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| Detailed Content | Time Ratio | Notes on Teaching |
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| | | *Example 1*<br>Given that $\cos 60^\circ = \frac{1}{2}$, find an approximate value of $\cos 61^\circ$ by using the method of small increments.<br>Let $f(x) = \cos x$, $f'(x) = -\sin x$<br>$x = 60^\circ = \frac{\pi}{3}$ radians<br>$x = 1^\circ = \frac{\pi}{180}$ radian<br>$\cos 61^\circ = \cos(\frac{\pi}{3} + \frac{\pi}{180})$<br>$\approx \cos(\frac{\pi}{3}) + (-\sin\frac{\pi}{3})(\frac{\pi}{180})$<br>$= 0.5 - \frac{\sqrt{3}}{2}(\frac{\pi}{180})$<br>$\therefore \cos 61^\circ \approx 0.485$<br>Teachers should remind students that the variable x in the function f(x)=cos x should be in radians.<br>*Example 2*<br>If the radius of a solid sphere is measured with a percentage error of 0.1% and the volume is calculated. Estimate the percentage error of the volume. |
| | ~~36~~<br>44 | |

## UNIT 6: Inequalities

*Specific Objectives:*

1. To understand the basic rules of inequalities.
2. To solve linear inequalities in one variable.
3. To solve quadratic inequalities in one variable.
4. To solve inequalities of the form $\frac{ax+b}{cx+d} \geq k$.
5. To solve linear and quadratic inequalities involving absolute values.
6. To prove simple absolute inequalities.

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| Detailed Content | Time Ratio | Notes on Teaching |
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| **6.1 Basic Rules of Inequalities** | 1* | Teachers should emphasize that if a–b is a positive number then a>b, and vice versa. With this fact, the following basic rules can be derived. For real numbers a, b, c:<br>(1) If a>b, and b>c, then a>c.<br>(2) If a>b, then a+c>b+c<br>(3) If a>b, then<br>(a) ac>bc for c>0.<br>(b) ac<bc for c<0.<br>(c) ac=bc for c=0.<br>Simple proofs of inequalities by using the basic rules should be introduced. |
| **6.2 Linear Inequalities in one Variable** | ~~1~~<br>1*+1 | Students should be reminded that the method of solving linear inequalities resembles that of solving linear equations. The only difference is that when an inequality is multiplied or divided by a negative number, then the inequality sign has to be reversed. |

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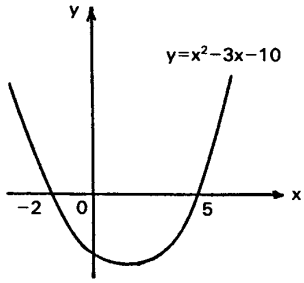
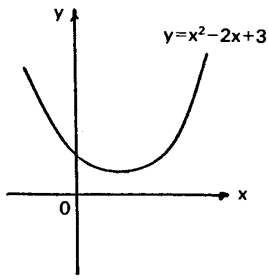
| | | It is a good practice for students to sketch the diagrams of the solutions. It is particularly helpful in solving compound inequalities where individual solutions are sketched on the same number line. The following two examples are typical.<br>*Example 1*<br>Solve the compound inequalities:<br>$7x-3<5x+1$ and $\frac{x}{2} < x + \frac{1}{2}$<br>*Example 2*<br>Solve the compound inequalities:<br>$\frac{2(x+1)}{5} < \frac{3(1-x)}{7}$ or $2 - \frac{4-5x}{4} < \frac{7x+1}{8}$ |
| **6.3 Quadratic Inequalities in One Variable** | ~~1* +2~~<br>1* +3 | There are several methods in solving quadratic inequalities such as graphical methods, factorization and tabulation.<br>The graphical solutions of quadratic equations have been learnt in lower forms. Graphs may be used to solve quadratic inequalities. For example: |

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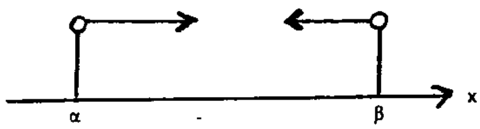
| | | From the graphs, it is obvious that the solution of:<br>$x^2-3x-10>0$ is $x<-2$ or $x>5$,<br>$x^2-3x-10 \le 0$ is $-2 \le x \le 5$ ,<br>$x^2-2x+3>0$ is all real values of x, and that<br>$x^2-2x+3<0$ has no real solution.<br>The method of solving quadratic inequalities is similar to that of solving quadratic equations. The terms are rearranged to make the right hand side zero, then the quadratic expression on the left hand side may be factorized (if possible).<br>The inequality can then be solved by using the fact that<br>(1) If $ab>0$, then $\begin{cases} a>0 \\ b>0 \end{cases}$ or $\begin{cases} a<0 \\ b<0 \end{cases}$<br>(2) If $ab<0$, then $\begin{cases} a>0 \\ b<0 \end{cases}$ or $\begin{cases} a<0 \\ b>0 \end{cases}$<br>It would be very helpful to memorize the following two results in which $\alpha<\beta$.<br>(1) For $(x-\alpha)(x-\beta)<0$, the solution is $\alpha<x<\beta$ and its graph is |



