UNIT 5: Momentum, Work, Energy, Power and Conservation Laws

Specific Objectives:

1. To recognize momentum, work, energy and power.

2. To understand and use the Conservation Laws of Momentum and Energy.

Detailed Content	Time Ratio	Notes on Teaching
5.1 Momentum and Conservation of Momentum	2	Newton's second law may take the form $$\vec{F} = \frac{\mathrm{d}(m\vec{v})}{\mathrm{d}t} = \frac{\mathrm{d}\vec{P}}{\mathrm{d}t}$$ where $\vec{P} = m\vec{v}$ is the momentum of the particle. In the absence of external force, the momentum $\vec{P} = m\vec{v}$ of the particle should remain constant. The conservation of momentum may be extended to a system of particles. Examples such as recoil of gun, collision of trucks (assuming that they couple together after impact) etc. should be provided to illustrate how the law of conservation of momentum can be used. However, detailed discussion of impact problems is not necessary here and can be left to Unit 6.
5.2 Work, Energy, Power and Conservation of Energy	3	Teachers should remind students of the fundamental concepts of work, energy and power. Students are expected to know that: 1. the work done by *F* on moving a particle from *a* to *b* along the positive direction of the x-axis is $\int_a^b F \, \mathrm{d}x$, 2. the kinetic energy of a particle of mass *m* moving with velocity *v* is $\frac{1}{2}mv^2$. 3. the gravitational potential energy of a particle of mass *m* at a height *h* above an arbitrary origin is *mgh*, and 4. the power of *F* is the rate of work done by *F*. After that, teachers may introduce the relation between work and energy for some mechanical systems. The following show two of them. 1. A particle moves along' a horizontal straight line under the action of a force *F*. The increase in K.E. of the particle is equal to the work done to the particle by *F*.

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		2. When an elastic string is being extended, the work done to the string is equal to the potential energy stored in the string. Students are expected to know that the P. E. stored in an extended string is $\frac{\lambda x^2}{2\ell}$ where λ is modulus, ℓ is the natural length and *x* is the extension. Finally, teachers should emphasize how to use the conservation of energy to solve mechanical problems, i.e. $$\begin{pmatrix}\text{Work done}\\ \text{to the}\\ \text{system}\end{pmatrix} = \begin{pmatrix}\text{change}\\ \text{in}\\ \text{K.E.}\end{pmatrix} + \begin{pmatrix}\text{change}\\ \text{in}\\ \text{P.E.}\end{pmatrix} + \begin{pmatrix}\text{Work done}\\ \text{against}\\ \text{friction}\end{pmatrix}$$ At this stage, students should have no problem to see that if there is no friction and no work done to the system, then the above expression can be reduced to $$\begin{pmatrix}\text{increase}\\ \text{in K.E.}\end{pmatrix} = \begin{pmatrix}\text{decrease}\\ \text{in P.E.}\end{pmatrix} \quad \text{or} \quad \begin{pmatrix}\text{decrease}\\ \text{in P.E.}\end{pmatrix} = \begin{pmatrix}\text{increase}\\ \text{in K.E.}\end{pmatrix}$$ *Example 1* A car of mass 1000 kg climbs a hill at a constant speed of 10 ms^{-1}. The inclination of the hill is 1 in 10. (a) find the work done by the car against gravitation in one minute. (b) If the total work done by the car in this time is 9×10 J, find the resistance to motion. In this example, students should be able to see that the work done by the car against gravitation in one minute is equal to the P.E. gained by the car in the same time interval. Then, by seeing that there is no K.E. change of the car, students should be able to use the conservation law of energy to find the work done against friction and hence the resistance to motion. *Example 2* The figure shows a smooth wire XYZ in a vertical plane. The straight portion YX and YZ of the wire are at right angles and YX is horizontal. Smooth rings A and B each of mass

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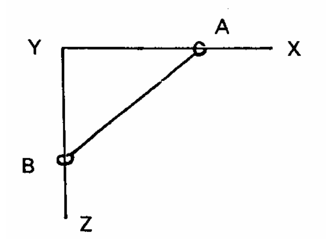
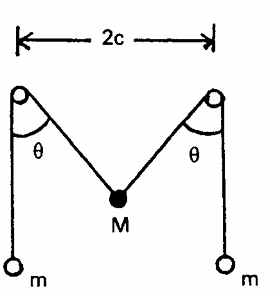
m are threaded onto YX and YZ respectively and connected by a tight inextensible string of length 5ℓ . The system is released from rest with A at a distance 4ℓ from Y. Calculate the speed B when A is at a distance 3ℓ from Y.



In this example, students may be guided to find the relation of the velocities of A and B at time t after the release, and then solve the problem by using the conservation law of energy.

Example 3

In the figure, the two pulleys are smooth and the system is released from rest so that M falls to a position in which the strings are inclined to the vertical at an angle ϕ.



Show that there is a loss of potential energy of amount

$Mgc(\cot\phi - \cot\theta) - 2mgc(\csc\phi - \csc\theta)$

In this example, students are expected to realize that the potential energy alone is not conserved in general. From the above result, students may be asked to deduce that if there is an equilibrium when the strings are each inclined to the vertical at an angle α, and the

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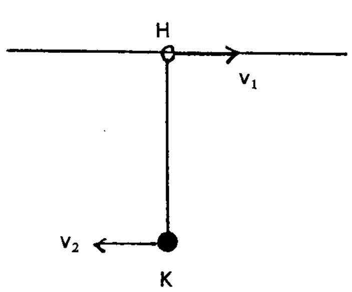
system is released from a position in which the strings are each inclined to the vertical at an angle θ, it will next come to instantaneous rest when the inclination is ϕ where

$$\tan^2\frac{1}{2}\alpha = \tan\frac{1}{2}\theta\tan\frac{1}{2}\phi \ .$$

Example 4

A light rod HK of length a connects a smooth ring H of mass m_1 to a particle K of mass m_2. The ring is threaded onto a thin smooth horizontal wire. The rod HK is held in a horizontal position and released from rest. Find the velocities of the particles at the instant when the rod HK becomes vertical.

Students should realize that at the instant when the rod becomes vertical, K is moving horizontally. The result follows immediately from the law of conservation of momentum and the law of conservation of energy.



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