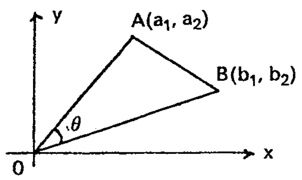
Unit A4: inequalities

Objectives: (1) To learn the elementary properties of inequalities.
(2) To prove simple absolute inequalities.
(3) To solve simple conditional inequalities.

22

Detailed Content	Time Ratio	Notes on Teaching
4.1 Absolute inequalities	6	Students are expected to use the symbols a > b and a ≥ b correctly. Elementary properties of inequalities including (i) For any real number x, $x^2 \geq 0$ (ii) a > b > 0 and n is a positive integer, then $a^n > b^n$ and $\sqrt[n]{a} > \sqrt[n]{b}$ (iii) If a > b > 0 and x > y > 0, then ax > by should be revised. Formal proofs of these properties are not required. However, students are expected to be able to deduce simple absolute inequalities from the elementary properties. The following techniques in proving absolute inequalities should be emphasized: *Example*: Prove that $E_1 \geq E_2$ Proof: $E_1 - E_2 = \ldots\ldots$ $= \ldots\ldots$ $= \ldots\ldots$ ≥ 0 $\therefore E_1 \geq E_2$
4.2 A.M. ≥ G.M.	4	The proof of A.M. ≥ G.M. may be provided up to four variables in the first instance and the general proof need not be emphasized. If required, teachers may apply backward induction. Students are expected to apply this result to n variables.
4.3 Cauchy—Schwarz's inequality	3	Students are expected to understand that the necessary and sufficient conditions for the quadratic form $ax^2 + bx + c$ to be positive for all real values of x are a > 0 and $b^2 - 4ac < 0$. Students should be able to apply this result to problems such as finding the range of C for which the expression $Cx^2 + 4x + C + 3$ is positive for all values of x. The above result may be used to prove the Cauchy–Schwarz's inequality.

23

Detailed Content	Time Ratio	Notes on Teaching
		A geometric interpretation of the Cauchy—Schwarz's inequality for n =2 may be given by using points on the coordinate plane. $\cos\theta = \dfrac{OA^2 + OB^2 - AB^2}{2OA \cdot OB}$ $= \dfrac{(a_1^2 + a_2^2) + (b_1^2 + b_2^2) - [(a_1 - b_1)^2 + (a_2 - b_2)^2]}{2\sqrt{(a_1^2 + a_2^2)}\sqrt{(b_1^2 + b_2^2)}}$ $= \dfrac{a_1b_1 + a_2b_2}{\sqrt{a_1^2 + a_2^2}\sqrt{b_1^2 + b_2^2}}$ $\cos^2\theta = \dfrac{(a_1b_1 + a_2b_2)^2}{(a_1^2 + a_2^2)(b_1^2 + b_2^2)}$  Since $\cos^2\theta \leq 1$, $(a_1b_1 + a_2b_2)^2 \leq (a_1^2 + a_2^2)(b_1^2 + b_2^2)$ Students are also expected to apply the Cauchy—Schwarz's inequality in solving simple problems.
4.4 Conditional inequalities	7	The concepts of intervals on the real number line should be revised. The definition and properties of the absolute value of a real number should be discussed. Students should be able to solve linear inequalities, quadratic inequalities and inequalities of higher degrees in x. Solutions of inequalities involving absolute values such as $\|ax^2 + bx + c\| \geq d$, $\|x - a\| + \|x - b\| \geq c$ and $(x - a)\|x - b\| \geq c$ are required. Teachers should also discuss with students inequalities of the form $\dfrac{P(x)}{Q(x)} \geq 0$, where P(x) and Q(x) are polynomials in x. Compound inequalities of the above inequalities should also be taught.
	20	

indicates the parts deleted