UNIT 8: Vectors in the Two-dimensional Space

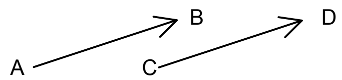
Specific Objectives:

1. To learn the concept and representation of vector quantities.
2. To learn some properties of and operations on vectors in two-dimensional space.
3. To understand the geometrical representation of vectors in two-dimensional space.
4. To apply the vector method particularly to the solution of some geometric problems.

84

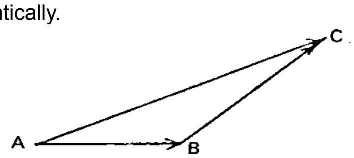
<table>
<tr><th>Detailed Content</th><th>Time Ratio</th><th>Notes on Teaching</th></tr>
<tr><td>8.1 Scalar and Vector Quantities Equality of Vectors, Zero Vector and Unit Vector</td><td><s>2</s>
3</td><td>Examples of scalar quantities such as mass, length, time and temperature and examples of vector quantities such as force, displacement, velocity and acceleration may be cited to illustrate the difference between scalar and vector quantities.

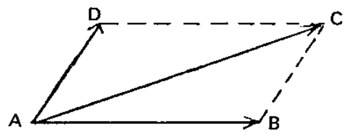
It is desirable to introduce the geometric representation of a vector (i.e. by a directed line segment) early as it would help students visualize the concept. Students should learn some common notations of vectors printed in text-books (such as a, $\overrightarrow{AB}$) as well as those used in writing (such as $\vec{a}$, $\underline{a}$); the corresponding notations for magnitude are ∣ a ∣, ∣ $\overrightarrow{AB}$ ∣, ∣ $\vec{a}$ ∣ and ∣ $\underline{a}$ ∣. Teachers should ensure students are in the habit of writing an arrow (→) over a letter e.g. $\vec{a}$ or writing the letter over a bar, $\underline{a}$ for a vector.

Teachers should emphasize to students that two vectors are equal if they have the same magnitude and direction. For example, if $\overrightarrow{AB} = \overrightarrow{CD}$ then ∣ $\overrightarrow{AB}$ ∣ = ∣ $\overrightarrow{CD}$ ∣and AB//CD.


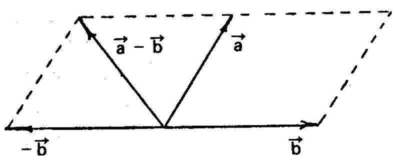
The idea of a zero vector that has zero magnitude and no specific direction should then be introduced. It is usually denoted by $\vec{0}$, or $\underline{0}$. The unit vector may be introduced as a vector, irrespective of direction, that has unit magnitude. It is usually used to specify direction.</td></tr>
</table>

85

<table>
<tr><th>Detailed Content</th><th>Time Ratio</th><th>Notes on Teaching</th></tr>
<tr><td>8.2 Sum and Difference of Vectors, Multiplication of a Vector by a Scalar</td><td><s>3</s>
4</td><td>Addition of vectors could best be illustrated by the triangle law, depicted diagramatically.


$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$
or the parallelogram law,


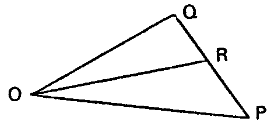
$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$
Practical examples such as the vector sum of two displacements and the resultant of two forces or velocities could be used. The polygon law as an extension of the triangle law could also be introduced.

The difference $\vec{a} - \vec{b}$ could be introduced as the sum of $\vec{a} + (-\vec{b})$.


The geometric meaning of a scalar multiple of a vector could then be introduced. Teachers should point out that when $\overrightarrow{AB} = k\overrightarrow{CD}$, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel and ∣ $\overrightarrow{AB}$ ∣= k∣ $\overrightarrow{CD}$ ∣; if k>0, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are in the same direction, whereas if k<0, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are in opposite direction. With the guidance of</td></tr>
</table>

indicates the parts deleted

86

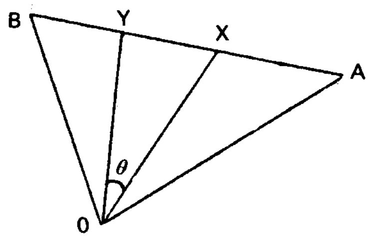
Detailed Content	Time Ratio	Notes on Teaching
		teachers, students should be able to discover the following rules illustrated with vector diagrams. $\vec{u}+\vec{v}=\vec{v}+\vec{u}$ $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$ $r(s\vec{u})=(rs)\vec{u}=s(r\vec{u})$ $(r+s)\vec{u}=r\vec{u}+s\vec{u}$ $r(\vec{u}+\vec{v})=r\vec{u}+r\vec{v}$ Examples like the following may be given. *Example 1* ABCD is a square and X, Y are mid-points of BC and CD. If $\overrightarrow{AX}=\vec{p}$ and $\overrightarrow{AY}=\vec{q}$, find (*a*) $\vec{p}$ and $\vec{q}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{BC}$; (*b*) $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\vec{p}$ and $\vec{q}$. *Example 2* If ABCD is a parallelogram, show that (*a*) $\overrightarrow{AB}+\overrightarrow{DB}+\overrightarrow{CB}=2\overrightarrow{DB}$; (*b*) $\overrightarrow{BC}+\overrightarrow{BA}+\overrightarrow{BD}+2\overrightarrow{AC}+2\overrightarrow{CB}-2\overrightarrow{AD}=\vec{0}$ *Example 3* In the diagram, m PR=n RQ where m, n are positive constants, show that $m\overrightarrow{OP}+n\overrightarrow{OQ}=(m+n)\overrightarrow{OR}$. 

