**UNIT 2: The Binomial Expansion**

*Specific Objectives:*

1. To learn the binomial expansion of $(1+x)^n$ when $n$ is a positive integer.

2. To study the expansion as an infinite series when n is not a positive integer and $\lvert x\rvert < 1$

| Detailed Content | | Time Ratio | Notes on Teaching |
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| **2.1** | **The expansion** $(1+x)^n = \sum_{r=0}^{n} C_r^n x^r$ **, when *n* is a positive integer.** | 3 | The formal proof of the expansion is not required. Determination of the greatest term and relations between coefficients are excluded. However, the properties of the binomial expansion should include<br>(a) the expansion contains $n + 1$ terms;<br>(b) the binomial coefficients $C_r^n$ are all integers.<br>The Pascal triangle should be studied in relation to the coefficients in the expansion. Students are expected to know<br>$\sum_{r=1}^{n}(ax_r \pm by_r) = a\sum_{r=1}^{n} x_r \pm b\sum_{r=1}^{n} y_r$<br>$\sum_{r=1}^{n}(x_r + y_r)^2 = \sum_{r=1}^{n} x_r^{\,2} + 2\sum_{r=1}^{n} x_r y_r + \sum_{r=1}^{n} y_r^{\,2}$ |
| **2.2** | **The expansion of** $(1+x)^n$ **when *n* is not a positive integer and** $\lvert x\rvert < 1$**.** | 5 | Students should learn what happens to the coefficient $\frac{n(n-1)\cdots(n-r+1)}{r!}$ of the general term i.e. the $(r + 1)$st term of the expansion; when $n$ is a positive integer, it vanishes when $r = n + 1$. If $n$ is not a positive integer, it will never vanish in the latter case, the expansion is given in the form<br>$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots$<br>To convince students that the infinite series converges only when $-1 < x < 1$ or $\lvert x\rvert < 1$, the following example may be given:<br>Consider the expansion $(1-x)^{-1} = 1 + x + x^2 + \cdots + x^r + \cdots$ |



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| | | Putting x = −1 we have expansion 1−1+1−1+1−1+ …<br>For odd number of terms, the sum =1 and for even number of terms, the sum = 0 and is never equal to L.H.S. which is equal to $\frac{1}{2}$.<br>The meaning of absolute value should also be taught, however informal discussion on the intuitive meaning of convergence and divergence is expected.<br>Expansions for particular values of $n$ should be studied, such as<br>(a) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$<br>(b) $(1-x)^{\frac{1}{2}} = 1 + \frac{x}{2} + \frac{1\cdot 3}{2!}\left(\frac{x}{2}\right)^2 + \frac{1\cdot 3\cdot 5}{3!}\left(\frac{x}{2}\right)^3 + \cdots$<br>(c) $(1+x)^{-\frac{1}{3}} = 1 - \frac{x}{3} + \frac{1\cdot 4}{2!}\left(\frac{x}{3}\right)^2 + \frac{1\cdot 4\cdot 7}{3!}\left(\frac{x}{3}\right)^3 + \cdots$<br>Students should be able to obtain expansion like $\frac{(1+x)^{\frac{1}{3}}}{(1-x)^2}$ in ascending powers of $x$ to a specified number of terms. |
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