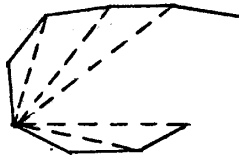
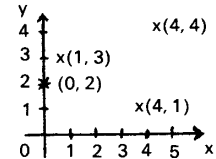


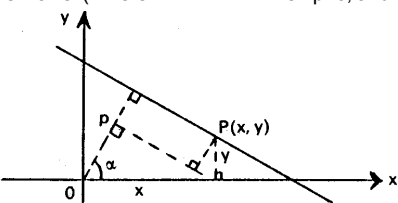
66

Detailed Content	Time Ratio	Notes on Teaching
		<p>With the above knowledge in hand, the area of any triangle on the coordinate plane may be taken to be the algebraic sum of the areas of three triangles each with a vertex at the origin, and may hence be calculated by the formula.</p> <p>Area of triangle: $= \frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3)$ where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the coordinates of the vertices of the triangle, taken in the counterclockwise direction.</p> <p>Students should be told that for convenience, the formula may be written in the form</p> $\text{Area of triangle} = \frac{1}{2} \begin{vmatrix} x_1 & \times & y_1 \\ x_2 & \times & y_2 \\ x_3 & \times & y_3 \\ x_1 & \times & y_1 \end{vmatrix}$ <p>where the terms x_1y_2, x_2y_3 and x_3y_1 are regarded as downward products and are positive, and the terms x_2y_1, x_3y_2 and x_1y_3 are regarded as upward products and are negative, and the area is equal to half the sum of these terms.</p> <p>In the final stage, the area of any n-sided polygon on the plane may be calculated by regarding it as composed of (n-2) triangles, as in the following diagram:</p> 

67

Detailed Content	Time Ratio	Notes on Teaching
		<p>The following formula should be obtained:</p> <p>Area of polygon $= \frac{1}{2}(x_1y_2 + x_2y_3 + \dots + x_ny_1 - x_2y_1 - x_3y_2 - \dots - x_1y_n)$ where (x_1, y_1), (x_2, y_2), (x_3, y_3), ..., (x_n, y_n) are the coordinates of the vertices, taken in the counterclockwise direction.</p> <p>Again, for convenience sake, students may write the formula in the form</p> $\text{Area of polygon} = \frac{1}{2} \begin{vmatrix} x_1 & \times & y_1 \\ x_2 & \times & y_2 \\ \dots & \times & \dots \\ x_n & \times & y_n \\ x_1 & \times & y_1 \end{vmatrix}$ <p>In real practice, say, to find the area of the quadrilateral with vertices at $(1, 3)$, $(4, 1)$, $(4, 4)$ and $(0, 2)$, teachers should constantly remind students that the order of the points in the formula must be counterclockwise in direction. Therefore, the area of the said polygon is</p> $\frac{1}{2} \begin{vmatrix} 0 & 2 \\ 4 & 1 \\ 4 & 4 \\ 1 & 3 \\ 0 & 2 \end{vmatrix} \quad \text{and NOT} \quad \frac{1}{2} \begin{vmatrix} 0 & 2 \\ 1 & 3 \\ 4 & 1 \\ 4 & 4 \\ 0 & 2 \end{vmatrix}$ 

Detailed Content	Time Ratio	Notes on Teaching
7.3 Inclinations and Slopes of Straight Lines	1*	<p>A sketch of the points before applying the formula is desirable.</p> <p>The slope of a straight line should be defined as the tangent of its angle of inclination to the positive direction of the x-axis, and the range of the inclination is from 0° to 180°.</p> <p>Hence the slope of a line joining the points (x₁, y₁) and (x₂, y₂) is given by</p> $m = \frac{y_2 - y_1}{x_2 - x_1}$
7.4 Angle between Two Straight Lines	3 4	<p>Given that the inclinations of two straight lines to the positive x-axis are α and β respectively, the angle between the two straight lines is equal to the difference between α and β. Since $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{m_1 - m_2}{1 + m_1 m_2}$,</p> <p>the angle between the two straight lines is given by</p> $\tan^{-1} \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $ <p>where the absolute signs are included to ensure that the acute angle between the lines is obtained.</p> <p>Now, students should have no problem to see that when two lines with slopes m₁ and m₂ respectively are parallel (i.e. the angle θ between them is zero) if and only if m₁=m₂. On the other hand, the two lines are perpendicular (i.e. θ=90°) if and only if m₁m₂ = -1.</p> <p><i>Example</i></p> <p>The acute angle between the lines x+y=5 and 2x-y=7 is given by $\tan^{-1} \left \frac{-1-2}{1+(-2)} \right = 71.6^\circ$</p>
7.5 Equations of Straight Lines	1*	<p>Teachers should stress to students that the equation of a straight line is in fact a relationship between the x and y coordinates of any point on the line. The different standard forms are:</p>

