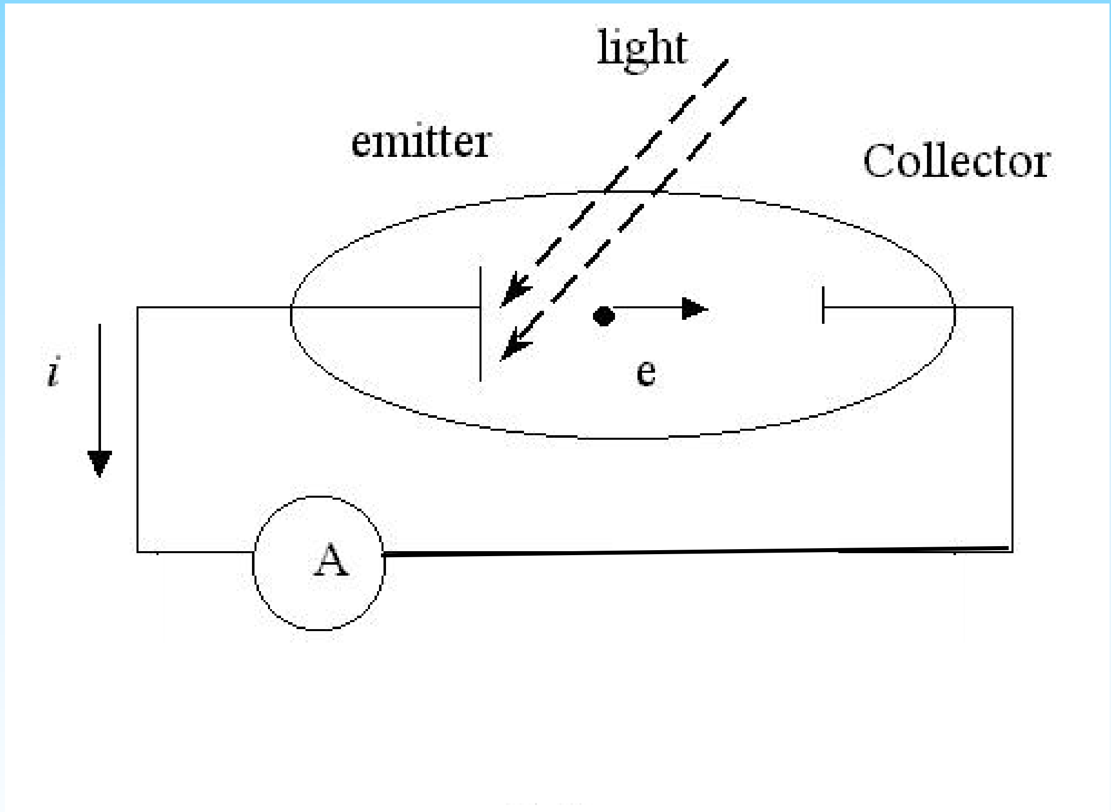
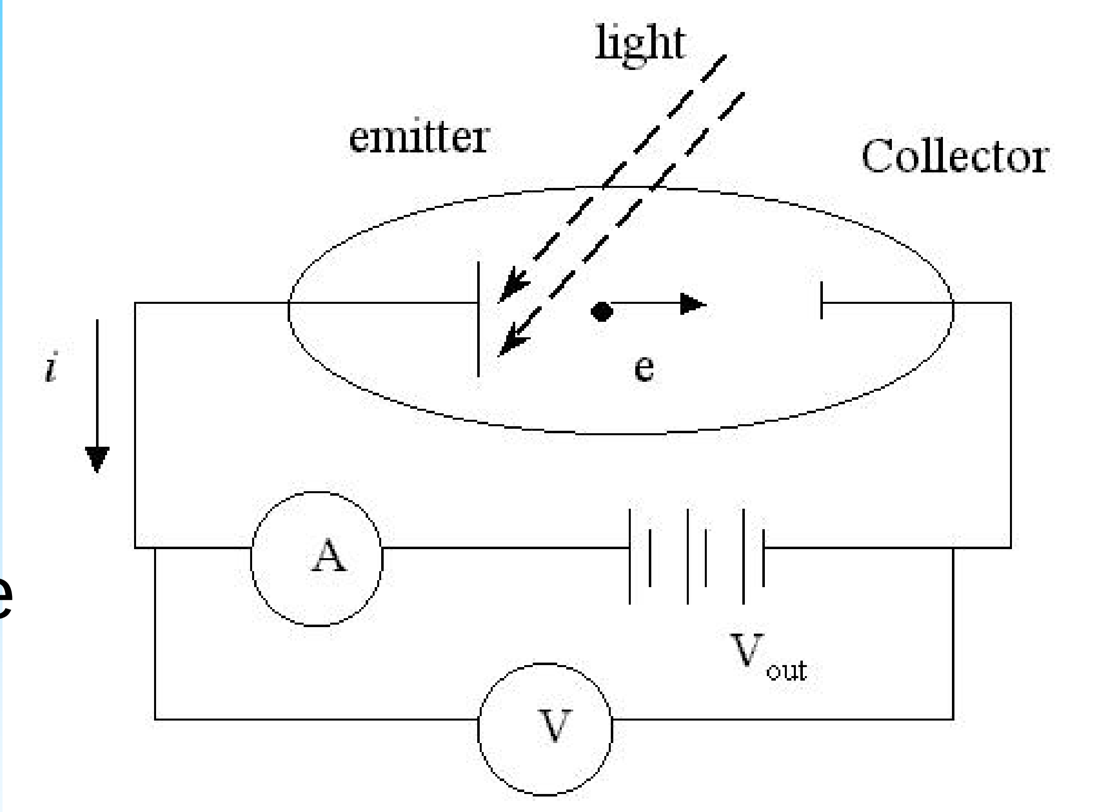


Photo-electric Effect

What can be measured?

i) Rate of electron emission

– i.e. electric current *i* for positive applied voltage upon the illumination of light with different wavelength.



ii) Maximum kinetic energy of the photoelectrons

– i.e. no current for large enough negative applied voltage (stopping potential).

Experimental results of Photoelectric Effect

i) Maximum kinetic energy of the electrons is independent of the intensity of the radiation, but the number of electrons emitted (current) is proportional to the intensity of light.

ii) No photoelectric effect if the frequency of the light is below a certain critical value ν_0. Above ν_0, photoelectric effect exists no matter how weak the intensity of the light is.

iii) The first photoelectron emitted is virtually instantaneously ($\sim 10^{-9}$s) after the light strikes the metal surface.

Einstein: “On a Heuristic Point of View about the Creation and Conversion of Light” *Ann. Physik 17, 132 (1905)* (Nobel Prize 1921)

i) Einstein suggested that somehow energy of the light is not distributed uniformly in the wave, but exists in a form of small “packages” called photons.

(Here wave is treated as particle!)

ii) Moreover, the energy carried by each photon is related only to the frequency of the light.

i.e. $$E = h\nu = hc/\lambda$$

Here, h = Planck’s constant ~ 6.57 x 10^{-34} Js.

(This was introduced by Planck earlier to explain Black Body radiation.)

Photo-electron emission Einstein's theory (1905)

iii) Intensity of light ~ **number of photons** carried by the EM wave.

→ total energy carried by the wave

$$E \sim \text{(intensity)} \times h\nu = nh\nu.$$

Einstein's interpretation of the photoelectric effect

i) Photoelectron is released as a result of the absorption of a photon by an electron which happens instantaneously when the photon hits the electron.

→ energy delivered to the electron = $h\nu$,

and the photoelectron is emitted if

$K = h\nu - \phi > 0$. (K is the kinetic energy of the electron.)

Photo-electron emission Einstein's theory (1905)

From

$$K = h\nu - \phi > 0,$$

It is clear that

a) K_{max} is independent of the intensity of the light.

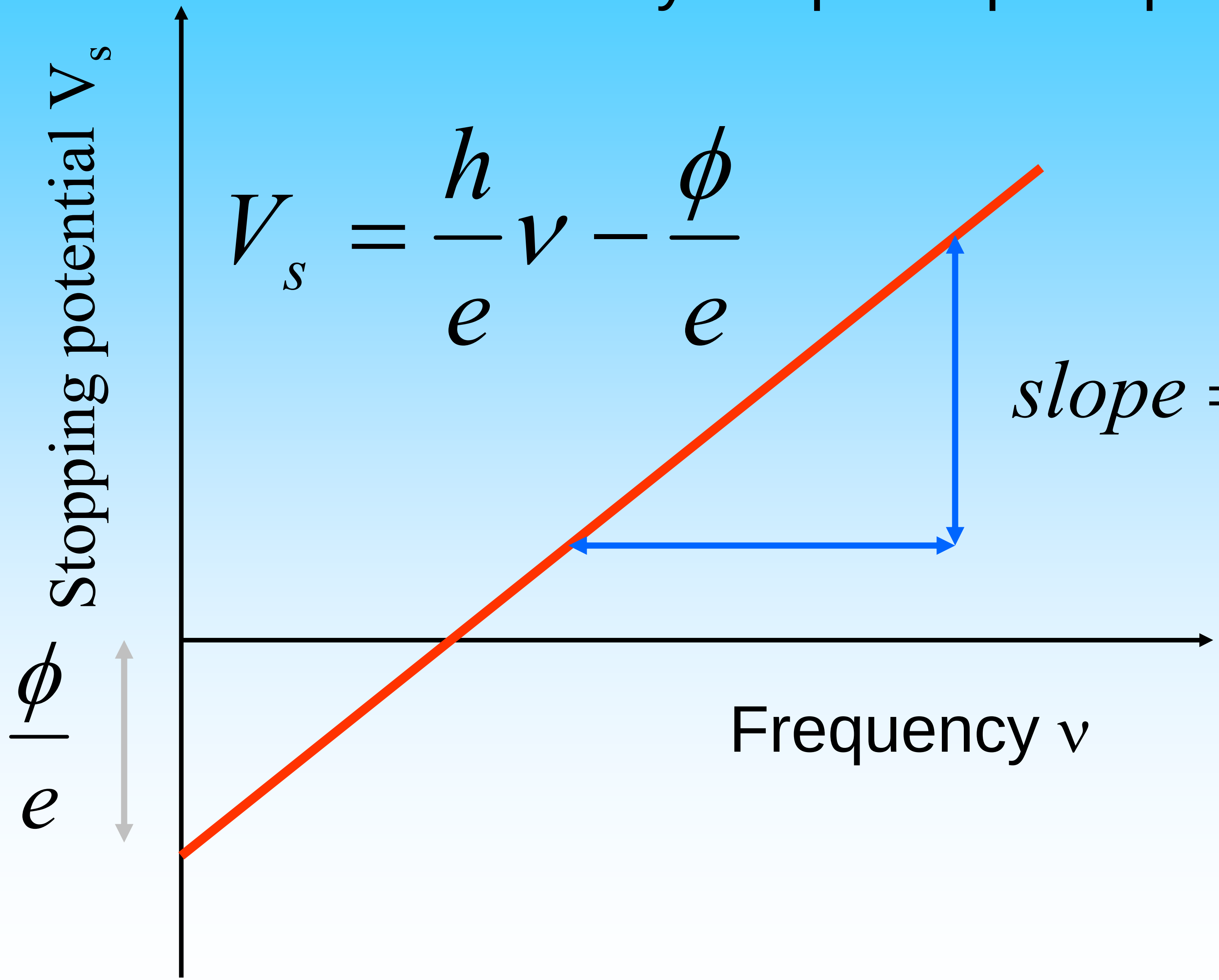
K_{max} can be determined by the stopping reversed potential:

