Comparison between the Additional Mathematics Curriculum and *Syllabuses for Secondary Schools – Additional Mathematics (Secondary 4 – 5) 1992*

This Additional Mathematics Curriculum Guide is a revised edition of the version published in 1992. Some topics have been deleted or trimmed. For ease of reference of teachers, these topics are enclosed in boxes like from the 1992 version, page for page. Notes and additional remarks are enclosed in boxes like to delimit the complexity of teaching.

3. SYLLABUS

UNIT 1: Principle of Mathematical Induction

Specific Objectives:

1. To understand the concept of mathematical induction.
2. To be familiar with the steps in the method of mathematical induction.
3. To apply the principle of mathematical induction to various fields.

Detailed Content	Time Ratio	Notes on Teaching
1.1 Concept of Mathematical Induction	2	Students should realize that some formulae are true only for positive integers. Examples include formulae for the sums of n terms of an A.P. and a G.P. While those formulae may be derived by other methods they may also be proved by a method which takes particular advantage of the integer nature of the variables in the formulae. This method is the method of mathematical Induction. Teachers may use some simple, but perhaps less familiar examples like $1^2+2^2+3^2+\ldots+n^2=\frac{1}{6}n(n+1)(2n+1)$ as an illustration. The formula is true for n=1, 2 and 3. By "induction", we expect it to be true for all positive integers n. But, how can we be sure of this? In particular, how can we know it is true for n=100 or 257, say? It is necessary to prove the validity.
1.2 Steps in the Method of Mathematical Induction	~~2~~ 3	The steps in a proof by mathematical induction are now developed, preferably using a simple example as the tool instead of starting with the general proposition P(n) right from the beginning. The results should be generalized afterwards. Teachers should emphasize the presentation of the proof by mathematical induction. The following shows an example.

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Detailed Content	Time Ratio	Notes on Teaching
		Example Prove $1+2+3+...+n=\frac{n(n+1)}{2}$ for all positive integers n. Let P(n) be the statement " $1+2+3+...+n=\frac{n(n+1)}{2}$ ". When n = 1, L.H.S.= 1 R.H.S.= $\frac{1(1+1)}{2}=1$ $\therefore$ L.H.S. = R.H.S. P(1) is true Suppose P(k) is true. i.e. $1+2+3+...+k=\frac{k(k+1)}{2}$ When n = k+1, L.H.S. = 1+2+3+…+k+(k+1) $=\frac{k(k+1)}{2}+(k+1)$ $=(k+1)[\frac{k}{2}+1]$ $=\frac{(k+1)[(k+1)+1]}{2}$ = R.H.S. $\therefore$ P(k+1) is also true. By the principle of mathematical induction, P(n) is true for all positive integers n.

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Detailed Content	Time Ratio	Notes on Teaching
		The introductory example should not involve too much manipulation; otherwise it will distract students' attention from understanding the basic argument. Other useful examples are $1\times 2+2\times 3+3\times 4+...+n(n+1)=\frac{1}{3}n(n+1)(n+2)$ and $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+...+\frac{1}{n(n+1)}=\frac{n}{n+1}$
1.3 Applications of Mathematical Induction	~~4~~ 5	Teachers may now introduce some less straight forward examples to illustrate the power of the method, for example, to prove that, for any positive integer n, (1) $1^2+3^2+5^2+...+(2n-1)^2=\frac{1}{3}n(4n^2-1)$ (2) $1\cdot n+2(n-1)+3(n-2)+...+(n-1)\cdot 2+n\cdot 1=\frac{1}{6}n(n+1)(n+2)$ (3) $\sin\alpha+\sin 2\alpha+\sin 3\alpha+...+\sin n\alpha=\frac{\sin(\frac{n+1}{2}\alpha)\sin(\frac{n}{2}\alpha)}{\sin\frac{\alpha}{2}}$ for α not equal to an integral multiple of 2π. (If students have learned the required trigonometric formulae, this example can be given; otherwise it can be used for revision later.) Apart from the summation of series, applications should also include (1) Proofs of divisibility, for example, (a) 7^n+3n-1 is divisible by 9, (b) 23^n-1 is divisible by 11,

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Detailed Content	Time Ratio	Notes on Teaching

(c) $a^{2n-1}+b^{2n-1}$ is divisible by a + b;

The proof of (c) may run as follows:

Let P(n) be " $a^{2n-1}+b^{2n-1}$ is divisible by a + b".

When n =1,

$a^{2n-1}+b^{2n-1}=a+b$

$\therefore$ P(1) is true.

Suppose P(k) is true,

i.e. $a^{2k-1}+b^{2k-1}=m(a+b)$.

