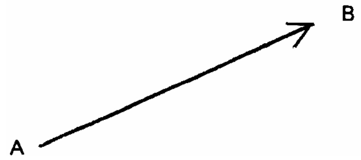
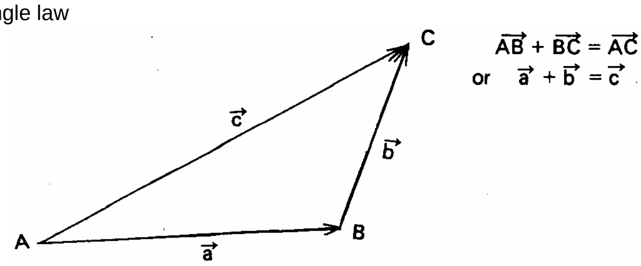
3. SYLLABUS

UNIT 1: Vectors

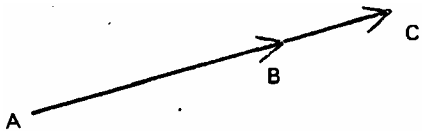
Specific Objectives:

1. To learn the nature of vectors and their basic properties in $\mathbf{R}^2$ and $\mathbf{R}^3$.
2. To be familiar with the basic operations of vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$.
3. To learn the differentiation and integration of vectors with respect to a scalar variable.
4. To apply the vector method to solve problems on the resolution and reduction of a system of forces in $\mathbf{R}^2$ and $\mathbf{R}^3$.
5. To apply the vector method to solve some kinematic problems in $\mathbf{R}^2$.

Detailed Content	Time Ratio	Notes on Teaching
1.1 Basic Knowledge Definition and notation of vectors, magnitude and direction of vectors, equal vectors, parallel vectors and unit vectors.	1	The fundamental concept of vector may have been dealt with in Secondary 5 Physics. Students are able to identify intuitively vectors as physical quantities that possess both magnitude and direction. Teachers should lay emphasis on the difference between scalar and vector quantities. Examples should be given to clarify the concepts. Students are expected to classify physical quantities into vectors (such as displacement, velocity, acceleration, force, impulse etc) and scalars (such as temperature, energy, volume, mass etc). At this stage, it should be emphasized that a vector quantity will change when either its magnitude or direction is changed. (An object travelling in uniform circular motion is a good practical example to illustrate the latter.) It is also essential that students should be acquainted themselves with the representation of a vector geometrically by a directed line segment. 

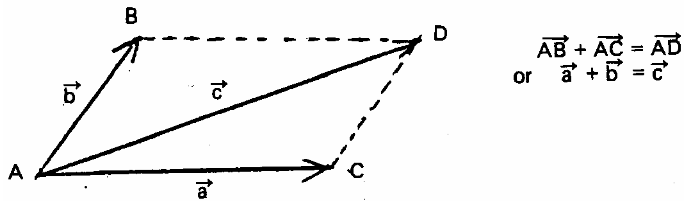
Detailed Content	Time Ratio	Notes on Teaching
		The current notations of vectors (such as $\overrightarrow{AB}$, **AB**, $\vec{a}$, **a**) and their magnitudes (such as $\lvert\overrightarrow{AB}\rvert$, |**AB**|, $\lvert\vec{a}\rvert$, |**a**|) should be introduced. Students are also expected to get the concepts of free vectors (e.g. wind velocity vector) and line-localized vectors (e.g. force vector). With the help of vector diagrams, teachers can guide students to grasp the essential features of equal vectors, parallel vectors and unit vectors. At the same time, teachers should remind students of the difference between equal vectors and parallel vectors. In the former, the vectors must have the same direction and equal magnitude, but in the latter, the vectors may have opposite directions and their magnitudes may not be equal. In case of unit vector, teachers should indicate that since its magnitude is 1, it is usually used to specify direction. Therefore, $\vec{a} = \lvert\vec{a}\rvert\hat{a}$ where $\hat{a}$ is the unit vector in the direction of $\vec{a}$.
1.2 Vector Addition (a) Triangle law and parallelogram law	3	Triangle law  Teachers should remind students that the end-point of the vector $\vec{a}$ must be coincident with the initial point of vector $\vec{b}$. Moreover, it should be noted that, in general, $\lvert\overrightarrow{AB}\rvert + \lvert\overrightarrow{BC}\rvert \neq \lvert\overrightarrow{AC}\rvert$. Teachers should also indicate that if the points A, B and C above are collinear, the triangle law is still valid although the triangle ABC has vanished. (Refer to the figure below.)

