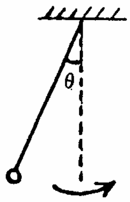
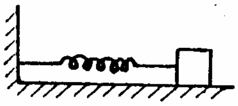
UNIT 9: Simple Harmonic Motion

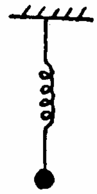
Specific Objectives:

1. To recognize simple harmonic motion.
2. To recognize damped and forced oscillation.
3. To solve related problems.

Detailed Content	Time Ratio	Notes on Teaching
9.1 Simple Harmonic Motion	12	Teachers may illustrate the concept of simple harmonic motion (S.H.M.) by using examples like simple pendulum and simple mass-spring system.







$\ddot{\theta} = -\frac{g}{\ell}\theta$ $\qquad$ $\ddot{x} = -\frac{\lambda}{m\ell}x$ $\qquad$ $\ddot{x} = -\frac{\lambda}{m\ell}x$

Students should know that any motion satisfying the equation of motion $\ddot{x} = -\omega^2 x$ is simple harmonic and they should be aware of the negative sign in the equation. Teachers may relate this unit with Topic Area II (Differential Equations).

After acquiring the relevant concept, teachers should guide students to obtain the fundamental formulae of S.H.M. which are. listed collectively below:

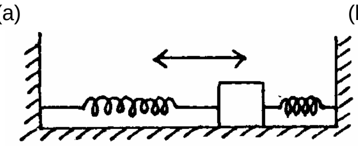
Acceleration	$\ddot{x} = -\omega^2 x$
Velocity	$\dot{x} = A\omega\cos(\omega t + \alpha) = \omega\sqrt{A^2 - x^2}$
Displacement	$x = A\sin(\omega t + \alpha)$
Period	$T = \frac{2\pi}{\omega}$

Detailed Content	Time Ratio	Notes on Teaching
		Teachers should remind students to compare the direction of acceleration with those of displacement and velocity.

Other daily-life examples that may lead to S.H.M. should also be discussed. The following show some of them.

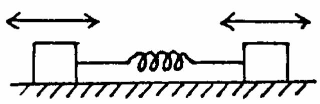
(1) A floating cylindrical cork oscillating vertically in water.
(2) Column of liquid oscillating in U-tube.
(3) More complicated mass-spring or mass-string systems such as those shown below.

(a)



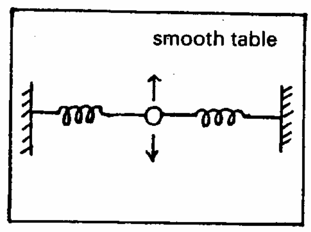
smooth surface

(b)

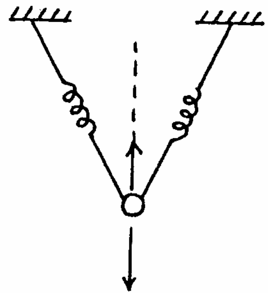


smooth surface

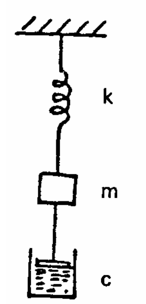
(c) (Small oscillation)

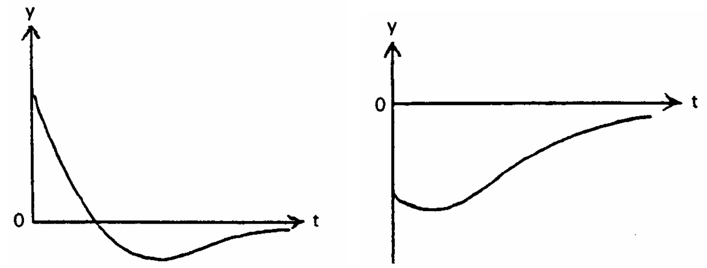
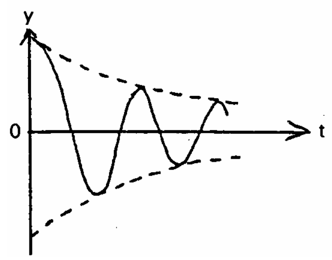
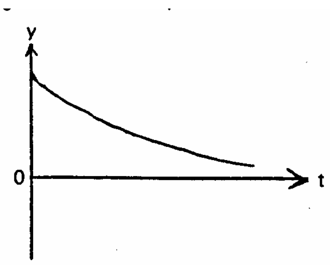


