UNIT 6: Differentiation

Specific Objectives:

1. To acquire general techniques of differentiation

	Detailed Content	Time Ratio	Notes on Teaching
6.1	**Basic differentiation rules**	4	The following rules should be taught. Their proofs may be provided for completeness. (a) $\frac{\mathrm{d}}{\mathrm{d}x}k = 0$, k is constant (b) $\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$; (c) $\frac{\mathrm{d}}{\mathrm{d}x}kf(x) = k\frac{\mathrm{d}}{\mathrm{d}x}f(x)$; (d) $\frac{\mathrm{d}}{\mathrm{d}x}[f(x) \pm g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) \pm \frac{\mathrm{d}}{\mathrm{d}x}g(x)$ (e) $\frac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = g(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x) + f(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}g(x)$ (f) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$, $g(x) \neq 0$
6.2	**Differentiation of composite functions and inverse functions**	4	Chain rule should be introduced in order to find the derivatives of composite functions. The derivatives of inverse functions can be treated as a special case of the derivatives of composite functions. Rigorous proofs on $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x}$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$ are not required however sufficient illustrations on the application of the chain rule are expected. Differentiation of simple implicit function is included while differentiation of parametric equations is not required.

	Detailed Content	Time Ratio	Notes on Teaching
6.3	**Differentiation of e^x and $\ln x$**	5	$\frac{\mathrm{d}e^x}{\mathrm{d}x} = e^x$ may be proved by assuming that it is legitimate to differentiate the infinite series $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$ term by term. The derivative of $\ln x$ may be treated as the derivative of the inverse function of e^x . When y is a complicated function of x and especially when it involves a variable as index, the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ may sometimes be more easily found by logarithmic differentiation. The following rules should be taught: (a) $\frac{\mathrm{d}}{\mathrm{d}x}(\ln x) = \frac{1}{x}$ (b) $\frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a$ (c) $\frac{\mathrm{d}}{\mathrm{d}x}\log_a x = \frac{\ln a}{x}$ Differentiation of functions like x^x , e^{x^2} and $\log_a\sqrt{x+1}$ is expected.
6.4	**Second derivative**	2	The symbol $f''(x)$ and should $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ be introduced to students. The other higher order derivatives may be omitted.
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