Detailed Content	Time Ratio	Notes on Teaching
7.6 The Normal Form	5	<p>(1) The general form $Ax+By+C=0$</p> <p>(2) The two-point form $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$</p> <p>(3) The point-slope form $y - y_1 = m(x - x_1)$</p> <p>(4) The slope-intercept form $y = mx + c$</p> <p>(5) The intercept form $x/a + y/b = 1$</p> <p>Students should be guided to derive the normal form of a straight line in terms of its distance p from the origin (p ≥ 0) and the inclination α of its normal to the positive x-axis. (Where 0 ≤ α < 2π when p > 0, and 0 ≤ α < π when p = 0)</p>  <p>With the above diagram, students should have no difficulty to prove that the coordinates (x, y) of any point P on the straight line satisfy the relationship $x \cos \alpha + y \sin \alpha - p = 0$, which is the equation of the straight line in normal form.</p> <p>Given the equation of a straight line in its general form $Ax+By+C=0$, students should be able to convert it to the form</p> $\frac{Ax + By + C}{\pm \sqrt{A^2 + B^2}} = 0$

70

Detailed Content	Time Ratio	Notes on Teaching
		<p>Which is in fact the normal form with the following correspondence</p> $\frac{A}{\pm\sqrt{A^2+B^2}} = \cos\alpha$ $\frac{B}{\pm\sqrt{A^2+B^2}} = \sin\alpha$ $\frac{C}{\pm\sqrt{A^2+B^2}} = -p$ <p>The rules to choose the appropriate sign in the denominator are: (1) If $C \neq 0$, the sign should be opposite to that of C. (2) If $C=0$, the sign should be the same as that of B.</p> <p><i>Example</i></p> <p>The line $x - \sqrt{3}y + 6 = 0$ is converted to $\frac{x - \sqrt{3}y + 6}{-2} = 0$ with $p=3$ and $\cos\alpha = -\frac{1}{2}$, $\sin\alpha = \frac{\sqrt{3}}{2}$, so $\alpha = 120^\circ$.</p> <p>Alternatively, the equation $Ax+By+C=0$ can be rearranged so that only a positive constant appears on the right-hand side and the whole equation is then divided throughout by $\sqrt{A^2+B^2}$. For example, the above equation is rearranged as $-x + \sqrt{3}y = 6$. Thus, $\frac{-x + \sqrt{3}y}{\sqrt{((-1)^2 + 3)}} = \frac{6}{\sqrt{((-1)^2 + 3)}}$ or $-\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 3$</p> <p>getting $p=3$, $\cos\alpha = -\frac{1}{2}$ and $\sin\alpha = \frac{\sqrt{3}}{2}$ as before.</p> <p>The distance from a point to a straight line may be obtained by considering the normal form of a straight line parallel to the given line and passing through the given point (x_0, y_0). The formula $d = \frac{Ax_0 + By_0 + C}{\pm\sqrt{A^2+B^2}}$ should be memorized.</p>

71

Detailed Content	Time Ratio	Notes on Teaching
7.7 Family of Straight Lines	3-5	<p>Students should also note that with proper choice of the sign in the denominator, the value obtained bears a sign which tells whether the given point is on the same side of the straight line as the origin or not. To obtain the absolute distance, we take the absolute value of the signed distance.</p> <p>For abler students, elementary treatment of the distance between two parallel lines and the angle bisectors of the lines can be discussed through examples so as to arouse their interest.</p> <p><i>Example 1</i></p> <p>Find the distance between the parallel lines $12x-5y-10=0$ and $12x-5y+16=0$.</p> <p>The distance between the two parallel lines may be obtained by taking any point on either of the lines and finding its distance from the other line. Suppose the point $(0, -2)$ on the first line is chosen, the distance of this point from the second line is given by $[(-5)(-2)+16] / \sqrt{(12^2 + 5^2)} = 2$, and this is the distance between the parallel lines.</p> <p><i>Example 2</i></p> <p>Find the angle bisectors of the lines $x+y-3=0$ and $x-7y+5=0$</p> <p>If $P(x,y)$ is a point on the angle bisector, then the distance from P to the two lines are equal. Hence we have</p> $\left \frac{x+y-3}{\sqrt{(1+1)}} \right = \left \frac{x-7y+5}{\sqrt{(1+49)}} \right $ <p>getting $x+3y-5=0$ or $3x-y-5=0$</p> <p>These are the equations of the angle bisectors of the given lines.</p> <p>This topic includes a family of parallel lines with slope m, a family of lines passing through a given point (a, b) and also a family of lines passing through the intersection of the given lines: $A_1x+B_1y+C_1=0$ and $A_2x+B_2y+C_2=0$. Their equations are respectively. $y=mx+k$ $y=k(x-a)+b$</p>