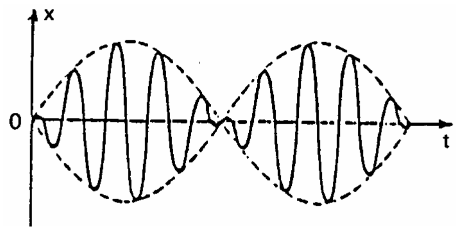
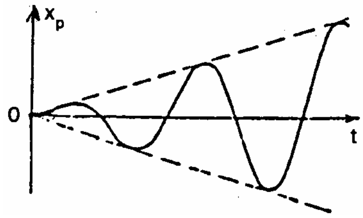
(d) (Small oscillation)

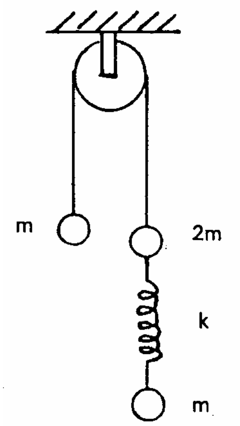


Adequate practice is very essential. The following shows some typical problems.

Detailed Content	Time Ratio	Notes on Teaching
		Example 1 A particle A of mass m hangs in equilibrium from a fixed point at the end of a light spring of modulus k. If another particle B of mass M is added to A and is then released, find the equation of motion and the amplitude of the resulting oscillation. *Example 2* A heavy particle suspended at the end of a light elastic string is performing a vertical S.H.M. of amplitude *a*. The maximum speed in the motion is $\sqrt{nga}$, where $n > 1$. The string is cut when the particle is at a height x above the centre 0 of the motion and is moving upwards with the string taut. Investigate the subsequent motion of the particle and find the greatest possible height reached by the particle.
9.2 Damped Oscillation	3	Teachers can introduce the concept of damped oscillations by adding to the mass-spring system a resistive force which is proportional to the speed of the body. (See figure) The equation of motion is in the form $$m\ddot{x} + c\dot{x} + kx = 0$$ where *m* is the mass of the body, *c* is the damping coefficient of the liquid and *k* is the spring constant.  The above equation is a second order differential equation and students should have no problem to solve it if they have learnt Unit 13. Otherwise, teachers may provide students with the solution directly and leave the proof to Unit 13. Teachers should lead students to discuss the nature of the roots of the above differential equation. This would give rise to the 3 cases: overdamping, underdamping and critical damping. The following figures show some forms for some typical initial conditions.

Detailed Content	Time Ratio	Notes on Teaching
		 Positive initial displacement Negative initial displacement Overdamped cases with negative initial velocity  Underdamping  Critical damping *Example* A particle of mass m is suspended from a fixed point by a string of natural length ℓ and Modulus of elasticity $5mn^2\ell$. When in motion, the particle is resisted by a force of magnitude 2*mn* times its sped. Initially, the particle is hanging in equilibrium and is then projected vertically downwards with speed V.

Detailed Content	Time Ratio	Notes on Teaching
		In this example, students are expected to show that the equation of motion is $\ddot{x}+2n\dot{x}+5n^2x=0$ where x is the downward displacement at time t. Teachers can then guide students to solve the equation and find the time when the particle is instantaneously at rest.
9.3 Forced Oscillation	5	The driving force $F(t)$ should be in one of the forms t^n, $\cos \omega t$, $\sin \omega t$, $e^{\mu t}$ or a linear combination of these functions. The following two cases should be discussed: (a) Undamped forced oscillation . The equation of motion is $m\ddot{x}+kx=F(t)$. Teachers can discuss with students how to get the general solution and the particular solution. The physical meanings of the solutions should be emphasized. For example, when $F(t) = F_0 \sin \omega t$, the following are the two cases of the particular solutions. Beats  Resonance 

Detailed Content	Time Ratio	Notes on Teaching
		(b) Damped forced oscillation Students should have no difficulty to get the equation of motion $m\ddot{x}+c\dot{x}+kx=F(t)$. The solution of the equation for a given $F(t)$ can be found directly. Moreover, in some cases, students are expected to derive the driving force under a given situation. The following shows an example. *Example* Two particles of masses m and 2m respectively, and attached by an inextensible string is hanging over a smooth fixed pulley as shown. A third particle of mass m is then attached to the heavier particle by a spring of modulus k. All particles are released from rest with the spring just unstretched. Investigate the subsequent motions of the particles. In this example, the driving force acting on the third particle is not directly known. Students are expected to find the force themselves with the information given. 
	20	