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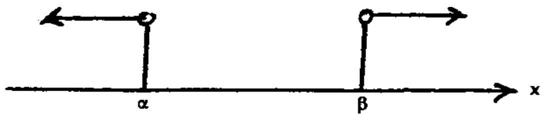
| | | (2) For (x-α)(x-β)>0, the solution is x<α or x>β and its graph is:<br><br>Students should be given due amount of practice in order to master the skill.<br>When the quadratic expression cannot be factorized, teachers should remind students to convert it into the form $(x+c)^2+d>0$ or $\left(x-\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(x-\frac{-b-\sqrt{b^2-4ac}}{2a}\right)>0$ and etc. In employing the method of tabulation for solving (x–α)(x–β)>0 etc, students are advised to construct a table and determine the sign of (x–α)(x–β) for x<α, α<x<β and x>β respectively.<br>The following examples could be introduced.<br>*Example 1*<br>Solve the inequality $3x^2>7x-1$<br>*Example 2*<br>Prove that for real values of x, the function $y=\frac{x-1}{x^2+3}$ must lie in the range $-\frac{1}{2}$ to $\frac{1}{6}$.<br>*Example 3*<br>Solve $x>\frac{2}{x}+1$ for each of the following cases:<br>(a) x>0<br>(b) x<0 |

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| | | Hence find the complete solution for the inequality $x>\frac{2}{x}+1$. |
| 6.4 Inequalities of the Form $\frac{ax+b}{cx+d}\geq k$ | 2 | It should be stressed that when multiplying an inequality of the form $\frac{ax+b}{cx+d}>k$ by a factor containing an unknown the cases for the factor being positive or negative should be considered separately. For example, to solve $\frac{x+2}{x-4}>1$ or $\frac{x+2}{x-4}-1>0$, the 2 cases:<br>x>4 and x<4 could be considered separately. (Again, students should note that $x\neq 4$.) However, it is more convenient to multiply the inequality by the square of the denominator, thus eliminating the denominator, i.e. $(x+2)(x-4)>(x-4)^2$. In fact, these inequalities can be solved by tabulation method.<br>*Example*<br>Solve $\frac{2x+5}{4x-3}>1$ |
| **6.5 Absolute values**<br><br>It is more appropriate to introduce the topic "Absolute values" in Unit 3 (Quadratic Equations and Quadratic Functions). | 1 | The definition $\|x\|=\begin{cases} x \text{ if } x\geq 0 \\ -x \text{ if } x<0 \end{cases}$ should be clearly stated.<br>The graph of y=\|x\| should be introduced.<br>To sketch graphs of simple functions involving absolute values, students should be reminded to remove at first the absolute value sign. For example, after removing the absolute value sign from $y=\|x^2-4\|$, the graph obtained will be that shown in figure 1. Since y is always non-negative in $y=\|x^2-4\|$, the required graph should be that as shown in figure 2. |



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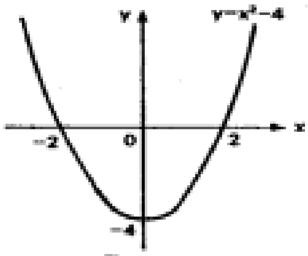
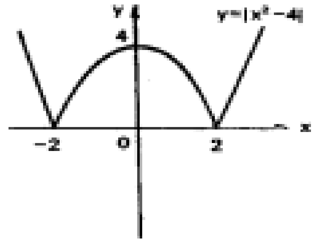
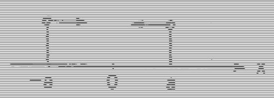
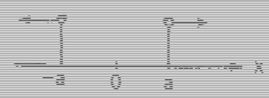
| | | <br>Figure 1<br><br>Figure 2 |
| 62 6.6 Inequalities Involving Absolute Values | 3 | These inequalities may be solved graphically or algebraically. It would be helpful for students to treat $\|x\|$ as the distance of x from zero on the number line. In this way, $\|x-y\|$ represents the distance between the two points x, y on the number line. It also helps to remember that:<br>(1) If $\|x\|<a$, then $-a<x<a$.<br>(2) If $\|x\|>a$, then $x<-a$ or $x>a$.<br>The following two graphs should be introduced for illustration<br>$\|x\|<a$<br><br>$\|x\|>a$<br> |