Detailed Content	Time Ratio	Notes on Teaching
7.8 Concept of Locus	4*+2 1*+3	<p>and $(A_1x+B_1y+C_1) + k(A_2x+B_2y+C_2)=0$ In each case, the equation of the family of straight lines involves a parameter k. N.B. In the case $(A_1x+B_1y+C_1)+k(A_2x+B_2y+C_2)=0$, the line $A_2x+B_2y+C_2=0$ is excluded. So teachers may use the form $\ell (A_1x+B_1y+C_1) + k(A_2x+B_2y+C_2)=0$ where ℓ, k are parameters to include the whole family of lines passing through the intersecting point of $A_1x+B_1y+C_1=0$ and $A_2x+B_2y+C_2=0$.</p> <p><i>Example 1</i> The family of lines parallel to the line $2x-3y+4=0$ is given by $2x-3y+h=0$, and the family of the lines passing through the point $(2, -3)$ is given by $y=k(x-2)-3$, where h and k are parameters.</p> <p><i>Example 2</i> Find the equation of the line passing through the intersection of $2x+y-1=0$ and $3x-2y-5=0$ and at unit distance from the origin. The equation of the required family of lines is $(2x+y-1)+k(3x-2y-5)=0$ where k is a parameter. Also k should satisfy the condition</p> $\frac{-1-5k}{\sqrt{(2+3k)^2+(1-2k)^2}} = 1$ <p>Teachers should use as many practical examples as possible to illustrate the concept of locus, e.g. paths of a moving point, a moving line, a moving area and moving objects in daily life such as the path of a train running along the rails, the path described by a point on the rim of a rolling cylinder and the trajectory of a football moving in the air. Activities on the geometrical construction of some simple loci may be carried out.</p> <p>Teachers could point out that a straight line is an example of locus in a plane and can be described by the equation of the straight line in the Cartesian plane.</p>

72

Detailed Content	Time Ratio	Notes on Teaching
7.9 Equation of a Circle, Intersection of a Straight Line and a Circle	7* 8*	<p><i>Example</i> Given two fixed points $A(1, 0)$ and $B(0, 3)$. Find the locus of a point P moving such that its distances from A and B are always equal. The condition $PA=PB$ will lead to a straight line equation. The coordinates of any point on the straight line satisfy the equation of the line and the equation of the line determines the position of the line in the plane.</p> <p>Students should be led to derive the equation of a circle with centre at the origin and radius r from the distance formula. After several examples, they should discover the equations take the form $x^2+y^2=r^2$.</p> <p>Given an equation in the above form, students should recognize that it is a circle with centre at the origin and radius equal to r.</p> <p>Teachers may then consider a more general situation, viz., the centre is not at the origin but with coordinates (h, k). After some practice, students will discover the equations can take one of the following two forms:</p> $x^2+y^2+Dx+Ey+F=0$ <p>or $(x-h)^2+(y-k)^2=r^2$</p> <p>Given an equation in either of the above forms, students should recognize that it is a circle. They should also know where the centre lies and what the length of its radius is.</p> <p>The equation of a circle passing through three non-linear points should also be discussed.</p> <p>Does a straight line usually cut a circle at two points? Teachers should discuss all the possible cases in relation to the roots of a quadratic equation, especially when the quadratic equation has a double root. The idea of tangency in geometry may be given in terms of the algebraic condition $b^2-4ac=0$.</p> <p>Due amount of examples should be provided and the following show some of them.</p>

73

Detailed Content	Time Ratio	Notes on Teaching
7.10 Equation of Tangents to a Circle	4 5	<p><i>Example 1</i> A diameter of a circle has end points at $A(x_1, y_1)$ and $B(x_2, y_2)$. Find the equation of the circle.</p> <p><i>Example 2</i> The vertices of $\triangle ABC$ are $A(-6, 5)$, $B(-3, -4)$, $C(2, 1)$. Find the equation of the circumscribed circle of $\triangle ABC$.</p> <p><i>Example 3</i> Does the line $L: x-y-2=0$ cut the circle $C: x^2+y^2-4x-2y+4=0$? Give reason.</p> <p>Given a circle with equation $x^2+y^2+Dx+Ey+F=0$, students should be guided to derive the equation of the tangent to the circle at a given point (x_1, y_1). The technique of differentiation will be helpful here. Students should be able to obtain the equation $x_1x+y_1y+\frac{1}{2}D(x+x_1)+\frac{1}{2}E(y+y_1)+F=0$.</p> <p>Teachers may then consider a more general situation, viz., the equations of tangents from an external point to the given circle. Students should be shown how this can be done by solving a linear equation of the line passing through the external point and the equation of the circle.</p> <p>Further discussion on finding the equation of tangent by using the fact that the distance from the centre to the tangent is equal to the radius is required.</p> <p><i>Example 1</i> Show that $A(1, 4)$ is a point on the circle $C: x^2+y^2-2x-2y-7=0$. Hence, find the equation of the tangent to C at A.</p> <p><i>Example 2</i> Find the equations of the tangents to the circle $x^2+y^2+6x-2y+5=0$ which are parallel to the line $2x+y+3=0$.</p>

