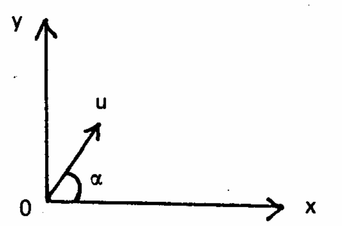
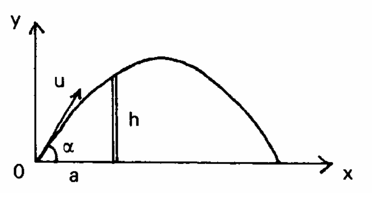
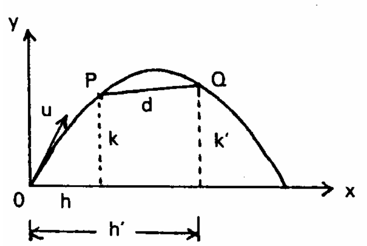
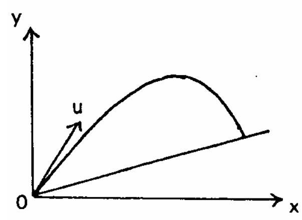
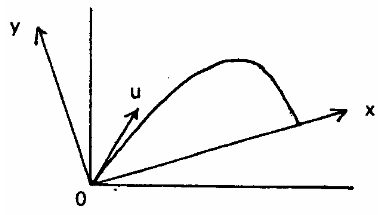
UNIT 7: Motion of Projectile under Gravity

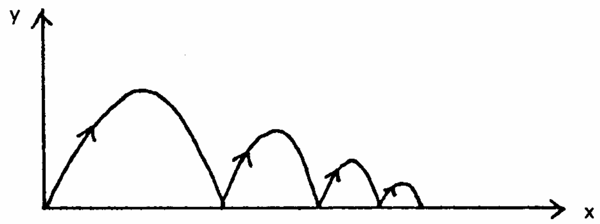
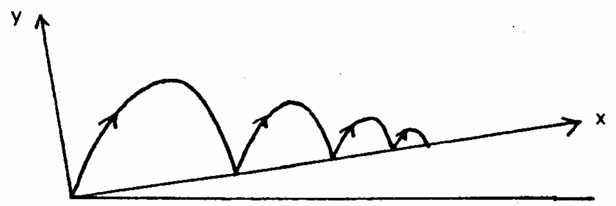
Specific Objectives:

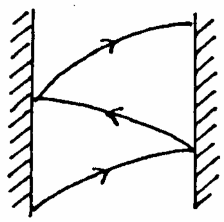
1. To understand the motion of projectile as a simple case of two-dimensional problems.
2. To recognize the path of a projectile as a parabola.
3. To solve related problems.

Detailed Content	Time Ratio	Notes on Teaching
7.1 Motion of Projectile	3	Students should be guided to obtain the equation of motion of a projectile under gravity in a vertical plane. The following results are expected. $\ddot{x} = 0$ and $\ddot{y} = -g$ $\dot{x} = u\cos\alpha$ and $\dot{y} = u\sin\alpha - gt$ $x = u\cos\alpha t$ and $y = u\sin\alpha t - \frac{1}{2}gt^2$  From these, students should be led to obtain the equation of trajectory, maximum height, time of flight, range and maximum range. Simple problems such as expressing the angle of projection a in terms of a given initial speed u and a given range R, expressing the initial speed u in terms of the maximum height H and the horizontal range R etc. should be given to students to ensure adequate practice.
7.2 Trajectory of Projectile	6	The relations $x = u\cos\alpha t$ and $y = u\sin\alpha t - \frac{1}{2}gt^2$ can be combined to form a quadratic function such as $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ which graph represents the path of a projectile, and $\frac{gx^2}{2u^2}\tan^2\alpha - x\tan\alpha + \left(y + \frac{gx^2}{2u^2}\right) = 0$.

Detailed Content	Time Ratio	Notes on Teaching
		Teachers should guide students to make use of the technique of solving quadratic equations to solve related problems. . *Example 1* A shuttlecock is struck by a badminton racket to the other side of the net which has a height h. It is projected with an initial velocity *u* at an angle of elevation α. Let the initial separation between the shuttlecock and the net be *a*. Show that the shuttlecock will cross the net if $h < \frac{u^2\sin^2\alpha}{2g}$  In this example, students are expected to distinguish the 3 cases: $h < \frac{u^2\sin^2\alpha}{2g}$ $h = \frac{u^2\sin^2\alpha}{2g}$ and $h > \frac{u^2\sin^2\alpha}{2g}$  Example 2 (a) A particle is projected at a point 0 with a speed of projection u as shown in the figure. If it passes through the point P(h, k), show that $u^2 \ge g\left(k + \sqrt{h^2 + k^2}\right)$.

Detailed Content	Time Ratio	Notes on Teaching
	3	If α_1 and α_2 are the two possible angles of projection, show that $\tan(\alpha_1+\alpha_2)=-\frac{h}{k}$ (b) If Q(h', k') is another point in the path such that the distance between P and Q is d, show that the minimum velocity with which the particle is projected so as to pass through both points P and Q is $\sqrt{g(d+k+k')}$. In (b), students should be able to make use of the results in (a) and the principle of conservation of energy to arrive at the result.
Range on an Inclined Plane	2	The emphasis here is on the choice of axes. The following two figures show the two possible ways.  In either case, students should be guided to obtain the range and the maximum range. They should also see that the range is maximum when the direction of projection bisects the angle between the vertical and the inclined plane.
Further Application of Projectile	6	At this stage, teachers should discuss with students projectile problems which also involve knowledge of other topics such as relative velocity and impact. *Example* Two particles P and Q are projected simultaneously with the same initial speed u from the same point in the same vertical plane.

Detailed Content	Time Ratio	Notes on Teaching
		Their angles of projection are α and β respectively and $\beta > \alpha$. (a) Show the relative speed of Q to P is $2u\sin\frac{1}{2}(\beta-\alpha)$. (b) If the trajectories intersect again at a point X, show that the time elapsed between P and Q passing through X is $\frac{2u\sin(\beta-\alpha)}{g(\cos\alpha+\cos\beta)}$ Other examples like the following should also be provided. 1. Impact with a horizontal plane  2. Impact with an inclined plane 

Detailed Content	Time Ratio	Notes on Teaching
		3. Impact with vertical walls  Two-dimensional problems involving resistive media should be avoided.
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