UNIT 3: Kinematics

Specific Objectives:

1. To understand the meaning of displacement, velocity and acceleration, and their corresponding angular quantities.
2. To understand resultant velocity and relative motion.
3. To recognize the radial and transverse component of velocity and acceleration.
4. To solve relevant practical problems.

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<tr><td>3.1 Displacement, Velocity and Acceleration</td><td>2</td><td>
Teachers should revise with students the meaning of displacement, velocity and acceleration. Nevertheless, teachers may try the approach which makes use of the knowledge of vector and calculus. For simplicity, teachers may restrict the motion along the x-axis and take the positive direction to be that of increasing x. In this way, students should have no difficulty to obtain the formulae

$$v=\frac{dx}{dt}=\dot{x}\ ,\quad a=\frac{dv}{dt}=\dot{v}=\frac{d^2x}{dt^2}=\ddot{x}\quad \text{or}\quad a=\frac{dv}{dt}=v\frac{dv}{dx}\ .$$

Teachers should remind students of the physical meanings when x, v and a are negative.

For constant acceleration, students should have no problem to derive the following formulae:

$$v=u+at$$

$$x=ut+\frac{1}{2}at^2$$

$$v^2=u^2+2ax$$

The motion of a particle in two dimensions should then be introduced. Teachers should remind students that by taking the components of the displacement, velocity and acceleration of the particle parallel to the x-axis and y-axis, the methods for motion in a straight line in each of these directions can be used. Denoting the displacement of the particle at a point by $\vec{r}=x(t)\vec{i}+y(t)\vec{j}$, students could be led to discover.

$$\vec{v}=\frac{d\vec{r}}{dt}=\dot{x}\vec{i}+\dot{y}\vec{j}$$

and $$a=\frac{dv}{dt}=\frac{d^2r}{dt^2}=\ddot{x}\vec{i}+\ddot{y}\vec{j}$$

from which the magnitude and direction could be easily derived.
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At this stage, the use of parametric equations for the motion of a particle whose path is described by an equation in the rectangular coordinates should be emphasized. Students are expected to use the techniques and knowledge of Section 1.10 in the solution of problems. Some of which are as follows.

Example 1

The velocity vector $\vec{v}$ (in ms^{-1}) of a particle P starting from a point 0 is given by $\vec{v}=80\vec{i}+(50+10t^2)\vec{j}$. The position vector relative to 0 and the acceleration vector of P at time t can be easily obtained by integrating and differentiating $\vec{v}$ respectively.

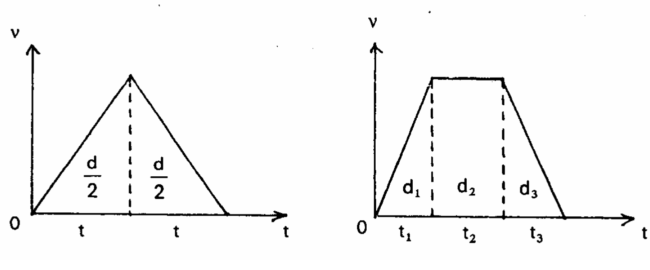
Example 2

The distance between 2 stations is d. A train starts from one station and stops at the next within a time t. If the maximum acceleration or deceleration is a and its highest speed cannot exceed v, show that the least possible value of t is

(a) $t=\frac{d}{v}+\frac{v}{a}$ if $d\geq\frac{v^2}{a}$

(b) $t=2\sqrt{\frac{d}{a}}$ if $d<\frac{v^2}{a}$

In this example, teachers may guide students to tackle the problem geometrically.



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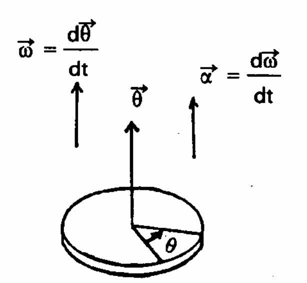
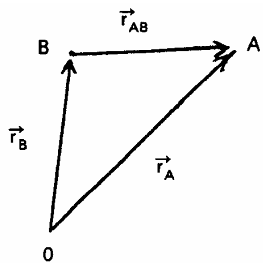
Detailed Content	Time Ratio	Notes on Teaching
3.2 Angular Displacement. Angular Velocity and Angular Acceleration	1	Teachers should briefly introduce the concepts of angular displacement, angular velocity and angular acceleration. Students are also expected to know the following three relations between the linear and angular quantities. $s = r\theta$ $\dot{s} = r\dot{\theta}$ (or $v = r\omega$) and $\ddot{s} = r\ddot{\theta}$ (or $a = r\alpha$)  The vectorial representation of angular motion may be illustrated with diagrams like that shown in the right. Students are expected to know that the right-hand rule is used to establish the positive sense, and as long as rotation is confined to a single plane, the rotation vectors $\vec{\theta}$, $\vec{\omega}$ and $\vec{\alpha}$ will be parallel to each other and can be considered as scalar quantities as the algebraic sign is sufficient to account for either sense of the vectors. *Example* A disc rotates about its axle with an acceleration given by $\ddot{\theta} = 2t$. Find its angular velocity and angular displacement at t = 3s if the initial conditions are θ = 0 rad and θ' = 0 rad/s. In this example, students are expected to obtain the results by integration.
3.3 Resultant Velocity	3	The emphasis here is on finding the resultant velocity of a particle. Students should be reminded that a triangle (or polygon) of velocities or component method could be used to get the result. Examples like a boat rowing straight across a flowing river, raindrops falling through a current of air etc. should be provided. In all these cases, students are expected to draw vector diagrams for finding the resultant velocities. Cases in which velocities are not perpendicular are also expected. The following shows an example.