Detailed Content	Time Ratio	Notes on Teaching



In this case, $|\overrightarrow{AB}|+|\overrightarrow{BC}|=|\overrightarrow{AC}|$

Parallelogram law



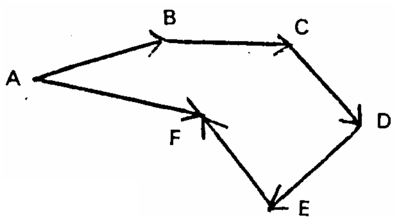
With the help of the above figure, teachers should again remind students that the initial points of the two vectors $\vec{a}$ and $\vec{b}$ must be coincident. The equivalence of the triangle law and the parallelogram law is worth discussing.

In either of the above cases, students should know that $\vec{c}$ is called the resultant of $\vec{a}$ and $\vec{b}$.

It is worthwhile for students to note that the triangle law is convenient for adding free vectors. However, we may apply the parallelogram law to add up line-localized vectors, when the lines of action are taken into account. Actually, in the above figure, the line AD is the line of action of the resultant of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

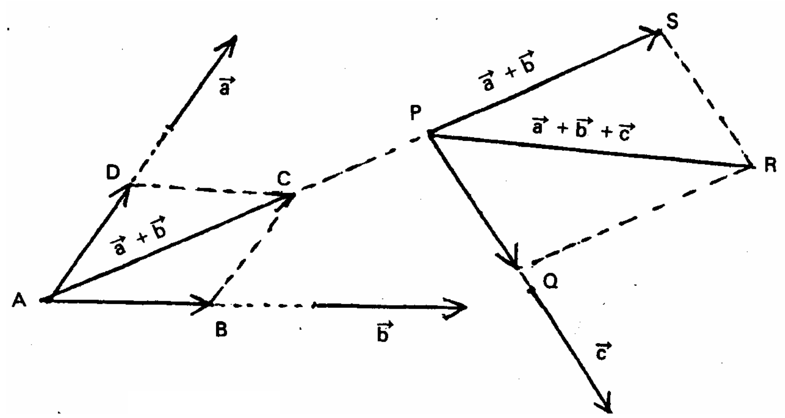
Detailed Content	Time Ratio	Notes on Teaching

Example 1
Addition of free vectors

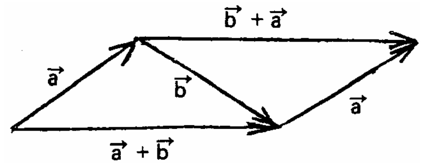
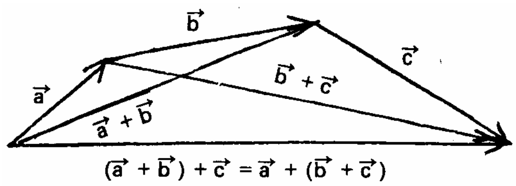


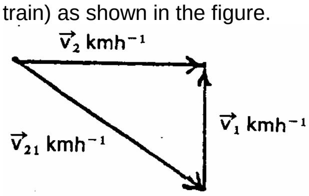
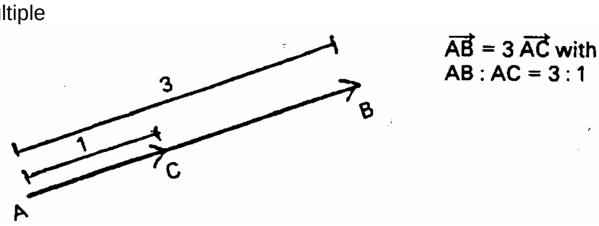
$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EF}=\overrightarrow{AF}$