When n = k+1, we have

$$
\begin{aligned}
a^{2(k+1)-1}+b^{2(k+1)-1} &= a^{2k+1}+b^{2k+1}\\
&= a^2\cdot a^{2k-1}+b^2\cdot b^{2k-1}\\
&= a^2\cdot a^{2k-1}+a^2\cdot b^{2k-1}-a^2\cdot b^{2k-1}+b^2\cdot b^{2k-1}\\
&= a^2(a^{2k-1}+b^{2k-1})-(a^2-b^2)b^{2k-1}\\
&= a^2(m)(a+b)-(a+b)(a-b)b^{2k-1}\\
&= (a+b)[a^2m-(a-b)b^{2k-1}]
\end{aligned}
$$

which is divisible by (a + b)

i.e. P(k+1) is true.

By the principle of mathematical induction, P(n) is true for all positive integers n.

(2) Proofs of simple inequalities, for example,

(a) $(1+x)^n \geq 1+nx$ if $x > -1$

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Detailed Content	Time Ratio	Notes on Teaching

(b) $(1+a_1)(1+a_2)\cdots(1+a_n)\geq 1+(a_1+a_2+\ldots+a_n)$

for positive numbers a_i (i=1, 2, …, n)

(c) $2^n > n^2$ for $n \geq 5$;

A possible proof for (c) is as follows:

Let P(n) be " $2^n > n^2$ for $n > 5$; ".

When n=5,

L.H.S. = 2^5 = 32

R.H.S. = 5^2 = 25

L.H.S. > R.H.S.

$\therefore$ P(5) is true.

Suppose P(k) is true,

i.e. $2^k > k^2$ where $k \geq 5$

When n = k+1,

$$
\begin{aligned}
\text{Consider } 2^{k+1}-(k+1)^2 &= 2^{k+1}-k^2-2k-1\\
&= 2\cdot 2^k-2k^2+k^2-5k+3k-15+14\\
&= 2(2^k-k^2)+k(k-5)+3(k-5)+14\\
&> 0
\end{aligned}
$$

$\therefore 2^{k+1} > (k+1)^2$

$\therefore$ P(k+1) is true.

By the principle of mathematical induction, P(n) is true for $n > 5$.

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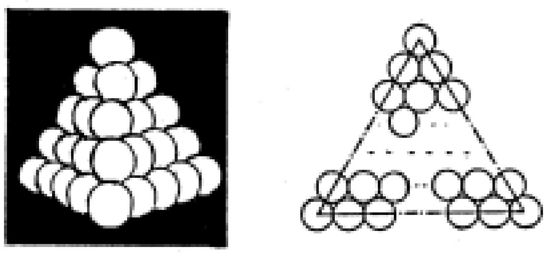
Detailed Content	Time Ratio	Notes on Teaching
		(3) Simple practical problems such as finding the total number of balls in a pyramidal heap of balls.  Teachers should remind students that for some propositions the starting values of may differ from 1 such as $2^n > n^2$ for $n > 5$.
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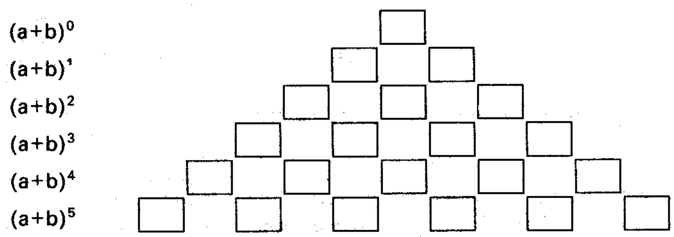
UNIT 2: Binomial Theorem for Positive Integral Indices

Specific Objectives:

1. To recognize the notations n! and C^n_r .
2. To learn to expand a binomial with positive integral index by the binomial theorem.

Detailed Content	Time Ratio	Notes on Teaching
2.1 The n! and C^n_r notations	1	The definitions of n! and C^n_r should be introduced. The idea of permutation and combination may only be mentioned to abler students. The definition of O! = 1 should also be mentioned. It is desirable to bring students' attention to the other forms of C^n_r , viz, ${}_nC_r$ and $\binom{n}{r}$. Students are expected to be able to verify $C^n_r = C^n_{n-r}$ and $C^n_{r-1} + C^n_r = C^{n+1}_r$. In the latter part, teachers should guide students to start from L.H.S. and end up with R.H.S. Examples like the one below can be given. *Example* Solve for n if $C^{18}_{n+1} = C^{18}_{2n+1}$.
2.2 The Pascal Triangle	1	Teachers can ask students to expand $(a + b)^2$, $(a + b)^3$, $(a + b)^4$ and $(a + b)^5$ by multiplication and fill the coefficients of the terms of the expanded expressions into the boxes below. 

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