

UNIT 10: Motion of a Particle in a Plane

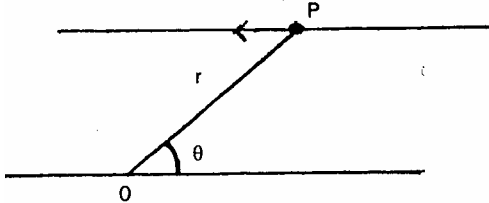
Specific Objectives:

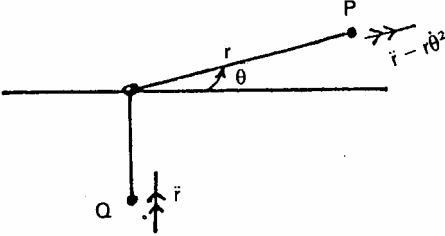
To solve problems involving motion of a particle in a plane by various techniques.

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Detailed Content	Time Ratio	Notes on Teaching
10.1 Motion of a Particle in a Plane	8	<p>In order to solve the general motion of particle in a plane. two coordinate systems, i.e. the Cartesian coordinate and polar coordinate systems, should be introduced to the students. It is also worthwhile for teachers to discuss with students the distinction of employing polar coordinates and Cartesian coordinates in solving such problems.</p> <p>Simple integration is worthwhile as the major solving technique for this kind of problems. However, students should be told that the skill learnt from the differential equations may also be employed where necessary.</p> <p>Some examples using Cartesian coordinates may be introduced to students. Below is one:</p> <p><i>Example</i></p> <p>A particle moving in a plane is subject to a force acting towards the x-axis and proportional to its distance from that axis. Initially, it is projected from the origin with a velocity v in a direction making an angle θ with the x-axis.</p> <p>In this problem, students are asked to set up force equations along the x- and y-direction, and to find the velocity and position of the particle at time t by direct integration or by using the technique of differential equation.</p> <p>Moreover, students should be reminded that projectile motion is a kind of motion that employs Cartesian coordinates.</p> <p>Before teaching students to handle the problems using polar coordinates, the knowledge of resolution of velocity and acceleration along and perpendicular to the radius vector learnt should be recalled. Sufficient practical examples in which the position, velocity and acceleration vectors (as well as their radial and transverse components) of an object which could be represented in polar coordinates should be given. However, teachers should note that velocity and acceleration components in tangential-normal form are not required.</p>

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Detailed Content	Time Ratio	Notes on Teaching
		<p>At this stage, problems involving the motion of particle(s) under a radial force and simple orbital motion are worthwhile to be discussed. However, it should be reminded that detailed knowledge of orbit problems need not be taught. It should be emphasized that setting up of force equations or velocity relations in radial and transverse directions is essential. Moreover, the law of conservation of angular momentum may also be employed where there are only radial forces in the system, but the elementary concepts of angular momentum and the corresponding conservation law should be taught when necessary. (The details may be left to Unit 11.) Students may also be told that such result may also be obtained by equating the acceleration in transverse direction to zero, i.e.</p> $r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$ <p>After solving the equation, students are also expected to interpret the results. The following show two examples:</p> <p><i>Example 1</i></p> <p>A particle P moves in a plane. Its position at time t is represented by (r, θ) and its velocity always makes an angle θ with PO.</p>  <p style="text-align: right;">polar axis</p> <p>Show that $\frac{dr}{d\theta} = -\frac{r}{\tan \theta}$</p>

Detailed Content	Time Ratio	Notes on Teaching
		<p><i>Example 2</i></p> <p>P and Q are two particles, each of mass m, connected by a light inextensible string of length $2a$, which passes through a small smooth hole O in a smooth horizontal table. P is free to slide on the table and Q hangs freely. Initially OQ is of length a, and P is projected from rest, at right angles to OP with velocity v_0.</p> <p>The following figure shows the particles' positions at time t:</p>  <p>In this problem, students are led to draw the above diagram and set up the force equations in radial direction of particle P and vertical direction of particle O. They should be told that the transverse component of the velocity of P may be found by employing the law of conservation of angular momentum and the velocity of P and O can be found by solving the above equations.</p>
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