87

Detailed Content	Time Ratio	Notes on Teaching
8.3 Representation of Vectors in Rectangular Coordinate System	~~2~~ 3	The unit vectors $\vec{i}$ and $\vec{j}$ should be introduced first and then the fact that any vector in the rectangular coordinate plane can be expressed in terms of $\vec{i}$ and $\vec{j}$ can be explained by means of examples. For any vector $\vec{u}=x\,\vec{i}\pm y\,\vec{j}$, the magnitude and direction given by $\lvert\vec{u}\rvert=x^2+y^2$ and $\theta=\tan^{-1}(\frac{y}{x})$ should be taught. The angle θ made with the positive x-axis can be determined by considering the quadrant in which (x, y) lies. For example, the vector $\vec{u}=\vec{i}+\vec{j}$ has magnitude $\lvert\vec{u}\rvert=\sqrt{2}$ and direction $\theta=\tan^{-1}(1)=45^\circ$, but the vector $\vec{v}=\vec{i}-\vec{j}$ has magnitude $\lvert\vec{v}\rvert=\sqrt{2}$ and direction given by $\theta=\tan^{-1}(-1)=315^\circ$. The ideas of equality of two vectors, addition and subtraction of two vectors should also be revised in the context of the rectangular coordinate system and ample exercises should be given.
8.4 Scalar Product of Two Vectors	~~4~~ 5	After learning the representation of $\vec{u}=x_1\vec{i}+y_1\vec{j}$ and $\vec{v}=x_2\vec{i}+y_2\vec{j}$, students are ready to learn the definition of scalar product or dot product of two vectors. $\vec{u}\cdot\vec{v}=\lvert\vec{u}\rvert\lvert\vec{v}\rvert\cos\theta$, where θ is the angle between $\vec{u}$ and $\vec{v}$, if $\vec{u}$, $\vec{v}$ are non-zero vectors $\vec{u}\cdot\vec{v}=0$ if either $\vec{u}=\vec{0}$ or $\vec{v}=\vec{0}$ and the other definition $\vec{u}\cdot\vec{v}=x_1x_2+y_1y_2$ may also be derived by noting that $\vec{i}\cdot\vec{i}=\vec{j}\cdot\vec{j}=1$ and $\vec{i}\cdot\vec{j}=\vec{j}\cdot\vec{i}=0$. Teachers should remind students to write the dot (·) between vectors in dot products. Teachers may then guide students to deduce the following properties of scalar products. $\vec{u}\cdot\vec{u}=\lvert\vec{u}\rvert^2$ $\vec{u}\cdot\vec{v}=\vec{v}\cdot\vec{u}$ $k(\vec{u}\cdot\vec{v})=(k\vec{u})\cdot\vec{v}=\vec{u}\cdot(k\vec{v})$ $\vec{u}\cdot(\vec{v}+\vec{w})=\vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w}$

indicates the parts deleted

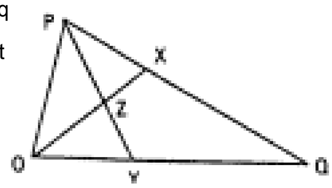
88

Detailed Content	Time Ratio	Notes on Teaching
		By extending the fact that two vectors are equal if and only if they have the same corresponding $\vec{i}$ and $\vec{j}$ components, to any two vectors expressed in terms of two non-parallel vectors $\vec{u}$ and $\vec{v}$, students should find it intuitively clear that if $a_1\vec{u}+b_1\vec{v}=a_2\vec{u}+b_2\vec{v}$ then $a_1=a_2$ and $b_1=b_2$. Teachers can provide some examples for illustration. The formal definition of a basis should not be introduced. Exercises like the following may be given. *Example 1* Given points A(1, 3), B(-2, 4), C(-1, 5), find (*a*) $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in terms of $\vec{i}$ and $\vec{j}$, (*b*) $\overrightarrow{BA}\cdot\overrightarrow{BC}$ and (*c*) $\angle$ ABC (correct to 0.1^0). *Example 2* In the figure, AX=XY=YB.  If $\overrightarrow{OA}=\vec{i}-\vec{j}$ and $\overrightarrow{OB}=10\vec{i}+5\vec{j}$, (*a*) express $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of $\vec{i}$ and $\vec{j}$. (*b*) find $\lvert\overrightarrow{OX}\rvert$ and $\lvert\overrightarrow{OY}\rvert$ and (*c*) find the value of cos θ.