| Detailed Content | Time Ratio | Notes on Teaching |
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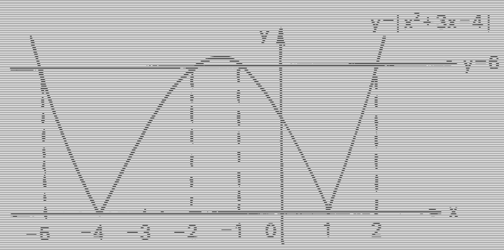
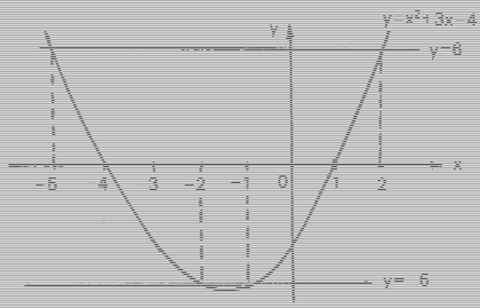
| 63 | | *Example*<br>Solve the inequality $\|x^2+3x-4\| \geq 6$.<br>The solution could be obtained from the graph $y=\|x^2+3x-4\|$.<br><br>Alternatively the solution may be obtained from the graph $y=x^2+3x-4$.<br><br>Algebraically, the inequality is equivalent to $x^2+3x-4 \leq -6$ or $x^2+3x-4 \geq 6$ which may be simplified to the solution: $x \leq 5$ or $-2 \leq x \leq -1$ or $x \geq 2$. |



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| Detailed Content | Time Ratio | Notes on Teaching |
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| 6.7 Absolute Inequalities | 1 | Complicated examples like $\|\|x+1\| - \|x-1\|\|<1$ could also be introduced. To deal with this type of inequalities, the absolute value sign may be removed by considering x in different intervals which are then treated independently. Teachers should illustrate clearly how the different intervals of x can be identified. The inequality in each interval of x is solved and then the individual solutions are combined. It is a good practice to sketch the individual solutions on the same number line to find out the overall solution.<br>Students should be reminded that $x^2 \ge 0$ for all real values of x. Teachers may derive the inequalities:<br>$a^2 + b^2 \ge 2ab$ and<br>$\frac{a+b}{2} \ge \sqrt{ab}$ , where a>0, b>0.<br>A usual method of proving x>y is to prove the inequality x−y>0.<br>With these two facts in hand, teachers can then discuss with students some simple absolute inequalities like:<br>$a^2 + b^2 + c^2 \ge ab + bc + ca$ . |
| | ~~3*+9~~<br>3*+4 | |

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**UNIT 7: Coordinate Geometry**

Specific Objectives:

1. To find areas of plane rectilinear figures in the coordinate plane.
2. To find the angle between two straight lines.
3. To understand the normal form of a straight line equation and to apply the knowledge in finding distances.
4. To find the equation of a circle and the points of intersection of a straight line and a circle.
5. To find the equations of tangents to a circle.
6. To find the equations of families of straight lines and circles.
7. To find the equations of a parabola, an ellipse and a hyperbola in standard position.
8. To acquire the concept of parametric equations and locus, and solve simple locus problem.

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| Detailed Content | Time Ratio | Notes on Teaching |
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| **7.1 Plane Rectanglular Coordinates and Distance Between Two Points** | 1* | Since students should have learnt some elementary concepts of plane rectangular coordinates in S1-3, only a brief revision is necessary at this stage. Distance formula and Section formula could be stated.<br>For Section formula, the external division should be emphasised, i.e. if R is a point on PQ produced such that PR:RQ = r:1, then r will have a negative value.<br>*Example*<br>The centre of a circle is at O(3,4). If A(1,1) is an end point of a diameter, find the coordinates of the other end point. |
| **7.2 Area of Rectilinear Figures** | 3 | Students should be led to discover that the area of a triangle with vertices at O(0,0), $A(x_1,y_1)$ and $B(x_2,y_2)$ may be expressed as the algebraic sum of two right-angled triangles and a trapezium, and is hence given in magnitude by $\frac{1}{2}(x_1y_2 - x_2y_1)$ . They should also note that if the vertices O, A and B are chosen in counterclockwise direction then the value obtained will be positive, otherwise a negative value will be obtained. |



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