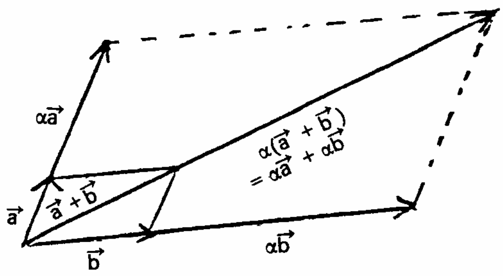
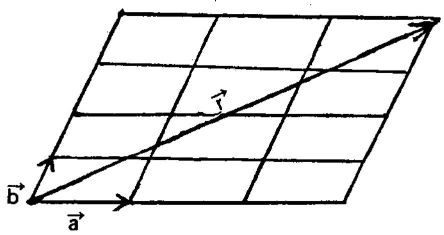
Example 2
Addition of line-localized vectors

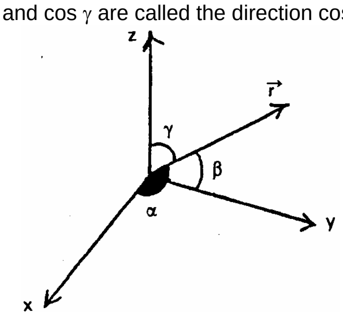


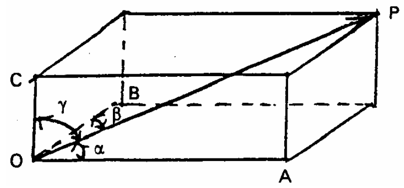
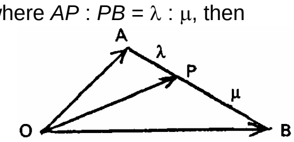
$\vec{a}+\vec{b}+\vec{c}=\overrightarrow{PR}$

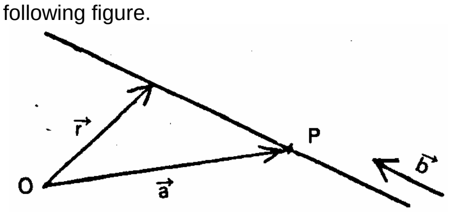
Detailed Content	Time Ratio	Notes on Teaching
		This example shows the addition of 3 coplanar vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. In the figure, $\overrightarrow{AD}=\vec{a}$, $\overrightarrow{AB}=\vec{b}$, $\overrightarrow{AC}=\vec{a}+\vec{b}$, and $\overrightarrow{AC}$ is the line of action of the resultant of $\vec{a}$ and $\vec{b}$. Also, $\overrightarrow{PS}=\vec{a}+\vec{b}$, $\overrightarrow{PQ}=\vec{c}$, $\overrightarrow{PR}=\vec{a}+\vec{b}+\vec{c}$, and $\overrightarrow{PR}$ is the line of action of the resultant of $\vec{a}$, $\vec{b}$ and $\vec{c}$.
(b) Properties of vector addition (i) Commutative law: $\vec{a}+\vec{b}=\vec{b}+\vec{a}$		Teachers may make use of simple vector diagrams to illustrate these properties. Commutative law 
(ii) Associative law: $(\vec{a}+\vec{b})+\vec{c}$ $=\vec{a}+(\vec{b}+\vec{c})$		Associative law of addition 
1.3 Zero Vector, Negative Vector and Vector Subtraction	2	Students should note that any vector of magnitude equals zero is a zero vector, which is denoted by $\vec{0}$. Teachers should emphasize that $\vec{0}$ is different from 0. The former is a vector while the latter is a scalar. Also, students are expected to recognize that a zero vector may assume any direction. At this stage, students should have no problem to deduce the relations $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{AA}=\vec{0}$ and $\vec{a}+\vec{0}=\vec{a}$ for any vector $\vec{a}$.