$$K_{max} = e\,V_s = h\nu - \phi$$

a) No photoelectric effect when $\nu < \nu_0$.

This was confirmed by Millikan in 1915. (Nobel Prize 1923).

Slope will give *h/e*, and if you know *e*, then you can determine *h* – very simple in principle!



Basic Properties of Atoms

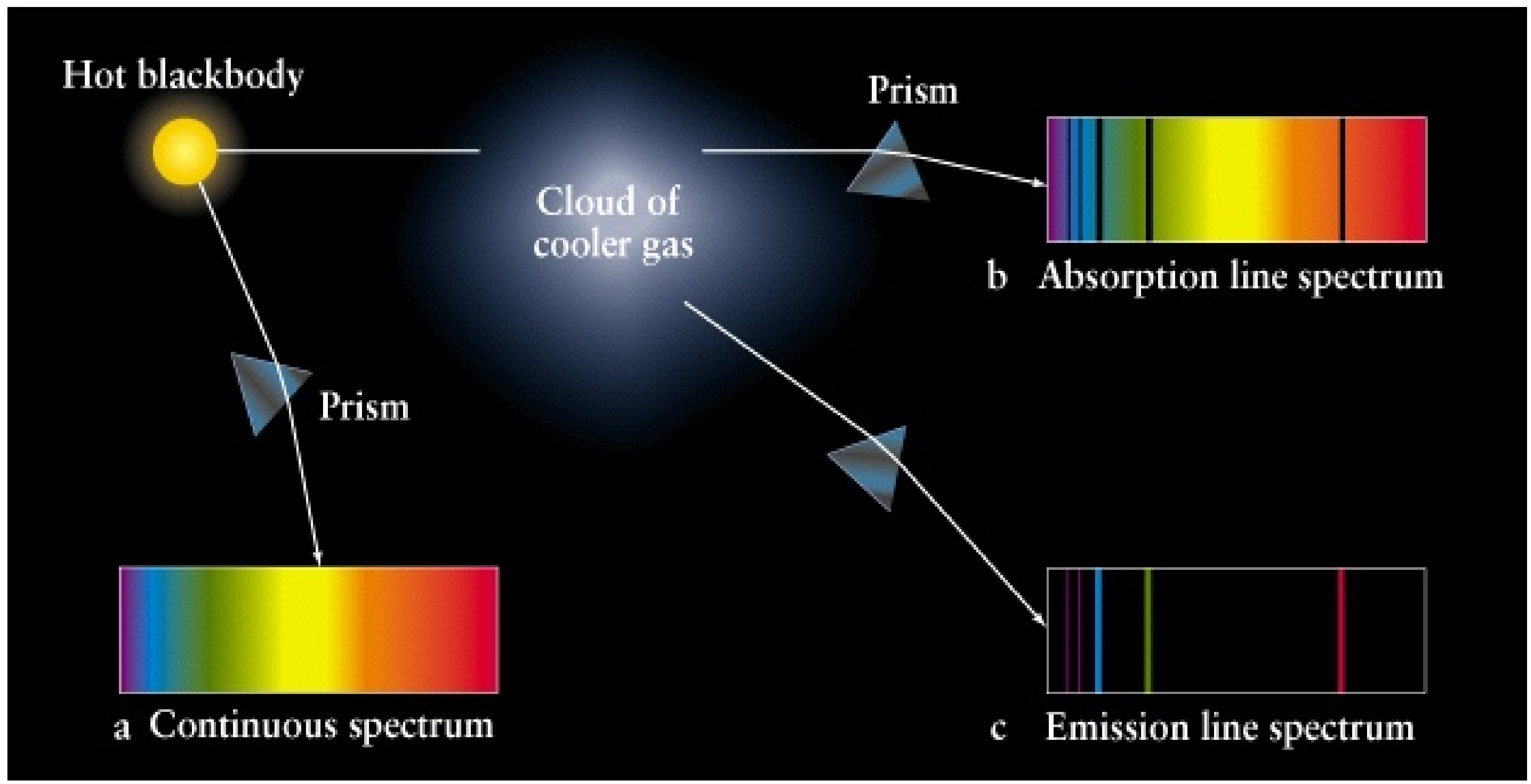
- Small: ~ 0.1 nm (10^{-10}m)

  A crude estimate:

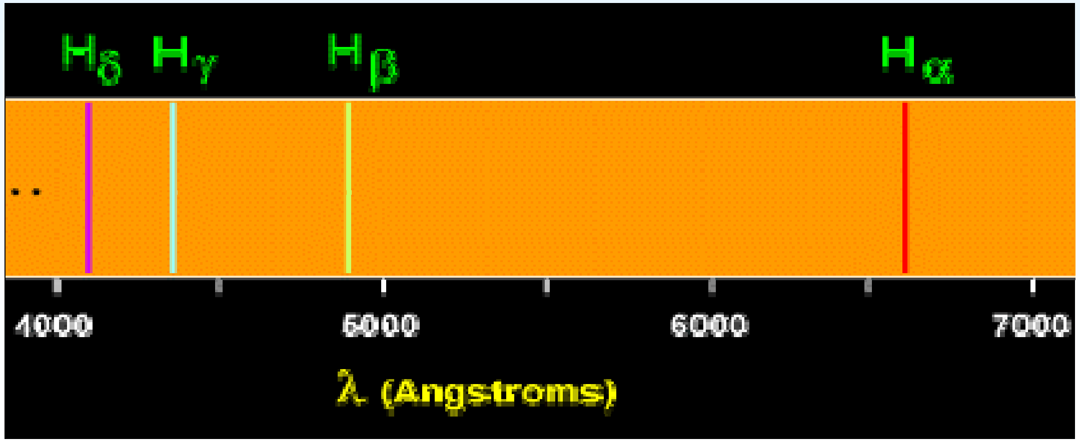
  - Iron has a density 8 g/cm^3 and molar mass 56 g.
  - So one mole (~$6x10^{23}$) of iron atoms has a volume of 7 cm^3.
  - One iron atom occupies a *max.* volume of $7/6x10^{23}$ ~ 10^{-23} cm^3.
  - The diameter is then about $10^{-23/3}$ ~ $2x10^{-8}$ cm=0.2nm.
  - It is impossible to *see* an atom using visible light (λ~500nm).

- Atoms are stable.
- Atoms contain negative charges:
  - external disturbance can expel electrons from atoms, e.g. Compton effect and photoelectric effect.
- An atom as a whole is neutral.
- Atoms can emit and absorb EM radiation.