89

Detailed Content	Time Ratio	Notes on Teaching
8.5 Applications of Vectors, Division of a Line segment, Parallelism and Perpendicularity	~~3~~ 5	*Example 3* It is given that $\overrightarrow{AB}=3\vec{i}-4\vec{j}, \overrightarrow{AC}=a\vec{i}-12\vec{j}$ and $\overrightarrow{AB}=-8\vec{i}+b\vec{j}$. Find: (*a*) a if A, B and C are collinear, (*b*) b if AD $\perp$ AB. The vector method can be applied to solving problems in various topics in the syllabus, for example, in coordinate geometry and trigonometry. Starting with the concept of the position vector of a point, teachers could guide students to derive the position vector of a point P, which divides AB in the ratio m:n as $\overrightarrow{OP}=\frac{1}{m+n}(n\vec{a}+m\vec{b})$ where $\vec{a}=\overrightarrow{OA}$ and $\vec{b}=\overrightarrow{OB}$ are position vectors of A and B respectively. In trigonometry, by considering the dot product of $\overrightarrow{AB}\cdot\overrightarrow{AC}$ and $\overrightarrow{BC}\cdot\overrightarrow{BC}$, teachers can lead students to derive the cosine formula $a^2=b^2+c^2-2bc\cos A$. Students are also expected to be familiar with the facts (1) If $\overrightarrow{AB}\cdot\overrightarrow{BC}$ =0 then AB $\perp$ BC. (2) If $\overrightarrow{AB}=k\overrightarrow{CD}$ then AB//CD. (3) If $\overrightarrow{AB}=k\overrightarrow{AC}$ then A, B, C are collinear. The vector method is particularly rich in applications to the proving of well-known results in geometry. For example, (1) The line segment joining the mid-points or two sides of a triangle is parallel to and equal to one half of the third side. (2) The line segment joining the centre of a circle to the mid-point of a chord is perpendicular to the chord

indicates the parts deleted

Detailed Content	Time Ratio	Notes on Teaching
		(3) An angle in a semi-circle is 90^0. (4) The diagonals of a rhombus bisect each other perpendicularly. (5) The altitudes of a triangle are concurrent. (6) The medians of a triangle are concurrent. Other examples may include the following. *Example 1* In the figure, $\overrightarrow{OP}=\vec{p}, \overrightarrow{OQ}=\vec{q}, 2\overrightarrow{PX}=\overrightarrow{XQ}$ and $3\overrightarrow{OY}=2\overrightarrow{YQ}$. (*a*) Express $\overrightarrow{OX}$ in terms of $\vec{p}$ and $\vec{q}$ (*b*) If $\overrightarrow{OZ}=\lambda\overrightarrow{OX}, \overrightarrow{PZ}=\mu\overrightarrow{PY}$, show that $5\lambda-6\mu=0$ and $2\lambda+3\mu-3=0$. (*c*) By solving for λ, find PZ:PY.  *Example 2* Show that if $\vec{u}$ and $\vec{v}$ are any vectors, then (*a*) $\lvert \vec{u}\cdot\vec{v} \rvert \le \lvert\vec{u}\rvert\,\lvert\vec{v}\rvert$ (*b*) $\lvert \vec{u}+\vec{v} \rvert \le \lvert\vec{u}\rvert+\lvert\vec{v}\rvert$.
	~~14~~ 20	

UNIT 9: Integration

Specific Objectives:

1. To recognize indefinite integration as a reverse process of differentiation.
2. To understand the properties of indefinite integrals.
3. To recognize some of the geometric and physical applications of indefinite integration.
4. To recognize and use some standard formulae of indefinite integration and the method of substitution.
5. To understand the underlying principle of definite integral as a limit of a sum.
6. To understand and apply the basic properties of definite integration.
7. To apply definite integration to find plane areas and volumes of solids of revolution.

Detailed Content	Time Ratio	Notes on Teaching
9.1 Indefinite Integral	~~1~~ 2	Indefinite Integration is introduced as the reverse process of differentiation, which leads to the simple properties. $\int (u \pm v)\,dx = \int u\,dx \pm \int v\,dx$ and $\int ku\,dx = k\int u\,dx$. The terms primitive function, integral sign, integrand and constant of integration should be introduced. Teachers should emphasize that the integral $\int u\,dx$ is only a notation and it doesn't mean that u is multiplied by dx.
9.2 Integration of Simple Functions and Simple Applications	~~4~~ 5	Ample examples and exercises should be provided to help students acquaint with the integration formulae; $\int x^n dx = \frac{1}{n+1}x^{n+1} + C (n \ne -1)$ (The case when n=0 should be discussed.) $\int \sin x\,dx = -\cos x + C$ $\int \cos x\,dx = \sin x + C$ $\int \sec^2 x\,dx = \tan x + C$ $\int \sec x \tan x\,dx = \sec x + C$

indicates the parts deleted