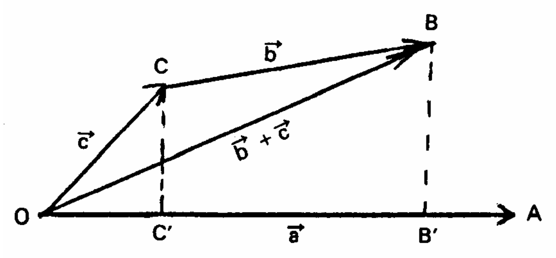
Detailed Content	Time Ratio	Notes on Teaching
		Intuitively, students can see that negative vectors are vectors having equal magnitude but opposite directions. With this concept, the vector subtraction $\vec{a}-\vec{b}$ can be introduced by considering it as the vector sum of the vector $\vec{a}$ and the negative of the vector $\vec{b}$, i.e. $\vec{a}+(-\vec{b})$. The relative velocity is a practical application of the vector subtraction. . *Example* An observer in a train moving at $\vec{v}_1\ \text{kmh}^{-1}$ due north sights a car moving at $\vec{v}_2\ \text{kmh}^{-1}$ due east. Then, the velocity of the car relative to the train, $\vec{v}_{21}\ \text{kmh}^{-1}$, is given by (velocity of car – velocity of train) as shown in the figure.  At this stage, detailed discussion of relative motion is not necessary. It may be left to Section 3.4.
1.4 Scalar Multiple and its Properties *(a)* Associative law $(\alpha\beta)\vec{a}=\alpha(\beta\vec{a})$		Again, teachers may employ simple vector diagrams to illustrate the meaning of scalar multiple and the related laws. The following are two examples. 1. Scalar multiple 

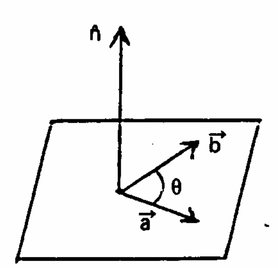
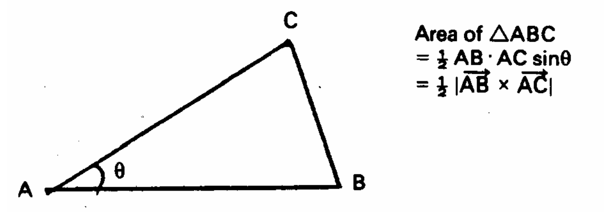
Detailed Content	Time Ratio	Notes on Teaching
(b) Distributive laws $\alpha(\vec{a}+\vec{b}) = \alpha\vec{a}+\alpha\vec{b}$ $(\alpha+\beta)\vec{a} = \alpha\vec{a}+\beta\vec{a}$		2. Distributive law of scalar multiple  After understanding the concept of scalar multiple, students should have no difficulty to deduce the following result. If $\vec{a}=\alpha\vec{b}$, then $\vec{a}$ is parallel to $\vec{b}$ for $\alpha \neq 0$. For α = 0, $\vec{a}=\vec{0}$.
1.5 **Components of Vectors** *(a)* Resolution of vectors	2	The resolution of vectors in $\mathbf{R}^2$ can be introduced with the following example.  In the example, $\vec{r}$ is resolved into two components $3\vec{a}$ and $4\vec{b}$ in the directions of $\vec{a}$ and $\vec{b}$ respectively. This can be generalized to $\vec{r}=\alpha\vec{a}+\beta\vec{b}$ where $\vec{a}$ and $\vec{b}$ are non-collinear vectors in $\mathbf{R}^2$ and $\vec{r}=\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar vectors in $\mathbf{R}^3$, for scalars α, β and γ.

Detailed Content	Time Ratio	Notes on Teaching
(b) The unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ (also denoted as $\hat{i}$, $\hat{j}$ and $\hat{k}$) and the resolution of vectors in the rectangular coordinate system.		The unit vectors in the directions of the positive *x*-, *y*- and *z*-axis are denoted by $\vec{i}$, $\vec{j}$ and $\vec{k}$ respectively. Any vector in $\mathbf{R}^2$ or $\mathbf{R}^3$ can be expressed in the form $\vec{r}=a\vec{i}+b\vec{j}+c\vec{k}$. Students are required to be familiar with the following properties of vectors in terms of $\vec{i}$, $\vec{j}$ and $\vec{k}$: $\lvert a\vec{i}+b\vec{j}+c\vec{k}\rvert=\sqrt{a^2+b^2+c^2}$; $\sum_{r=1}^{n}(x_r\vec{i}+y_r\vec{j}+z_r\vec{k})=\left(\sum_{r=1}^{n}x_r\right)\vec{i}+\left(\sum_{r=1}^{n}y_r\right)\vec{j}+\left(\sum_{r=1}^{n}z_r\right)\vec{k}$; $\lambda(a\vec{i}+b\vec{j}+c\vec{k})=(\lambda a)\vec{i}+(\lambda b)\vec{j}+(\lambda c)\vec{k}$
(c) Direction ratios and direction cosines		Students should be reminded that the two vectors $\vec{r_1}=a_1\vec{i}+b_1\vec{j}+c_1\vec{k}$ and $\vec{r_2}=a_2\vec{i}+b_2\vec{j}+c_2\vec{k}$ are parallel if $\vec{r_1}=\alpha\vec{r_2}$ or $a_1 : b_1 : c_1 = a_2 : b_2 : c_2$. A numerical example can help the teachers easily achieve the purpose. From this, students can be guided to discover that the direction (relative to the axes) of the vector $\vec{r}=a\vec{i}+b\vec{j}+c\vec{k}$ is completely defined by the ratio *a* : *b* : *c* which is called the direction ratios of $\vec{r}$. In the figure below, the angles α, β, γ determine the direction of $\vec{r}$ relative to the axes. cos α, cos β and cos γ are called the direction cosines of $\vec{r}$. 