Spectrum: Absorption and Emission



Hydrogen Spectrum
in the visible: 4 lines



Johann Balmer (1885) and Johannes Rydberg (1888)

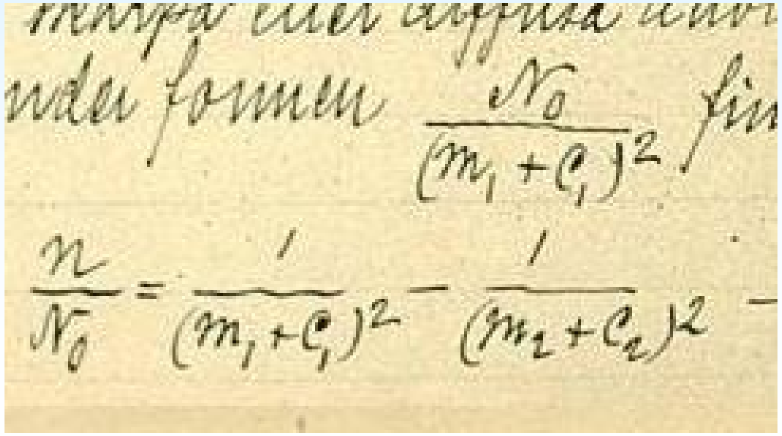
Balmer 1825 -1898

* Balmer (a mathematics teacher) noticed the pattern of the four visible lines of hydrogen. He found the formula that can account for the wavelengths of those four lines to very high accuracies using n = 3,4,5 and 6.

$$\frac{1}{\lambda} = 109,563\ \mathrm{cm}^{-1}\left(\frac{1}{4} - \frac{1}{n^2}\right),$$

* Rydberg generalized Balmer's formula and applied to alkali metals and other hydrogen like spectrum:

Rydberg 1854-1919

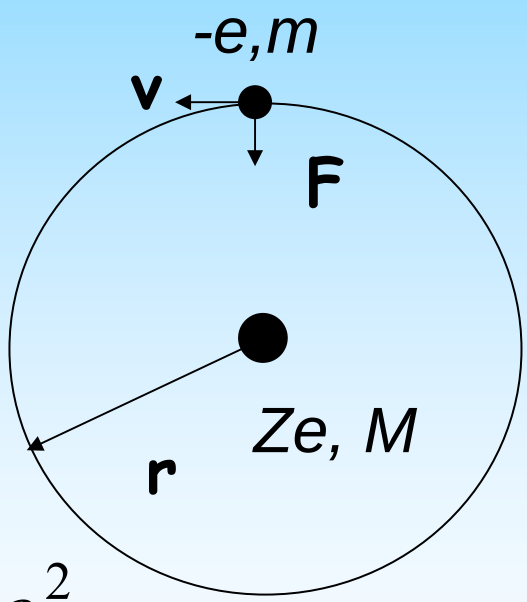


$$\frac{1}{\lambda_{\mathrm{vac}}} = R_{\mathrm{H}}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The Bohr's Atomic Model

- Following Rutherford's proposal on the atomic nucleus, Niels Bohr in 1913 suggested that atom is like a small planetary system: negative charged electrons circulating about the positive charged nucleus like planets circulating about the sun.

Coulomb attraction plays the role of gravitational attraction:



$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

Kinetic energy *K*: $K = \frac{1}{2}mv^2 = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$

The Bohr's Atomic Model

Potential energy *U*: $$U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Total energy = *E* = *K*+*U*: $$E = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$$

Problems with this pure classical mechanical model.

- The electron orbit is not stable: from classical electro-magnetism, an accelerating charge *must* continuously radiates EM waves. As a consequence, electron loses energy and spirals in toward the nucleus.
- Since ***r*** or ***v*** is arbitrary and can take on any value (a continuous range), hence the emission is expected to exhibit a continuous spectrum instead of line (discrete) spectrum.

To overcome these difficulties, Bohr made a bold and daring *hypothesis* *(postulate)* :

There are certain special states of motion (orbits), called *stationary states*, in which the electron may exist without radiating electromagnetic energy.

In the stationary states, the angular momentum ***L*** is given by:

$$L = n\hbar , \quad n = 1,2,3,.... \quad \hbar = h / 2\pi$$

i.e. in the stationary states angular momentum ***L*** is quantized in units of $\hbar$!

For circular orbits, ***r*** is always perpendicular to ***p***.
So the angular momentum ***L*** **(=** ***r*** *x* ***p*****)**, is simply

rp **=** mvr .

The Bohr's Atomic Model

The postulate that angular momentum is quantized implies:

$$mvr = n\hbar \quad \Rightarrow \quad v = \frac{n\hbar}{mr}$$

From classical mechanics (*F*=*ma*), ***v*** and ***r*** are related by

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

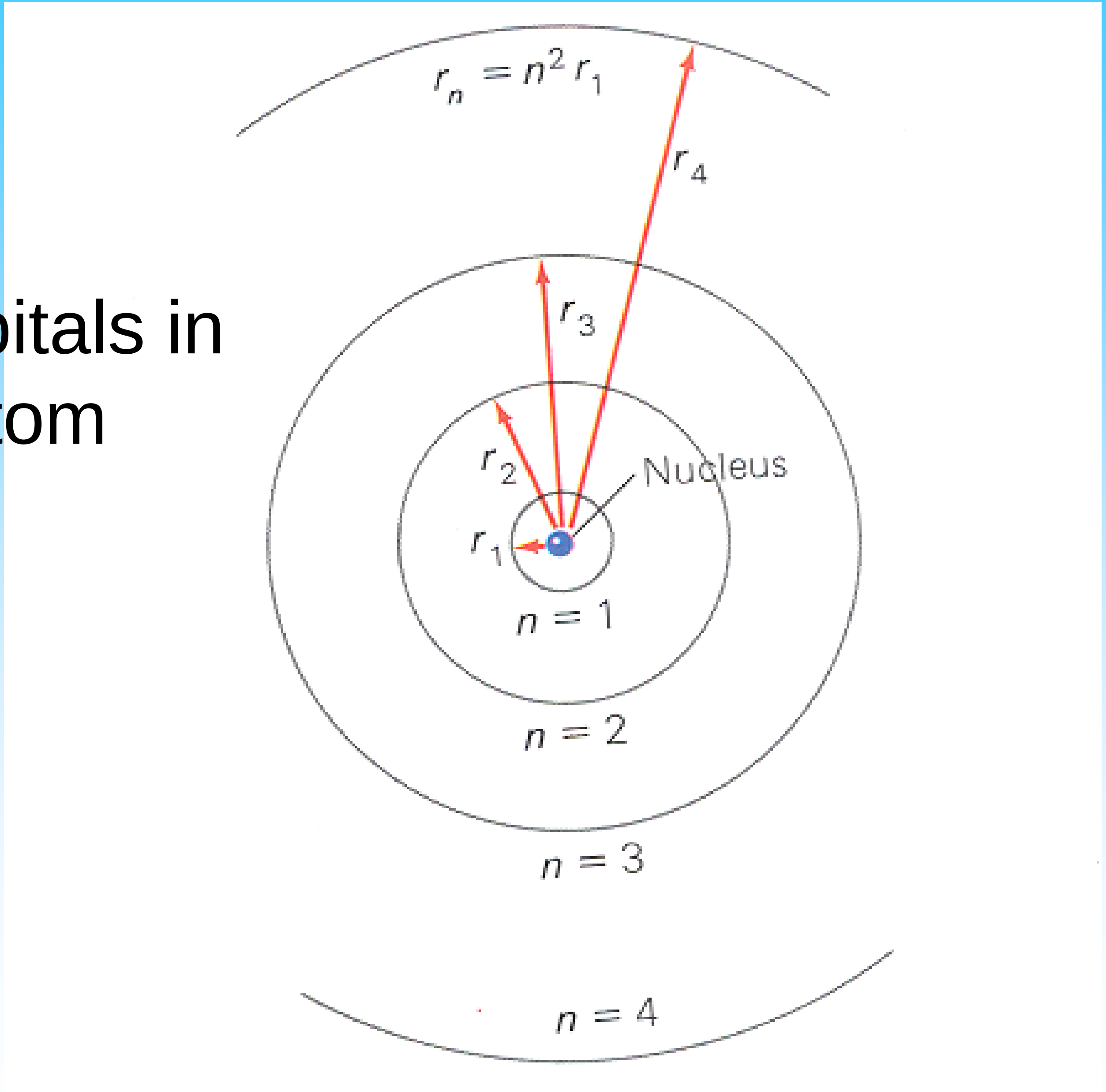
Taken all together, r_n is also quantized!!!

$$r_n = \frac{4\pi\varepsilon_0\hbar^2}{me^2}n^2 \equiv a_0 n^2$$

where a_0 is Bohr radius =0.0529nm

The Bohr's Atomic Model

Electron orbitals in hydrogen atom



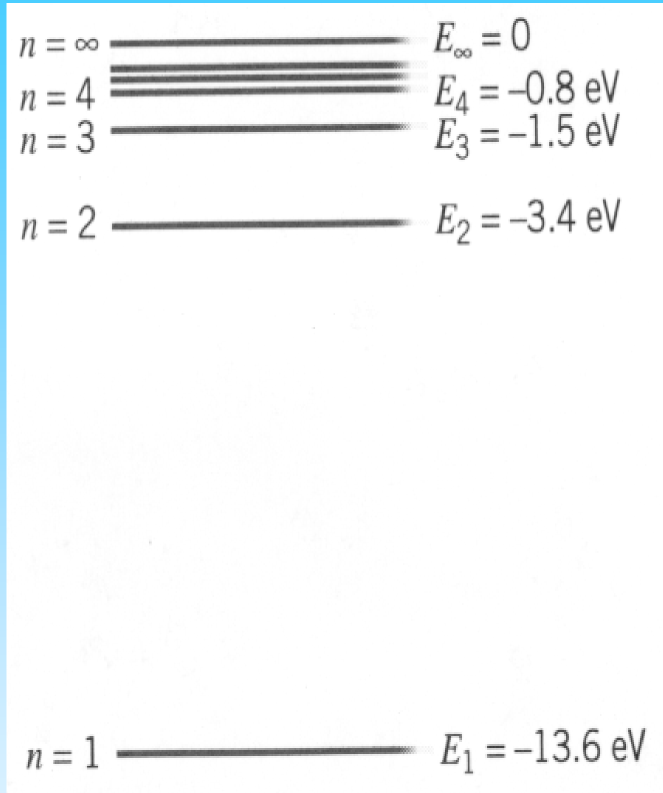
http://www.walter-fendt.de/ph11e/bohrh.htm

The Bohr's Atomic Model

Since the total energy of a circular orbit is $E = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$,

the total energy E is also quantized according to:

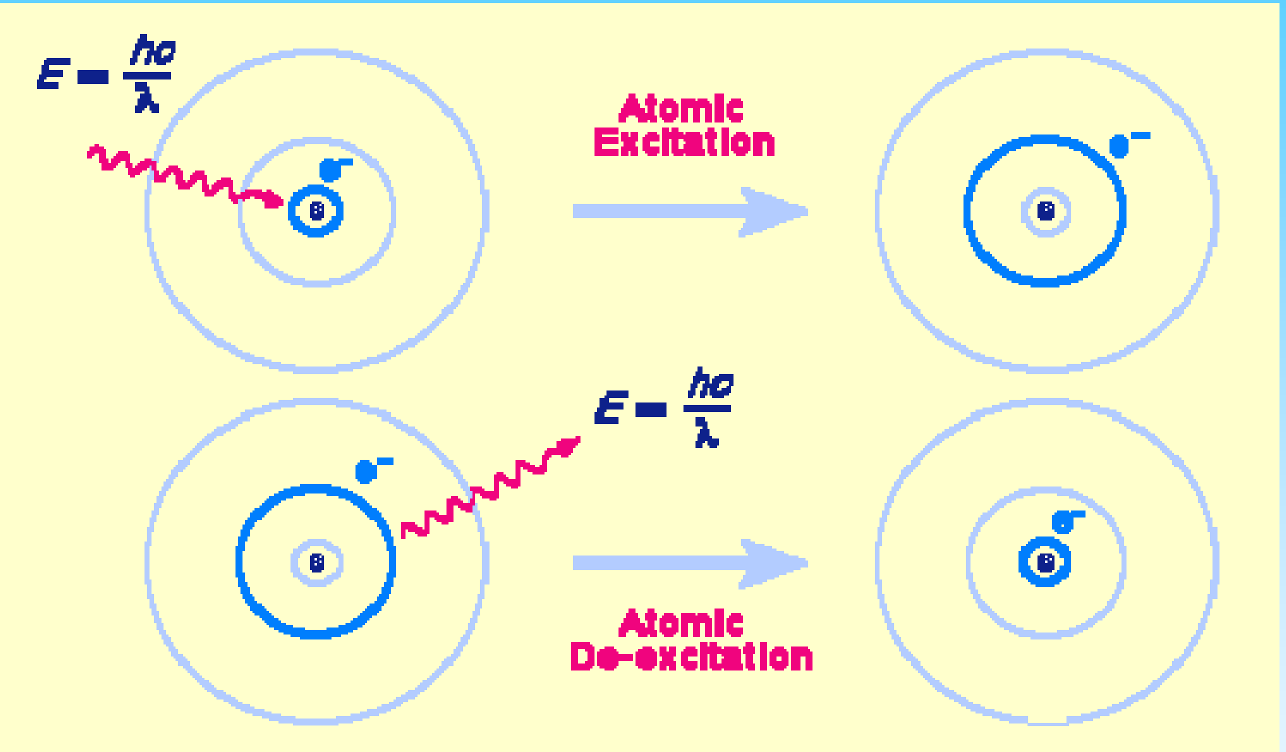
$$E_n = -\frac{e^2}{8\pi\varepsilon_0(a_0 n^2)} = -\frac{me^4}{32(\pi\varepsilon_0\hbar)^2}\frac{1}{n^2} = \frac{-13.6\,eV}{n^2}$$



- The state with n =1 has the lowest energy and it is called the ground state.
- The ground state of H atom has $E_1 = -13.6$ eV ; $r_1 = a_0$.
- When $n = \infty$, $r = \infty$, $E_\infty = 0$, i.e. the electron is unbound (free).
- Thus $|E_n|$ is the energy needed to ionize an electron in a state n. $|E_n|$ is also called the binding energy.

In the Bohr model how do atoms absorb and emit energy?

Additional postulate:



www.colorado.edu/physics/2000/quantumzone/bohr.html
http://66.99.115.200/atom-webexhibit.htm

- An atom can also be excited by electron (or photon) bombardment to higher energy states (excited states).

Transition Energies

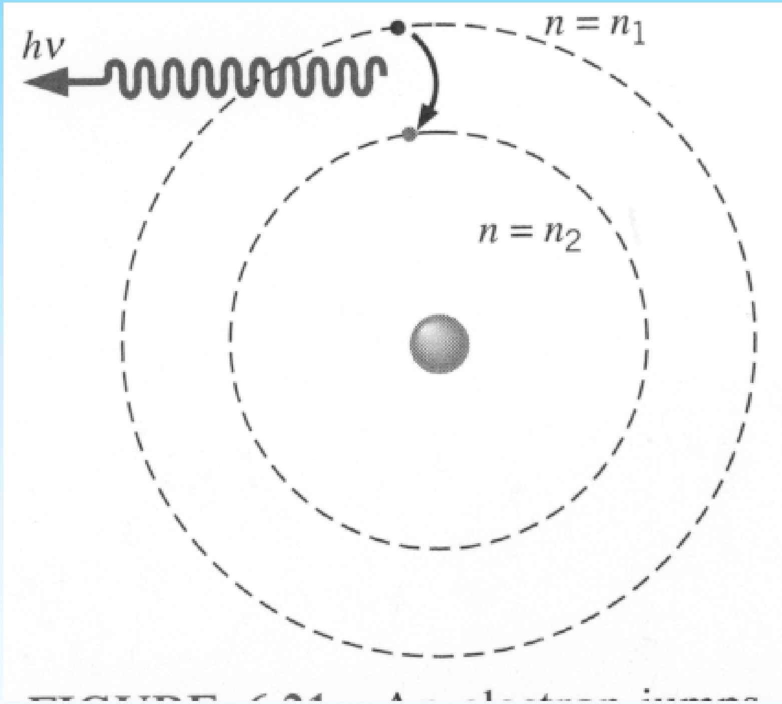
- An electron in a stationary state *n* does not radiate.
- But it can emit a photon when it "jumps" from a higher energy state n_1 to a lower energy state n_2 .
- The energy of the emitted photon is simply:

$$h\nu = E_{n_1} - E_{n_2}$$

$$\Rightarrow \quad \nu = \frac{me^4}{64\pi^3\varepsilon_0{}^2\hbar^3}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\frac{1}{\lambda} = \frac{me^4}{64\pi^3\varepsilon_0{}^2\hbar^3 c}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

Rydberg constant ($R = 1.097\times10^7 m^{-1}$)



The Importance of the Bohr's Paper: The Conceptual Breakthroughs

1. The stationary states with definite energies: these energies do NOT correspond to the frequencies of the emitted light.

2. Light emission corresponds to going from one stationary state *m* to another *n*: $h\nu = E_m - E_n$

The Bohr's Atomic Model

Successes:

- Reproduced the Rydberg-Balmer Formula
- Give the correct value of the Rydberg constant (using known values of fundamental constants of *e*, *m* and *h*)
- Even worked for ionized helium atom: a helium nucleus with a single electron in orbit.

Shortcomings:

- It was difficult to justify the postulate.
- Path of electrons are exactly known – violation of the Uncertainty Principle.
- Bohr's model gives ground state (n = 1) angular momentum $L = h/2\pi$; experiment shows L = 0 for ground state.

Bohr's model was soon replaced by Quantum Mechanics!!!

http://www.colorado.edu/physics/2000/quantumzone/schroedinger.html

http://dbhs.wvusd.k12.ca.us/webdocs/Chem-History/Bohr/Bohr-1913a.html

Conceptual Difficulties: Wave **or** Particle

- Classical waves are continuous function of space and time, have polarizations, can produce interference and diffractions, and can be summed using the superposition principle. (e.g. Maxwell Equations for EM waves)
- Classical particles are discrete objects with definite energy and momentum, and satisfy Newton's Law. (or Theory of Relativity)
- One **OR** the other, **NEVER both**!

Wave Properties of Light

There is no question that light is wave:

- Maxwell Equations give a wave equation for light.
- Light can have interference: principle of superposition
- X-ray diffraction

Particle Properties of Light

- Black Body radiation + photoelectric effect suggest that the energy of light is quantized in 'packets' (photons).

$$E = h\,\nu = h\,c\,/\,\lambda$$

- The problem is, how real are these light packets?
- Can these light packets (photons) be treated as real particles?

Question:

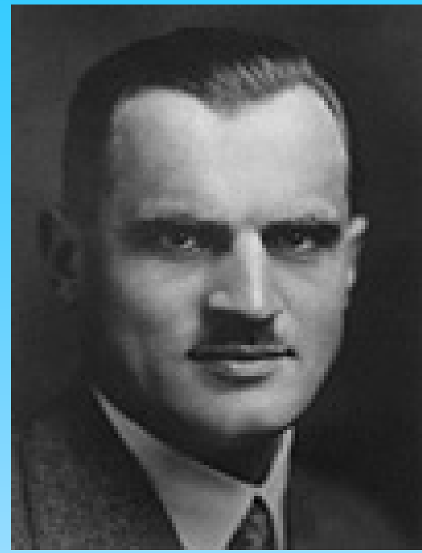
If light behaves like particle, does it have momentum?

Recall from Special Relativity, $E = pc$ if light behaves like particle.

