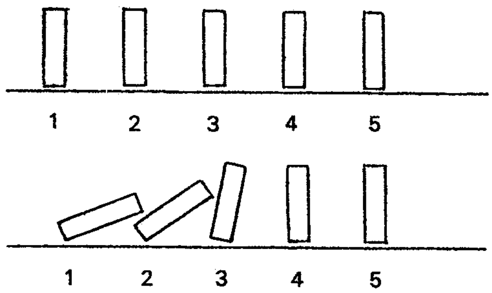
Unit A3: Mathematical Induction

Objective: (1) To understand the Principle of Mathematical Induction.
(2) To apply the Principle of Mathematical Induction to prove propositions involving integers.
(3) To be able to modify the Principle of Mathematical Induction to suit different purposes.

Detailed Content	Time Ratio	Notes on Teaching
3.1 The Principle of Mathematical Induction and its applications	6	As an introduction, students may be asked to guess the formula for the sum of the first n odd positive integers by considering $1 = 1$ $1 + 3 = 4$ $1 + 3 + 5 = 9$ …………….. …………….. After the proposition $1 + 3 + 5 + . . . + (2n - 1) = n^2$ is established, students should be led to understand that they should not claim this result is true by considering only a finite number of cases. An illustration of the use of mathematical induction should then follow. The Principle of Mathematical Induction should be formally written on the board. Teachers may find it easier to explain the Principle by referring to a game of dominoes:  Examples should be done on the applications to the summation of series, divisibility and proving inequalities. The Principle of Mathematical Induction may

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3.2 Other common variations of the Principle of Mathematical Induction and their applications **Excluding Backward Induction**	~~6~~ 5	sometimes modified to suit different cases. Examples should also be used to illustrate that both conditions of the Principle must be satisfied to prove a proposition. Further applications include the proofs of (i) the binomial theorem for positive integral indices (ii) De Moivre's theorem for a positive integer n (iii) some propositions involving determinants and square matrices (iv) Leibniz's Theorem and some propositions involving the nth derivative. As further development, teachers may discuss with the students cases where the Principle has to be modified. *Example:* $x^n + y^n$ is divisible by x + y for all positive odd integers n. *Example:* The Fibonacci sequence is defined as follows: $a_0 = 0, a_1 = 1$ $a_{n+1} = a_{n-1} + a_n$ for all natural numbers n. Prove that $a_n = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n]$ for all n Teachers should point out that a variation of the Principle is required for the proof of these examples. A few more examples on sequences defined by recurrence relations may be discussed.
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indicates the parts deleted