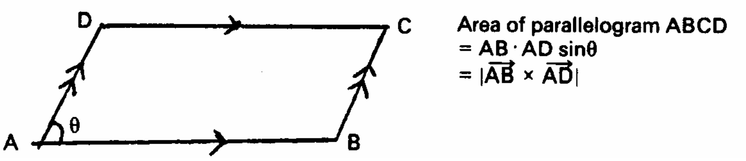
Detailed Content		Time Ratio	Notes on Teaching
			The concept of direction cosines can be clearly illustrated by using a model of rectangular cuboid as shown below.  Students are also expected to deduce the following relations. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ $\dfrac{\vec{r}}{\lvert\vec{r}\rvert}=\cos\alpha\,\vec{i}+\cos\beta\,\vec{j}+\cos\gamma\,\vec{k}$
1.6	**Position Vectors and Vector Equation of a Straight Line**	2	Students are required to represent a point *P* in space by its position vector $\overrightarrow{OP}$ where *O* is the origin of a Cartesian coordinate system. They should note that if *P* is a point on the line segment *AB* where *AP* : *PB* = λ : μ, then $\overrightarrow{OP}=\dfrac{\lambda\overrightarrow{OB}+\mu\overrightarrow{OA}}{\lambda+\mu}$ 

Detailed Content		Time Ratio	Notes on Teaching
			Teachers should lead students to recognize that a straight line can be fully specified when the position of a point on the line and the direction of the line are known. Basing on this idea, students should be able to deduce the vector equation of a line ($\vec{r}=\vec{a}+\lambda\vec{b}$ for a scalar λ) from the following figure.  At this stage, teachers are advised to emphasize to students the meanings of the vectors $\vec{a}$ and $\vec{b}$. (The former represents the position of the given point *P* on the line while the latter the direction of the line.) Once the concepts are clarified, students should have no problem to see that the two lines $\vec{r}_1=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}_2=\vec{a}_2+\lambda\vec{b}_2$ are 1. parallel if $\vec{b}_1$ is parallel to $\vec{b}_2$; 2. perpendicular if $\vec{b}_1$ is perpendicular to $\vec{b}_2$, and the lines intersect each other if there exist λ' and μ' such that $\vec{a}_1+\lambda'\vec{b}_1=\vec{a}_2+\mu'\vec{b}_2$. Also, the fact that the angle between the lines is equal to the angle between $\vec{b}_1$ and $\vec{b}_2$ is obvious.
1.7	**Scalar Product** *(a)* Definition	2	In introducing the definition, teachers should point out to students that the name 'scalar' is used because the product defined in this way gives a scalar quantity. Students are also expected to know the other name for scalar product, i.e. dot product. Hence, $\vec{a}\cdot\vec{b}$ is read as ' $\vec{a}$ dot $\vec{b}$ '.

