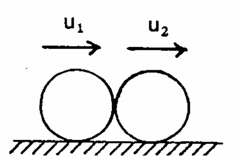
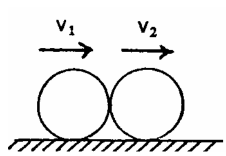
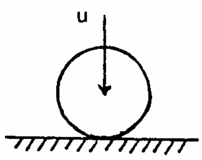
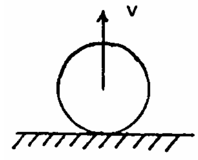
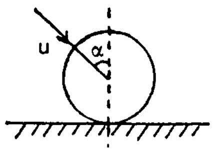
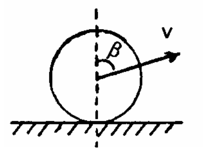
UNIT 6: Impact

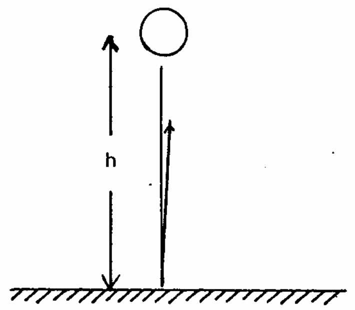
Specific Objectives:

1. To distinguish between elastic and inelastic impacts.
2. To understand Newton's Law of Restitution.
3. To apply Newton's Law of Restitution to solve problems of direct and oblique impacts.

Detailed Content	Time Ratio	Notes on Teaching
6.1 Impulse	1	The impulse of a force may be defined as the product of the force and the time t for which it acts. With this definition and starting from Newton's Second Law of Motion, it is not hard to arrive at the relationship Ft = mv − mu. Thus, students should have no problem to see that the impulse of a force is equal to the change in momentum which it produces. Students are also expected to realize the meaning of impulsive force. Examples include the blow of a hammer, the impact of water on a surface, the impact of a bullet on a target, the collision of balls etc. Teachers should revise with students the Principle of Conservation of Linear Momentum and remind them this principle is usually used in dealing with problems in which impacts or impulsive forces occur. The above-mentioned examples can be used for illustration, but problems involving impulsive tensions are not necessary.
6.2 Impact of Elastic Bodies	1	Teachers should explain clearly the meaning of direct impact and oblique impact, but the manipulation of relevant problems is not necessary here and should be left to Section 6.3, 6.4 and 6.5.  Before impact  After impact

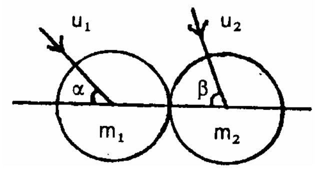
Detailed Content	Time Ratio	Notes on Teaching
		Newton's experimental law (i.e. $\frac{v_1 - v_2}{u_1 - u_2} = -e$) should be introduced at this stage. The positive constant *e* is known as coefficient of restitution or coefficient of elasticity. Teachers should remind students of the negative sign adhering to *e* in the law. (If the law is introduced as $\frac{v_2 - v_1}{u_1 - u_2} = e$, then teachers should remind students of the sequences of subtraction in the numerator and the denominator.) The different values of *e* for different bodies should be discussed. In particular, bodies with *e* = 0 are said to be perfectly inelastic while those with *e* = 1 are said to be perfectly elastic. For other elastic bodies, 0 < *e* < 1. Students are expected to know in perfectly elastic impact, kinetic energies are conserved while in perfectly inelastic impact, the two bodies after impact will adhere and move with a common velocity. The imperfectly elastic impact is in between the two extreme cases.
6.3 Direct Impact	4	Teachers should emphasize that unless the impact is perfectly elastic, kinetic energy is not conserved. This fact can further be verified by guiding students to develop the expression: $$\begin{pmatrix}\text{Loss in kinetic energy}\\\text{due to direct impact}\end{pmatrix} = \frac{1}{2}\cdot\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2(1 - e^2)$$ Clearly, the loss is zero if *e* = 1. Teachers should not encourage students to memorize the expression. Instead, they should encourage students to derive it when necessary. As a matter of fact, in most numerical cases, it is not hard to find directly the velocities of the bodies after impact. The loss can then be obtained easily by subtracting the kinetic energy after impact from that before. Various types of examples should be provided to acquaint students with the technique. Typical examples including finding the velocities after impact, the kinetic energy loss due to impact, the momentum transferred from one sphere to the other after impact etc.