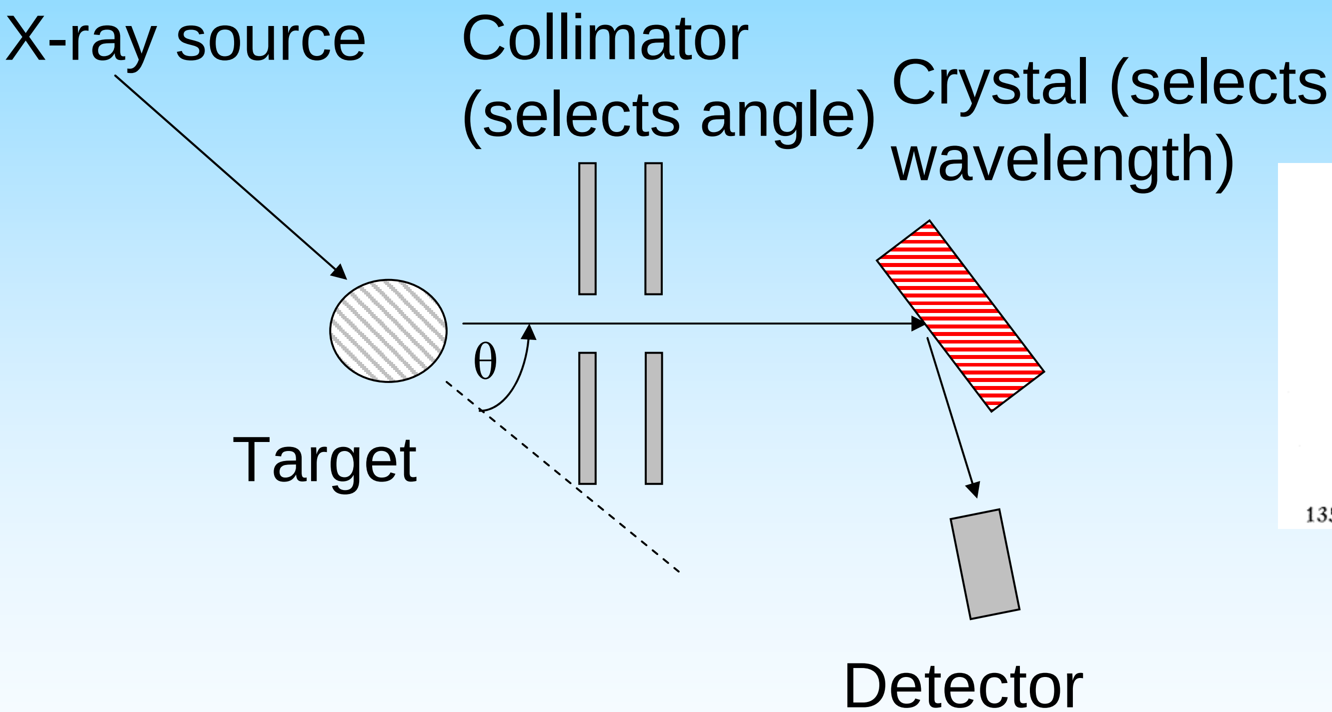
$$p = E/c = nh/\lambda$$

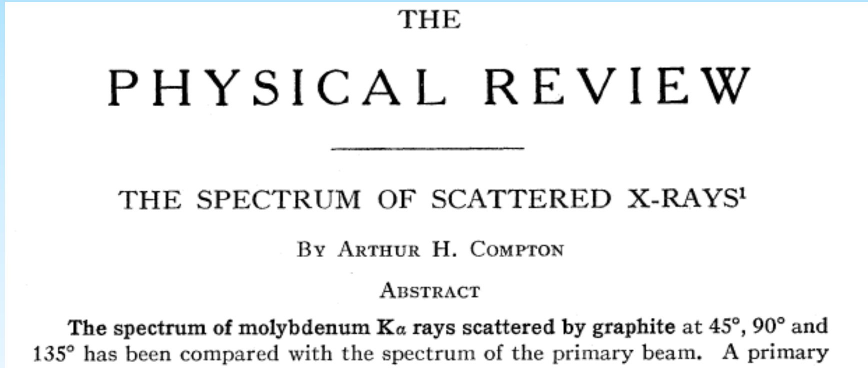
Compton Scattering

With Bragg crystal spectrometer technique, X-ray sources with definite energy can be used for precise experiment.



A. H. Compton

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PHYSICAL REVIEW

THE SPECTRUM OF SCATTERED X-RAYS[1]

By Arthur H. Compton

Abstract

The spectrum of molybdenum Kα rays scattered by graphite at 45°, 90° and 135° has been compared with the spectrum of the primary beam. A primary

A.H. Compton, *Phys. Rev.* **22** 409 (1923)

Result: peak in scattered X-ray radiation shifts to lower energy (longer wavelength) than the source. The amount of shift depends on θ (but not on the target material).

Compton Scattering result:

Two peaks: one shifted, and one not shifted.

The unshifted peak is scattering from the nucleus, which is much heavier than the electron. The shift is therefore much smaller.

How much smaller?

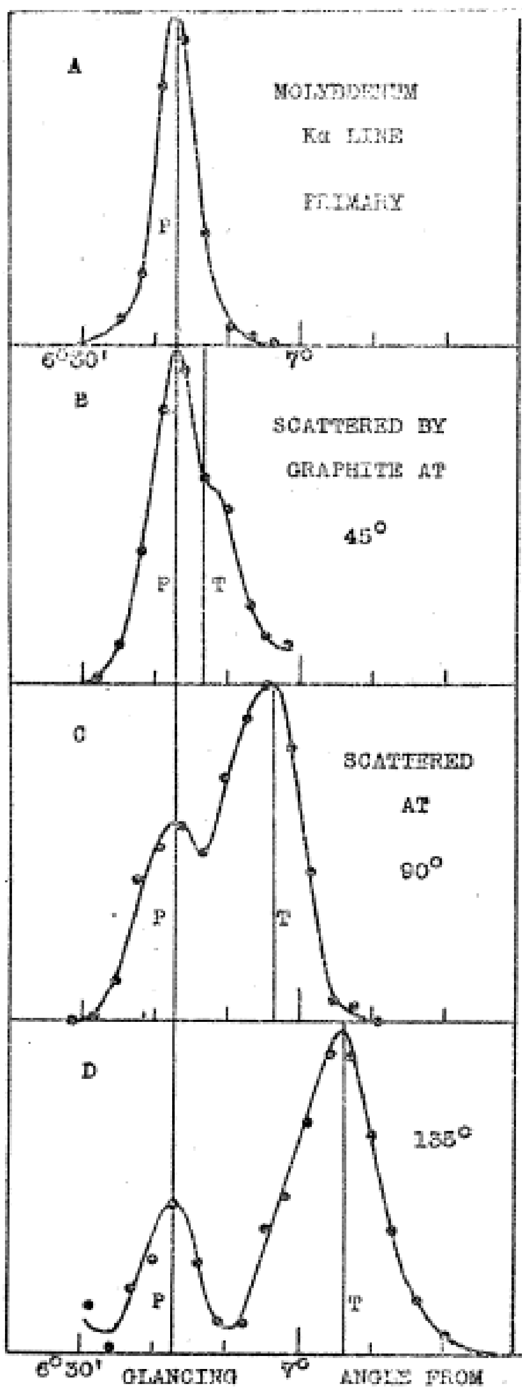


Fig. 3

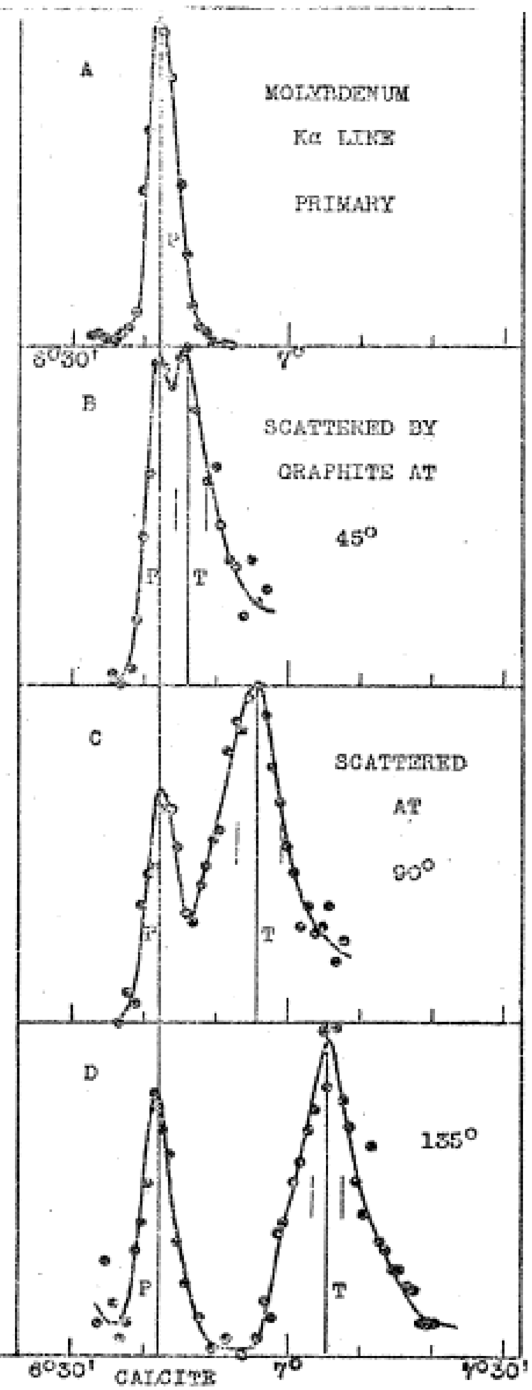
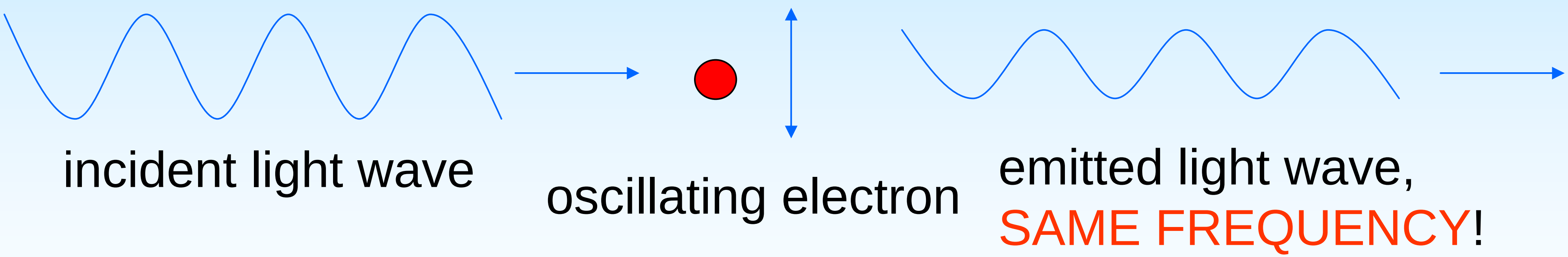


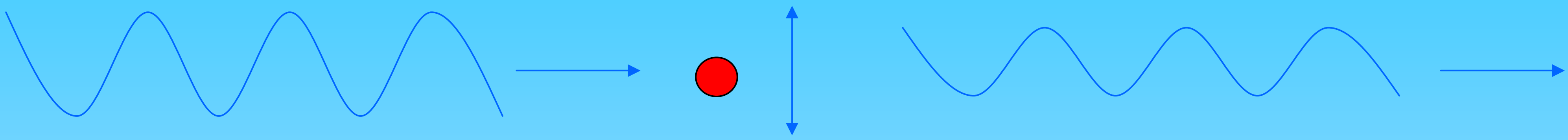
Fig. 4

Compton Scattering

- Classically, the electron oscillates at the driving frequency of the incident light wave. The emitted light wave might have different amplitude – but should be the same frequency as the incident light.
- Change in wavelength of scattered light is completely unexpected classically.