Detailed Content	Time Ratio	Notes on Teaching
(b) Properties of scalar product		Students are expected to be familiar with the following commutative law and distributive law of scalar product. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ The former can be easily proved from the definition while the latter can be illustrated by using the following figure. 
(c) Scalar product in Cartesian components		Students are expected to verify themselves: $\vec{i}\cdot\vec{i}=\vec{j}\cdot\vec{j}=\vec{k}\cdot\vec{k}=1$ $\vec{i}\cdot\vec{j}=\vec{j}\cdot\vec{k}=\vec{k}\cdot\vec{i}=0$ Afterwards they can be asked to prove themselves that the scalar product of two vectors is given by the sum of the products of their corresponding components, i.e. $\vec{a}\cdot\vec{b}=x_1x_2+y_1y_2+z_1z_2$ where $\vec{a}=x_1\vec{i}+y_1\vec{j}+z_1\vec{k}$ and $\vec{b}=x_2\vec{i}+y_2\vec{j}+z_2\vec{k}$
(d) Orthogonality		At this stage, teachers can ask students what happens to the scalar product of two vectors if they are orthogonal. The following answers are expected. $\vec{a}\cdot\vec{b}=0$ $x_1x_2+y_1y_2+z_1z_2=0$

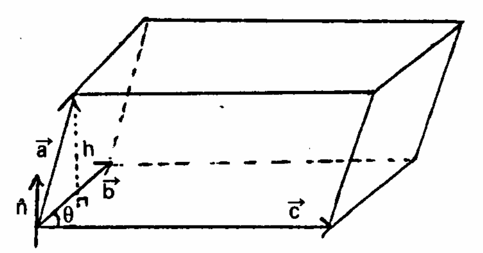
Detailed Content	Time Ratio	Notes on Teaching
		Teachers should provide students with examples involving application of scalar product. For example, in plane geometry, the theorems ‘The perpendiculars from the vertices of a triangle to the opposite sides are concurrent.’ and ‘The perpendicular bisectors of the sides of a triangle are concurrent.’ can be proved by using scalar product.
1.8 Vector Product *(a)* Definition $\vec{a}\times\vec{b}=\vert\vec{a}\vert\vert\vec{b}\vert\sin\theta\hat{n}$	2	In introducing the definition, teachers should emphasize the 'vector' feature of the product which is different from the scalar product introduced in Section 1.7. The other name for vector product, cross product, is also introduced and $\vec{a}\times\vec{b}$ is read as ‘a cross b’. The right-handed system used for the determination of the product direction (i.e. in the direction of the unit vector $\hat{n}$ in the definition) should be clearly explained. The figure shown will be helpful.  Simple applications of vector product can be introduced to arouse students’ interest. The following are two examples. 1 . *Area of triangle* 

Detailed Content	Time Ratio	Notes on Teaching
		2. *Area of Parallelogram*



Detailed Content	Time Ratio	Notes on Teaching
(b) Properties of vector product		Students are expected to know the following properties. $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$ $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ (distributive property) Formal proofs of these may be omitted.
(c) Vector product in Cartesian components		Students should be able to see that $\vec{a}\times\vec{b} = (y_1z_2 - y_2z_1)\vec{i} + (x_2z_1 - x_1z_2)\vec{j} + (x_1y_2 - x_2y_1)\vec{k}$ where $\vec{a} = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$ and $\vec{b} = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$ The determinant expression of vector product, i.e. $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}\vec{i} - \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix}\vec{j} + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}\vec{k}$ Where $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ only serves for simplicity and its introduction is optional.
(d) Perpendicular vectors and parallel vectors		Teachers should guide students to deduce the following results. 1. $\vec{a}$, $\vec{b}$ are perpendicular if $\lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{a}\rvert\times\lvert\vec{b}\rvert$ 2. $\vec{a}$, $\vec{b}$ are parallel if $\lvert\vec{a}\times\vec{b}\rvert = 0$

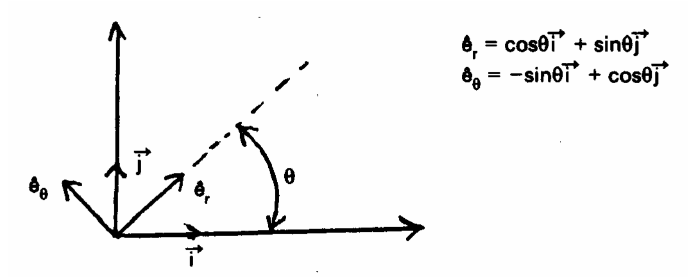
Detailed Content	Time Ratio	Notes on Teaching
1.9 Triple Product *(a)* Scalar triple product	2	By considering the volume of a parallelepiped (i.e. $bc\sin\theta h$), teachers can introduce the scalar triple product $\vec{a}\cdot(\vec{a}\times\vec{b})$ (or simply $\vec{a}\cdot\vec{b}\times\vec{c}$). However, students should note that the volume of a parallelepiped is actually given by $\lvert\vec{a}\cdot\vec{b}\times\vec{c}\rvert$.



