UNIT 13: Bernoulli, binomial, geometric and Poisson distributions and their applications

Specific Objectives:

1. To understand the concept of a random variable and a probability function
2. To learn the probability function for the four different distributions.
3. To recognize the mean and variance of the distributions.
4. To apply the formulae to practical problems.

	Detailed Content	Time Ratio	Notes on Teaching
13.1	**Random variable, probability function, and discrete probability distribution**	2	To begin with this unit, the definition of random variable should be introduced. Next, students should acquire 'preliminary understanding, for the meaning of discrete probability distribution. Probability function should also be defined (i.e. A function which assigns a probability $f(x)$ to each random variable x is called a probability function or probability distribution.)
13.2	**Bernoulli distribution**	2	Teachers should emphasize that Bernoulli distribution applies when an experiment has only two possible outcomes, namely, 'failure' or 'success'. The probability function of Bernoulli distribution is given by $f(x\,;p)=p^x(1-p)^{1-x}$ for $x = 0, 1$ where p is the probability of success.
13.3	**Binomial distribution**	3	It should be emphasized that when n independent Bernoulli trials are carried out, the probability distribution becomes a Binomial distribution which is given by $f(x\,;n\,,p)=\binom{n}{r}p^x(1-p)^{n-x}$ for $x = 0, 1, 2, \ldots, n$. Use of Binomial distribution table is not expected.
13.4	**Geometric distribution**	3	The probability function of this distribution is $f(x\,;p)=p(1-p)^{x-1}$ for $x = 1, 2, \ldots$ Students should be able to distinguish the probability function of geometric distribution from that of Bernoulli distribution. The former gives the probability of getting the 1st success in the last of x trails, while the latter gives the probability of getting x successes in one trial.

<table>
<tr><th></th><th>Detailed Content</th><th>Time Ratio</th><th>Notes on Teaching</th></tr>
<tr><td>13.5</td><td>Poisson distribution</td><td>3</td><td>It should be made clear to students that a Poisson distribution is actually a binomial distribution under the limiting condition that $n \to \infty$ and $p \to 0$ with $np = \lambda$ = constant. Under this restriction, the limiting form of the Binomial distribution is
$f(x\,;\lambda)=\frac{\lambda^x e^{-x}}{x\,!}$ for $x = 0, 1, 2, \ldots$
A random variable having this probability distribution is said to have the Poisson distribution.
The proof of the probability function and use of Poisson distribution table are not expected.</td></tr>
<tr><td>13.6</td><td>Means and variances</td><td>3</td><td>Knowledge of formulae for their means and variances is expected but proofs of these formulae should not be emphasized.
<table>
<tr><th>Distribution</th><th>Mean</th><th>Variance</th></tr>
<tr><td>Bernoulli (p)</td><td>p</td><td>$p(1-p)$</td></tr>
<tr><td>Binomial (n, p)</td><td>np</td><td>$np(1-p)$</td></tr>
<tr><td>Geometric (p)</td><td>$\frac{1}{p}$</td><td>$\frac{1-p}{p^2}$</td></tr>
<tr><td>Poisson(λ)</td><td>λ</td><td>λ</td></tr>
</table></td></tr>
<tr><td>13.7</td><td>Applications of Bernoulli, binomial, geometric and Poisson distributions</td><td>4</td><td>Throughout this unit, examples and discussions are the essential features. Attention should be paid to examples from daily life.</td></tr>
<tr><td></td><td></td><td>20</td><td></td></tr>
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