Compton Scattering



incident light wave

oscillating electron

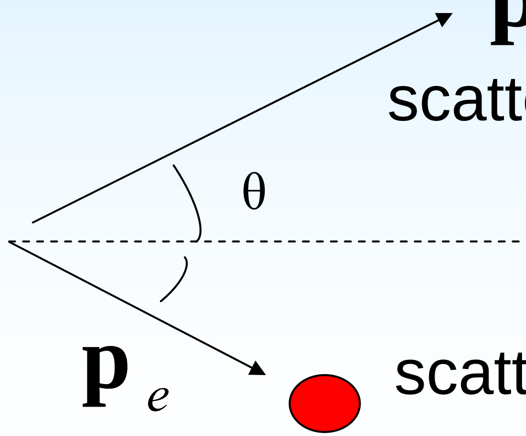
emitted light wave,
SAME FREQUENCY!

Compton's explanation:

"billiard ball" collisions between particles of light (X-ray photons) and electrons in the material

Before

After



Compton Effect

$$\lambda' - \lambda = (h/m_e c)(1 - \cos\theta)$$

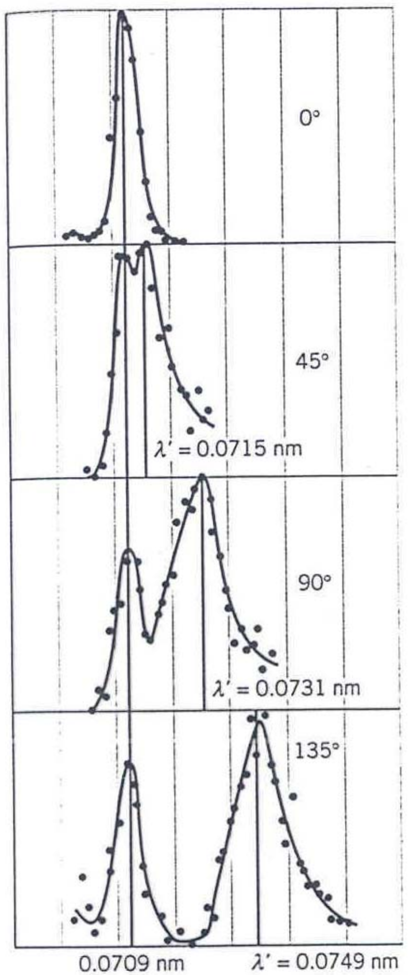


FIGURE 3.20 Compton's original results for X-ray scattering.

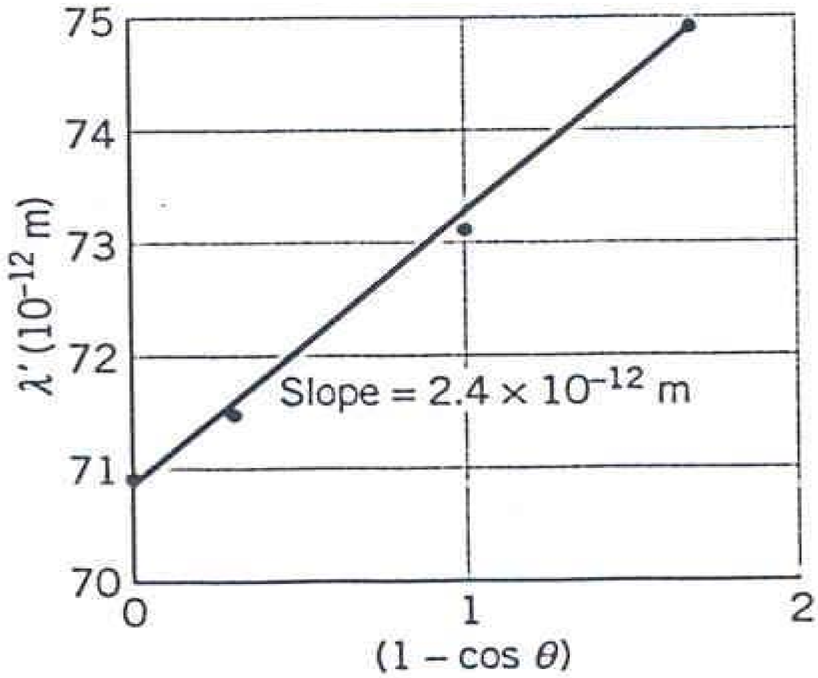


FIGURE 3.21 The scattered X-ray wavelengths λ′, from Figure 3.20, for different scattering angles. The expected slope is 2.43×10^{-12} m, in agreement with the measured slope of Compton's data points.

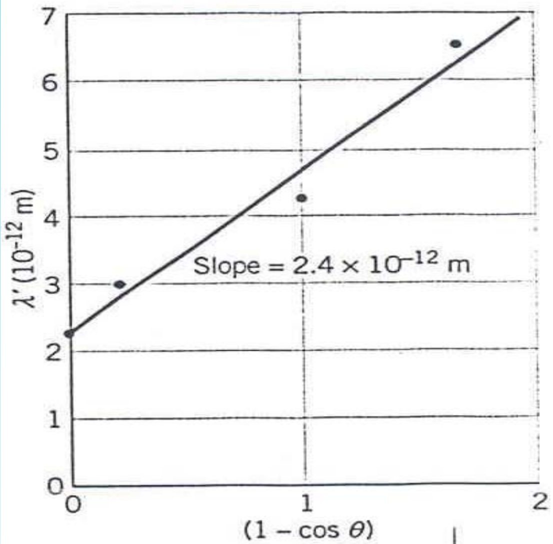


FIGURE 3.22 Compton's results for gamma-ray scattering. Even though the wavelengths are roughly two orders of magnitude smaller than the X-ray wavelengths, the slope is the same as in Figure 3.21, which the Compton formula, Equation 3.44, predicts.

γ-ray

Light is particle-like!

Wave-Particle Duality of Light

" There are therefore now two theories of light, both indispensable, and … without any logical connection."
Einstein 1924

Evidence for wave-nature of light

- Diffraction and interference

Evidence for particle-nature of light

- Photoelectric effect
- Compton effect

We need both to explain what we observe experimentally.

Problem: Is light Particle or Wave?

- Some experiments (Young double slits, etc..) indicate that light is wave.

- Other experiments (Photoelectric effect, Compton scattering, etc..) indicate light is a collection of particles!

→**Both particle and wave nature exist in light !!!**

If so, when is it behaving as particle and when behaving as wave?

Matter Wave: Louis de Broglie (1924)

Prince Louis-Victor de Broglie

We know that quantization of radiation (light) requires:

$$E = h\nu \quad \text{and } p = h\nu/c = h/\lambda \qquad (\lambda\nu = c)$$

Now, de Broglie simply made the **bold** and sweeping **hypothesis** that the same equations should also govern particles with energy *E* and momentum *p* and propose a matter wave with the properties:

$$\nu = E/h \quad \text{and } \lambda = h/p$$

For this, he was awarded the Nobel Prize in 1927. But in 1924, he must first convince his Thesis Committee to grant him his PhD.

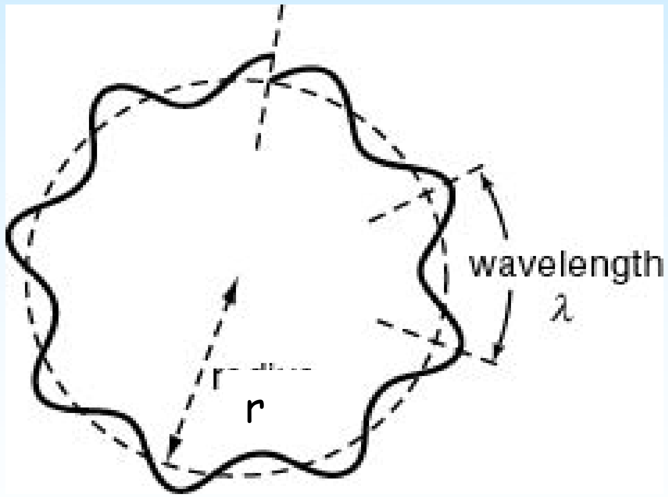
De Broglie's Matter Wave Hypothesis: Why is this such a breakthrough?

- This hypothesis is for **ALL matters** –not just some special quantization rule for some specific situation (as in the Bohr hydrogen atom.) Thus, this can be tested by doing experiments on any material particles.
- With the same two relations, this also places radiation and matter on equal footings ("unification"). The same complementary duality of particle/wave now apply to everything: atoms, protons, electrons, X-rays, light or radio waves.
- How did de Broglie know he got it right?