74

Detailed Content	Time Ratio	Notes on Teaching
7.11 Family of Circles	4 5	<p><i>Example 3</i> Two tangents are drawn from the point $M(-4, 4)$ to touch the circle $x^2+y^2-6x-6y-7=0$ at P and Q. Find, (a) the equations of the tangents; (b) $\angle PMQ$; (c) the length of each tangent segment; (d) the area of $\triangle MPQ$.</p> <p>Students should know that the solutions of some kinds of problems are simplified by using the equation of a family of circles. The following 3 cases should be discussed:</p> <ol style="list-style-type: none"> Family of concentric circles with centres at (a, b). The equation is $(x-a)^2+(y-b)^2=k$ ($k>0$). Family of circles passing through the points of intersection of the straight line $Ax+By+C=0$ and the circle $x^2+y^2+Dx+Ey+F=0$. The equation is $x^2+y^2+Dx+Ey+F+k(Ax+By+C)=0$. Family of circles passing through the points of intersection of two given circles with equations respectively equal to $x^2+y^2+D_1x+E_1y+F_1=0$ and $x^2+y^2+D_2x+E_2y+F_2=0$ The equation is $x^2+y^2+D_1x+E_1y+F_1+k(x^2+y^2+D_2x+E_2y+F_2)=0$. where $k \neq -1$. Teachers should explain to students that this equation will reduce to the equation of the common chord of the two circles if $k = -1$. If, in addition, the two circles touch each other, the common chord will become the common tangent of the circles. Examples such as the following can be introduced. <p><i>Example 1</i> Show that the equation $x^2+y^2-4x+2y+F=0$, where F is a constant, represents a family of concentric circles.</p>

75

Detailed Content	Time Ratio	Notes on Teaching
		<p><i>Example 2</i></p> <p>A circle passes through the points of intersection of the line $x+y=1$ and the circle $x^2+y^2-2x-2y+1=0$. If its centre is on the line $3x-y=3$, find its equation.</p> <p><i>Example 3</i></p> <p>A circle C passes through the point Q(1, 2) and the points of intersection of the circles $x^2+y^2-3x+2y-2=0$ and $x^2+y^2+x+3y-10=0$. Find the equation of C and the equation of the tangent to C at Q.</p>

76

77

Detailed Content	Time Ratio	Notes on Teaching
78		
Detailed Content	Time Ratio	
79		

Detailed Content	Time Ratio	Notes on Teaching
80		
Detailed Content	Time Ratio	
81		

82

Detailed Content	Time Ratio	Notes on Teaching
7.13 Simple Parametric Equations and Locus Problems	4 5	<div style="background-color: #cccccc; width: 100%; height: 100%; margin-bottom: 10px;"></div> <p>Parametric equation is a way of representation of the equation of a curve. The coordinates of any point (x,y) on the curve are related to a parameter t.</p> <p>Teachers may demonstrate that some parametric equations can be transformed into the equation $f(x, y)=0$ using the method of elimination. For example, $x=at^2$ and $y=2at$ can be rewritten as $y^2=4ax$. </p> <p>Students should recognize the parametric equations of straight lines and circles conics. Furthermore, teachers should emphasize that a function $f(x, y)=0$ can have more than one parametric representation.</p> <p>Problems involving loci in parametric forms should be provided to help students master the concept and skill. The following shows some of them.</p> <p><i>Example 1</i> Find an equation connecting x and y for each of the following parametric equations:</p> <p>(a) $x=t^2-2t, y=t^2+2.$</p> <p>(b) $x=\tan \alpha + \cot \alpha, y=\tan \alpha - \cot \alpha$</p>

83

Detailed Content	Time Ratio	Notes on Teaching
		<p><i>Example 2</i> By putting $y=tx$, find the parametric equations for the curve $x^3+y^3=6xy$.</p> <p><i>Example 3</i> Given the equation of a parabola ^{curve} $y^2=4ax$.</p> <p>(a) Prove that $P_1(at_1^2, 2at_1)$ and $P_2(at_2^2, 2at_2)$ are points on the parabola ^{curve}.</p> <p>(b) If M is the midpoint of P_1P_2, express the coordinates of M in terms of t_1 and t_2.</p> <p>(c) If OP_1 is perpendicular to OP_2 where O is the origin, find the equation of the locus of M.</p>
	11*+39 12*+35	