Detailed Content	Time Ratio	Notes on Teaching
		Example A cargo ship leaves a port and heads N50°E at a speed of 25 kmh^{-1} with respect to still water, while a westward sea current drifts at 4.5 kmh^{-1}. What is the resultant velocity of the cargo ship? In this example, apart from resolving the velocities in the north and west direction, students could also find the resultant velocity by using the sine rule and cosine rule.
3.4 Relative Motion	8	Teachers should revise with students the concept of position vector. The idea of relative motion can be introduced by vector approach. Referring to the figure, the position vector of A relative to B is $\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$ so that $\dot{\vec{r}}_{AB} = \dot{\vec{r}}_A - \dot{\vec{r}}_B$ i.e. the velocity of A relative to B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$  It is often convenient to introduce the idea of relative velocity of A to B by 'bringing B to rest' by adding the velocity $-\vec{v}_B$ to both A and B, so that the velocity of A relative to B is $\vec{v}_A - \vec{v}_B$. Relative velocity problems can be solved by using knowledge of trigonometry and vector. Problems involving the interception and the shortest distance between 2 objects are typical examples. *Example 1* A ship is moving due south at 50 kmh^{-1} and from it a second ship B appears to be moving SW at 75 kmh^{-1}. Calculate the velocity of B.

Detailed Content	Time Ratio	Notes on Teaching

In this example teachers can guide students to draw the vector triangle of velocities and use cosine rules and sine rule to find the velocity of B.

Example 2

At noon, a ship S, at the origin is streaming with a velocity vector $10\vec{j}$. Meanwhile, a second ship S_2 which has a position vector $\vec{r} = -10\vec{i} - 10\vec{j}$ is streaming with a velocity vector $20\vec{i} + 25\vec{j}$.

In this example, students are guided to write the vector equations in time *t* of the paths of one ship relative to the other. After that, the position vectors of the two ships at closest approach and the distance of closest approach were investigated.

Example 3

A satellite is falling with constant speed u kmh^{-1} on an east-west path inclined at a fixed angle θ to the horizontal. A ship travelling due west with constant speed V kmh^{-1} sights, from a point 0, the satellite at a height of h km and a horizontal distance d km on a bearing NE from 0.

(a) Write the expressions for

- (i) the position vector of the satellite;
- (ii) the velocity of satellite relative to 0.

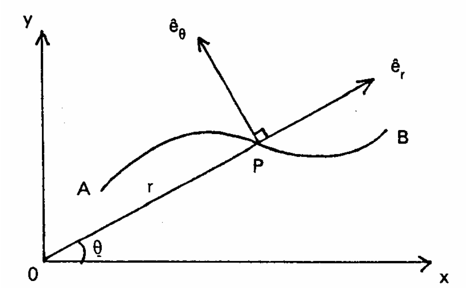
(b) Find the shortest distance between the ship and the satellite.

This problem may help students integrate what they have learnt in vectors and the concept of relative velocity.

Finally, teachers should also mention to students that relative acceleration of A to B can be defined in a way similar to that of relative velocity, i.e. $\overrightarrow{a_{AB}} = \overrightarrow{a_A} - \overrightarrow{a_B}$.

3.5 Resolution of Velocity and Acceleration Along and Perpendicular to Radius Vector	4	Teachers can introduce the radial and transverse components of velocity and acceleration by considering the motion of a particle in a plane using polar coordinates.

Detailed Content	Time Ratio	Notes on Teaching



Suppose P is the position of a particle at time *t*, APB is the path of the particle and the polar coordinates of P are (r, θ).

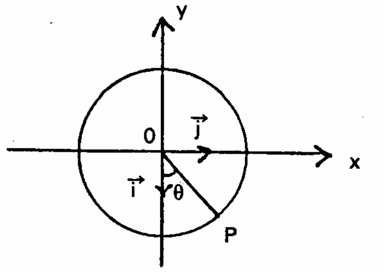
Then $\overrightarrow{OP} = \vec{r} = r\hat{e}$ where $\hat{e}_r$ and $\hat{e}_\theta$ are the unit vectors in the directions parallel and perpendicular to $\vec{r}$ respectively.

In Section 1.11, students have learnt that $\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta$ and $\dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r$. By direct differentiation, students should have no problem to get the following results

$\dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ and $\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta})\hat{e}_\theta$

Tabulating the results will facilitate students in recognizing the expression

	Radial Component	Transverse component
Velocity	$\ddot{r}$	$r\dot{\theta}$
Acceleration	$\ddot{r} - r\dot{\theta}^2$	$r\ddot{\theta} + 2\dot{r}\dot{\theta}$ or $\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta})$

Detailed Content	Time Ratio	Notes on Teaching
2.5 Equilibrium of Rigid Bodies	12	*Example 1* A particle of unit mass moves in a plane such that its polar coordinates at any point are given by $$r(t)=1+t \quad \text{and} \quad \theta(t)=\frac{\pi}{1+t}$$ where $t \geq 0$. Find the velocity, the radial and transverse component of the acceleration at time t = 1. *Example 2* A point P describes a circle of centre O and radius a as shown. Its position vector r> = OP at time t is given by $\vec{r}=a\hat{e}_r$ where $\hat{e}_r=\cos\theta\vec{i}+\sin\theta\vec{j}$ If $\hat{e}_\theta=-\sin\theta\vec{i}+\cos\theta\vec{j}$ Show that $\dot{\vec{r}}=a\dot{\theta}\hat{e}_\theta$ and $\ddot{\vec{r}}=a\ddot{\theta}\hat{e}_\theta-a\dot{\theta}^2\hat{e}_r$  Although polar coordinates are often appropriate for the solution of dynamical problems associated with central orbits, detailed knowledge of orbit problems is not necessary.
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