Detailed Content		Time Ratio	Notes on Teaching
6.4	**Impact of a Smooth Sphere on a Smooth Surface**	3	Teachers should remind students that Newton's experimental law is still valid. In this case, students should have no difficulty to get the formula $\frac{v}{u} = -e$ where u and v are the velocity of the sphere just before and after impact respectively (see figure).  Before impact  After impact If the impact is not normal to the plane as shown in the figure below,  Before impact  After impact the horizontal and vertical component of the motion of the sphere should be considered separately. Newton's experimental law is then reduced to $\frac{v\cos\beta}{u\cos\alpha} = e$. Teachers should emphasize that if the sphere and the plane are both smooth, then the horizontal component of the velocity of the sphere remains unchanged after impact, i.e. $u\sin\alpha = v\sin\beta$

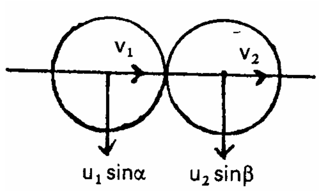
Detailed Content		Time Ratio	Notes on Teaching
			Examples such as finding the time that elapses between the release of a ball and the instant when it finally ceases to bounce can be provided.  Teachers should also discuss with students the case when the plane is not fixed. Basically, this type of prob1em can be manipulated with a method similar to that used in direct impact of two spheres. Examples such as investigating the motion of a small bead in a smooth and closed straight tube which is moving with a velocity u on a smooth table can be provided. 
6.5	**Oblique Impact**	6	For oblique impact, teachers should indicate to students how the problems can be solved by resolving the velocities into components and applying Newton's experimental law along the line of centres of the two spheres.

Detailed Content	Time Ratio	Notes on Teaching

Teachers should remind students the components of the velocities of the two spheres perpendicular to the line joining the two centres are unaltered by the impact provided that the two spheres are smooth.



Before Impact



After Impact

$$\frac{v_2 - v_1}{u_2 \cos\beta - u_1 \cos\alpha} = -e$$

$$m_1 u_1 \cos\alpha + m_2 u_2 \cos\beta = m_1 v_1 + m_2 v_2$$

The following different situations should be discussed.

(1) $m_1 = m_2$

(2) $m_1 = m_2$ and $e = 1$

(3) $u_2 = 0$

At this stage, students should be able to get the following result.

Loss in kinetic energy = $\frac{1}{2} \cdot \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2 (1 - e^2)$

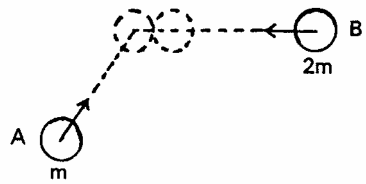
Special cases such as $m_1 = m_2$, $e_1 = e_2$, $u_1 = u_2$ and $\alpha = \beta$ should be discussed.

Examples should be provided.

Detailed Content	Time Ratio	Notes on Teaching

Example 1

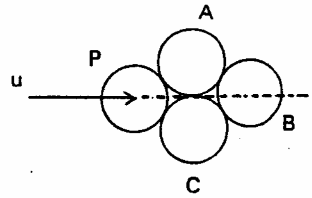
Referring to the figure, B is brought to rest by impact. Find e if the kinetic energy of A is unchanged.



Another possible question in this problem is to find the ratio of the increase in kinetic energy of A to the original kinetic energy of B if $e = \frac{2}{3}$ say.

Example 2

In the figure, all the spheres are identical. A, B and C are at rest, and P hits A and C symmetrically with velocity u. Find the speed of B after impact if $e = \frac{1}{2}$ for all impacts.



For students with lower ability, teachers may ask them to find firstly the speeds of A and C assuming that B is absent before calculating the speed of B.

	Time Ratio	
	15	