By requiring the orbit to have integral number of λ, i.e.,

$$2\pi r = n\lambda = nh/p \quad n=1,2,3.... ,$$

Bohr's quantization rule $L = rp = nh/2\pi$ follows!



de Broglie in 1924

- He submitted his PhD Thesis. But the thesis committee members were unsure. One of the examiners (Langevin) asked for an extra copy of the thesis to be sent to Einstein. Einstein wrote " …the younger brother of de Broglie has undertaken a very interesting attempt….I believe it is a first feeble ray of light on this worst of our physics enigmas." So, de Broglie got his PhD.
- If particles were waves, why had not anyone notice before?
- What is this de Broglie wavelength? $\lambda = h/p = h/mv$?

E.g. a baseball (200g) traveling at 30 m/sec, what is its de Broglie wavelength?

$$\lambda_B = \frac{h}{mv} = \frac{6.6 \times 10^{-27}\ gcm^2/\sec}{200\ g\ 3 \times 10^4\ cm/\sec} = 10^{-21}\ cm$$

How small is this compare to separation of atoms in crystals?

De Broglie Wavelength of Moving Objects

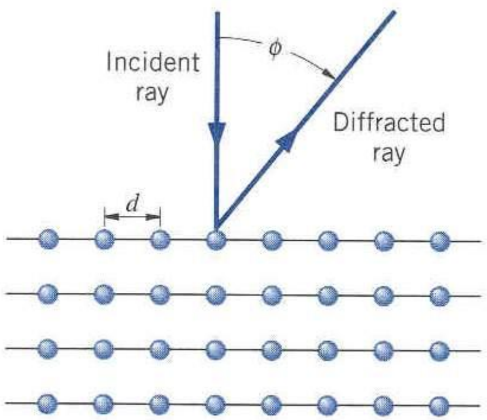
$$\lambda = h/m\mathbf{v}$$

Substance	**Mass (g)**	**Speed, v, (m/s)**	**λ (m)**
slow electron	9×10^{-28}	1.0	7×10^{-4}
fast electron	9×10^{-28}	5.9×10^{6}	1×10^{-10}
alpha particle	6.6×10^{-24}	1.5×10^{7}	7×10^{-15}
one-gram mass	1.0	0.01	7×10^{-29}
baseball	142	25.0	2×10^{-34}
Earth	6.0×10^{27}	3.0×10^{4}	4×10^{-63}

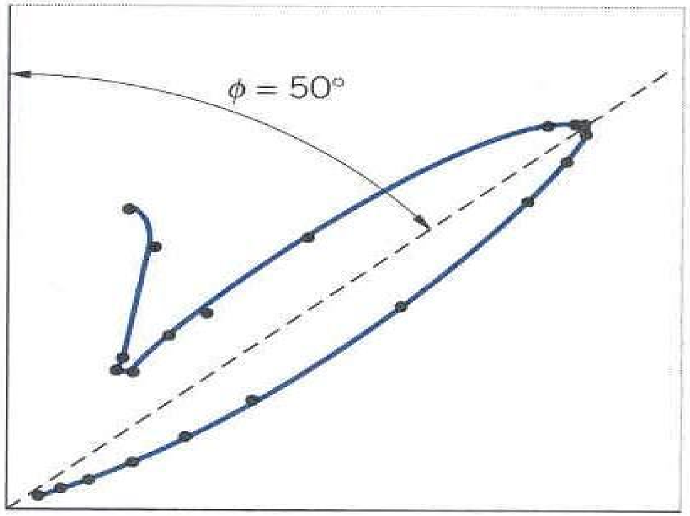
How can we prove matter wave is real?

- The most direct way, as demonstrated by Laue for X-Ray, is by diffraction.
- According to the de Broglie relation, if one uses electrons, to have wavelength comparable to the atomic separation in crystals, the electrons cannot have energy more than 150 eV.
- Several groups in Europe tried but failed to see the diffraction from electrons.
- At Bell Labs, Clinton Davisson and Lester Germer were studying the properties of the cathodes for vacuum tubes …which led to their discovery of the electron diffraction.

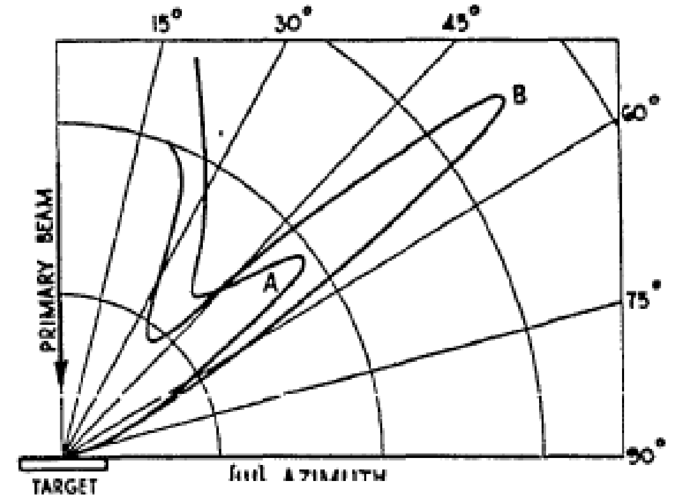
Davisson and Germer (1926)

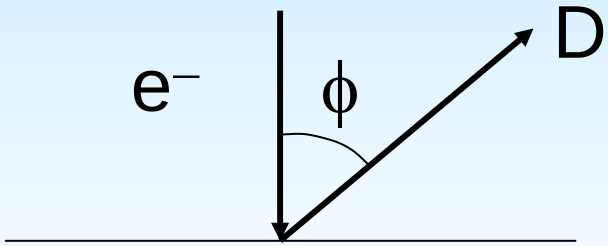


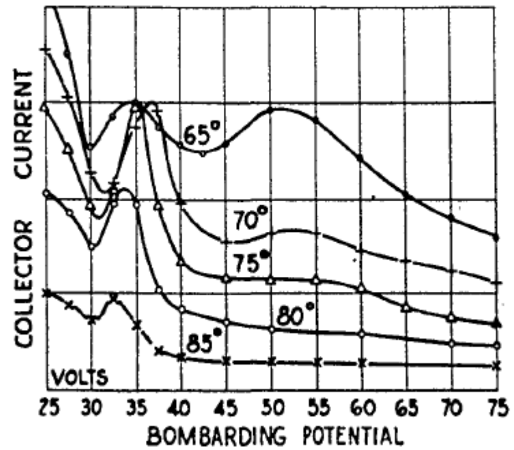
The crystal surface acts like a diffraction grating with spacing d.



Results of Davisson and Germer. Each point on the plot represents the relative intensity when the detector in Figure 4.4 is located at the corresponding angle ϕ measured from the vertical axis. Constructive interference causes the intensity of the reflected beam to reach a maximum at $\phi = 50°$ for $V = 54$ V.







Davisson and Germer's data

$$d \sin\phi = n\lambda = nh/p$$

Electrons and X-rays diffract the same way! That Matter Wave is real!