Detailed Content	Time Ratio	Notes on Teaching
		The same approach can be used to show that each of the products $\vec{b}\cdot\vec{c}\times\vec{a}$ and $\vec{c}\cdot\vec{a}\times\vec{b}$ has the same value as $\vec{a}\cdot\vec{b}\times\vec{c}$. Also, by using the commutative property, students should have no problem to see that $\vec{a}\cdot\vec{b}\times\vec{c} = \vec{a}\times\vec{b}\cdot\vec{c}$. Students should also know that the condition for 3 vectors to be coplanar is $\vec{a}\cdot\vec{b}\times\vec{c} = 0$. For students who have learnt determinant, the following formula may also be introduced. $\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$
(b) Vector triple product		Teachers should emphasize that the brackets in the vector triple product like $\vec{a}\times(\vec{b}\times\vec{c})$ are essential to determine which product is taken first. In order to show that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$ and $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$, teachers are advised to choose appropriate Cartesian axes (by rotation if necessary) so that $\vec{a}$, $\vec{b}$ and $\vec{c}$ can be expressed in the forms:

Detailed Content	Time Ratio	Notes on Teaching
		$\vec{a}=a_1\vec{i}$, $\vec{b}=b_1\vec{i}+b_2\vec{j}$, $\vec{c}=c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$ From the above results, students should be able to find that $\vec{a}\times(\vec{b}\times\vec{c})\neq(\vec{a}\times\vec{b})\times\vec{c}$.
1.10 Vector Function, Differentiation and Integration	2	
(a) Vector as a function of a scalar variable		Students are expected to be familiar with notations like $\vec{r}(t)$, $\vec{v}(\theta)$ etc, where $\vec{r}$ and $\vec{v}$ are vector functions of the scalar variables *t* and θ respectively.
(b) Differentiation of a vector function with respect to a scalar variable		Students should be able to differentiate vector functions in component form, i.e. when $\vec{r}(t)=f(t)\vec{i}+g(t)\vec{j}+h(t)\vec{k}$ $\frac{\mathrm{d}}{\mathrm{d}t}\left[\vec{r}(t)\right]=f'(t)\vec{i}+g'(t)\vec{j}+h'(t)\vec{k}$ They are also expected to be familiar with the differentiation of scalar multiples, scalar products and vector products: $\frac{\mathrm{d}}{\mathrm{d}t}[\lambda\vec{a}]=\frac{\mathrm{d}\lambda}{\mathrm{d}t}\vec{a}+\lambda\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}$ $\frac{\mathrm{d}}{\mathrm{d}t}[\vec{a}\cdot\vec{b}]=\frac{\mathrm{d}\vec{a}}{\mathrm{d}t}\vec{b}+\vec{a}\frac{\mathrm{d}\vec{b}}{\mathrm{d}t}$ $\frac{d}{dt}[\vec{a}\times\vec{b}]=\frac{d\vec{a}}{dt}\times\vec{b}+\vec{a}\times\frac{d\vec{b}}{dt}$
(c) Integration of a vector function with respect to a scalar variable		Teachers should emphasize to students that integration of a vector function is the reverse process of differentiation. In this way, students should have no problem to carry out integration like $\int\vec{r}(t)\ \mathrm{d}t=\int f(t)\ \mathrm{d}t\ \vec{i}+\int g(t)\ \mathrm{d}t\ \vec{j}+\int h(t)\ \mathrm{d}t\ \vec{k}+\vec{c}$ where $\vec{r}(t)=f(t)\vec{i}+g(t)\vec{j}+h(t)\vec{k}$ and $\vec{c}$ is a constant vector.

Detailed Content	Time Ratio	Notes on Teaching
1.11 Vectors in Polar Coordinates	2	Knowledge of the radial and transverse components of a vector in polar coordinates is introduced. The radial and transverse unit vectors, $\hat{e}_r$ and $\hat{e}_\theta$ are defined and expressed in Cartesian form as shown below.