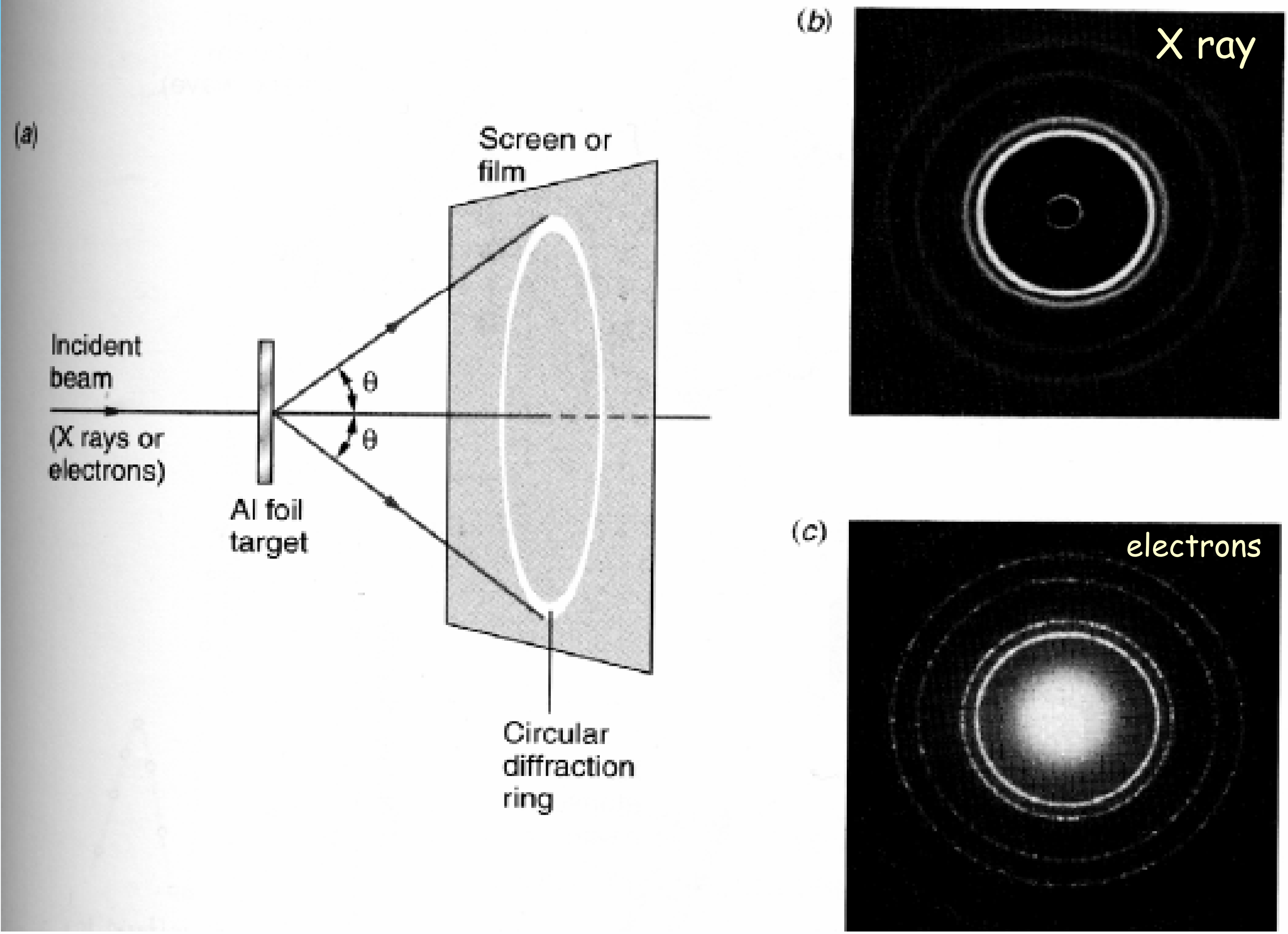
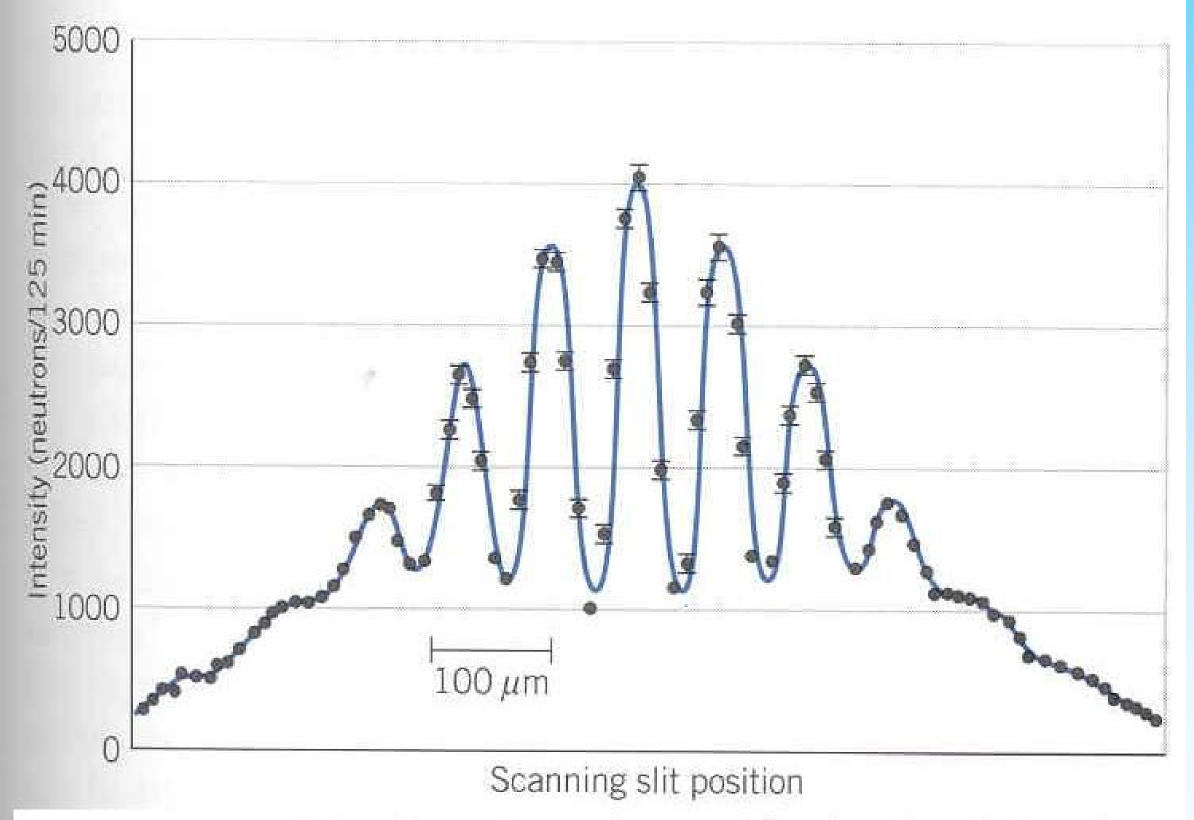


Fig. 5-8 (*a*) Schematic arrangement used for producing a diffraction pattern from a polycrystalline aluminum target. (*b*) Diffraction pattern produced by x rays of wavelength 0.071 nm and an aluminum foil target. (*c*) Diffraction pattern produced by 600-eV electrons (de Broglie wavelength of about 0.05 nm) and an aluminum foil target. The pattern has been enlarged by 1.6 times to facilitate comparison with (*b*). [*Courtesy of Film Studio, Education Development Center.*]

Neutron Diffraction Experiment

Recently interference effect in neutron beam was also observed in a double-slit experiment in 1991.



Intensity pattern observed for double-slit interference with neutrons. The spacing between the maxima is about 75 μm. [*Source*: R. Gähler and A. Zeilinger, *American Journal of Physics* **59**, 316 (1991).]

Particle-Wave Duality: What does it mean?

If your experiment is designed to measure its particle property, it behaves as particle (Compton Scattering);

and if your experiment is designed to measure its wave property, it behaves as wave (Laue Diffraction).

But we never measure BOTH properties at the same time: always one OR the other. (Bohr's Principle of Complementary)

Wave-Particle Duality

Problem:

How should we understand the co-existence of **particle and wave nature (duality) in particles**?

In particular, what does the wave-nature of particle mean for a **particle at rest (p = 0, $\lambda \rightarrow$ infinity) ?**

Note that wave is '**extended**', i.e. spread out over space. However particle is '**localized**'.

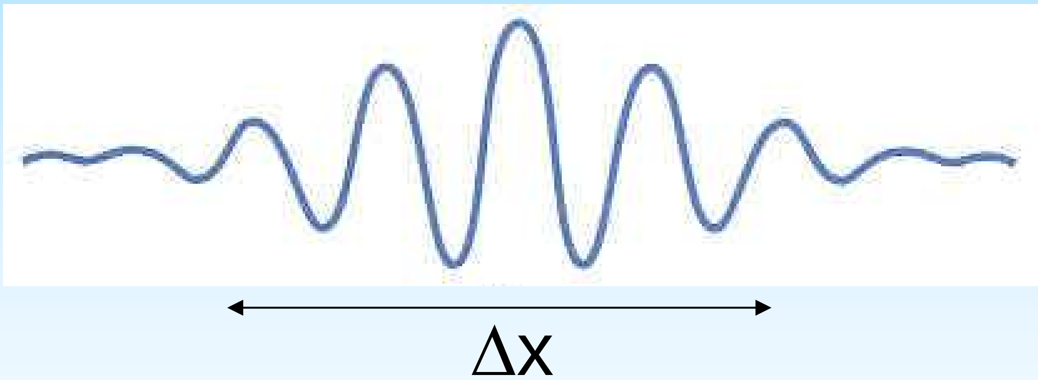
How can one construct a wave that is localized in space?

Wave packet !!!

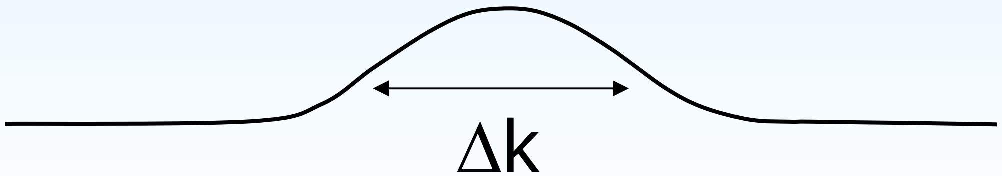
Wave Packet

It can be shown rigorously (**Fourier analysis**) that if we continue to add waves of different wavelengths with properly chosen amplitudes and phases, we can eventually achieve something like **a wave localized in a region of order Δx**, when **waves within a range of wave-vectors $\Delta k \sim (\Delta x)^{-1}$** are added up.

x-space



k-space



Note that $\Delta x \, \Delta k \sim 1$, Uncertainty Principle!

Wave Packet

Now, let's assume that electrons are also described by wave-packets as photons;

Questions:

What is the **physical meaning of the 'size'** of the a wave-packet **Δx** in this case?

For example, what does it mean when 2 electrons are described by wave-packets with, say, different sizes?

Is **Δx = size of the electron**?

Answers:

No, since **electrons are always observed with the same 'size' in any measurement**.

So, what are the other possibilities?

Wave Packet

Recall that in the case of light,

classically: $I \sim E^2$,

photon : $I \sim nh\upsilon$, n: number of photon per unit volume (density)

Combining the two

$|$amplitude of wave$|^2$ ~ density of photon

Born:

Proposed that similar results can be defined for electrons or other 'matter-waves', except that he replaced '**density**' by '**probability**'.

Wave Packet

i.e.

The probability of finding the 'particle' at any point in space is proportional to the absolute square of the amplitude of the corresponding de Broglie wave at the point.

$$\textbf{probability} \sim |\Psi|^2 \sim |A|^2$$

In the wave-packet description, the probability of finding the particle is large in the central region of size Δx where the wave amplitude is large, while it is small outside!

Note that the amplitude of the matter-wave has no physical meaning!

Quantum Mechanics

Now, it is clear that all we need is an equation that enables us to calculate this probability or the wave function $\Psi(x,t)$; i.e. the Schrodinger equation.

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2}+U\Psi(x)=E\Psi(x)$$

The equation can be solved with boundary conditions.
Then $|\Psi(x)|^2$ will give the probability of finding the particle at position x.