When $\hat{e}_r$ and $\hat{e}_\theta$ are vector functions of the time *t*, the above expressions can then be differentiated with respect to *t* to arrive at the following results.

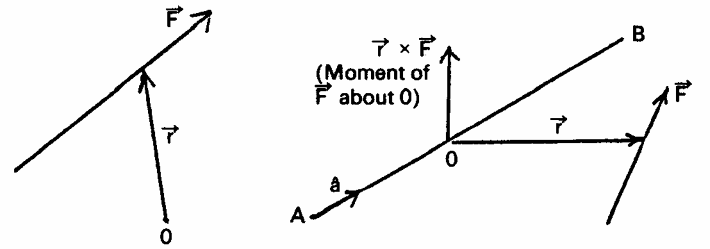
$$\frac{\mathrm{d}\hat{e}_r}{\mathrm{d}t}=\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{e}_\theta$$

$$\frac{\mathrm{d}\hat{e}_\theta}{\mathrm{d}t}=\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{e}_r$$

Detailed discussion of the position, velocity and acceleration vectors presented in polar coordinates may be left to Section 3.5. However, it is worthwhile, at this stage, for teachers to discuss with students the distinction of employing polar coordinates and Cartesian coordinates in solving problems such as the one shown below.

Example
The position of a particle moving in a plane is given by polar coordinates (r, θ). At time *t*, $\theta=\omega t$ where ω is a constant. The locus of the particle is determined by the polar equation $r=ae^{\theta}$ where *a* is a constant.

Detailed Content		Time Ratio	Notes on Teaching
1.12	**Application of Vectors** *(a)* Force as a vector	6	Students are expected to develop their skills in tackling problems related to vectors and their applications. Students are going to deal with forces in vector form. They should know how to find the resultant force of a system of forces. Knowledge of vector addition mentioned in Section 1.2 is recalled. The moment of a force in vector form about a point and about a line in $\mathbf{R}^3$ are introduced.



Moment of $\vec{F}$ about 0 $=\vec{r}\times\vec{F}$

Moment of $\vec{F}$ about line *AB* $=\left(\vec{r}\times\vec{F}\cdot\hat{a}\right)\hat{a}$

By considering the total moment of a system of Coplanar forces about a point or about a line in $\mathbf{R}^3$, students are able to identify the line of action of the resultant force of the system of forces in $\mathbf{R}^3$. The following are two examples.

Example 1
(*a*, *b*, 0). (0, *b*, *c*) and (*a*, 0, *c*) are the Cartesian coordinates of the vertices A, B and C respectively of a triangle. Forces of magnitude and direction equal to $\overrightarrow{BC}$, $\overrightarrow{AC}$ and $3\overrightarrow{BA}$ are set along the sides of the triangle. .

In this example, students may first be led to express the forces in vector form. After that they should be able to find the resultant of the forces by simple vector addition. Finally, by comparing the total moments of the forces about the origin and the moment of the resultant force about the origin, students may be asked to work out the line of action of the resultant force.

Detailed Content	Time Ratio	Notes on Teaching
		Example 2 Two forces, $\vec{F_1}=-\vec{i}+\vec{j}-\vec{k}$ and $\vec{F_2}=2\vec{i}+3\vec{j}$ act through points with position vectors $\vec{r_1}=\vec{i}+\vec{j}+\vec{k}$ and $\vec{r_2}=-\vec{i}-2\vec{j}+\vec{k}$ respectively. Find the force $\vec{F_3}$ needed to bring the system to equilibrium and the vector equation of its line of action. Nevertheless, complex problems involving forces and moments may be left to Unit 2.
(b) Kinematics in R^2		Problems should be introduced to investigate the relative motion of two bodies through vector approach. Teachers should ensure that students have acquired adequate knowledge of physical situations based on which students are capable of presenting displacement velocity and acceleration in vector form. Angular displacement, angular velocity and angular acceleration may also be involved in the kinematic problems. Students are also expected to employ the knowledge learnt in Section 1.10, i.e. the differentiation and integration of a vector function with respect to a scalar variable to tackle the problems. However, in-depth study of the topics may be left to Unit 3.
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