Unit A5: The Binomial Theorem for Positive Integral Indices

Objective: (1) To learn and apply the binomial theorem for positive integral indices.
(2) To study the simple properties of the binomial coefficients.

Detailed Content	Time Ratio	Notes on Teaching
5.1 The binomial theorem for positive integral indices	3	Students should learn how to evaluate n! and C_r^n. The binomial theorem for positive integral indices may be proved by the Principle of Mathematical Induction. Discussions concerning the notation C_r^n should be related to its use as a binomial coefficient. The Pascal's triangle in relation to the coefficients C_r^n in the binomial expansion may be discussed. Students are not expected to know the general binomial theorem.
5.2 Application of the binomial theorem for positive integral indices	5	Students should be able to expand expressions using the binomial theorem for positive integral indices. The determination of a particular term or a particular coefficient in a binomial expansion should also be taught. Students are expected to be able to find the greatest term and the greatest coefficient in a binomial expansion. Applications to numerical approximation should be discussed.
5.3 Simple properties of the binomial coefficients	5	Students should know that both the notations C_r^n and $\binom{n}{r}$ may be used to represent the binomial coefficients. Discussions should include simple properties of the binomial coefficients and the relations between these coefficients such as $C_0^n + C_1^n + C_2^n + \ldots + C_n^n = 2^n$; $\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{n}^2 = \frac{(2n)!}{(n!)^2}$ and similar relations. N.B. Permutation and combination may be used to introduce the binomial theorem but problems concerning permutation and combination are not required. Problems involving the use of differentiation and integration may be taught after students have learnt calculus.
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Unit A6: Polynomials and Equations

Objective: (1) To learn the properties of polynomials with real coefficients in one variable.
(2) To learn division algorithm, remainder theorem and Euclidean algorithm and their applications.
(3) To resolve rational functions into partial fractions.
(4) To learn the properties of roots of polynomial equations with real coefficients in one variable.

Detailed Content	Time Ratio	Notes on Teaching
6.1 Polynomials with real coefficients in one variable	5	Students are expected to know the general form of a polynomial with real coefficients in one variable and the following terms: degree of a non-zero polynomial, leading coefficient, constant term, monic polynomial, null or zero polynomial. Definitions of equality, sum, difference and product of two polynomials should also be studied. Also from definition, it is clear that for non-zero polynomials f(x), g(x) deg { f(x) g(x) } = deg f(x) + deg g(x). and deg {f(x) + g(x)} $\leq$ max {deg f(x), deg g(x)} The greatest common divisor (G.C.D.) or highest common factor (H.C.F.) of two non-zero polynomials should be defined. Students should clearly distinguish between division algorithm and Euclidean algorithm. By the division algorithm, the remainder theorem can be proved. Since students have studied the remainder theorem in lower forms, more difficult problems on this theorem can -be given. The Euclidean algorithm is a method of finding the G.C.D. of two polynomials. Some problems on finding the G.C.D. of two polynomials should be given as exercise.
6.2 Rational functions	4	A rational function should be defined first. Students may come across partial fractions the first time. Teacher may quote a simple example such as $\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$. The fractions $\frac{1}{x}$ and $\frac{1}{x+1}$ are called partial fractions. The rules for resolving a proper rational function into partial fractions should be clearly stated and examples studied. It should be emphasized and illustrated by examples that if a given rational function is improper, it should first be expressed as the sum of a polynomial and a proper fraction. Applications of partial fractions should be studied.

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Detailed Content	Time Ratio	Notes on Teaching
		Examples: 1. Express $\dfrac{x^2-8x+9}{(x+1)(x-2)^3}$ in partial fractions. 2. Resolve $\dfrac{x^4}{(x-a)(x^2+a^2)}$ into partial fractions. 3. Evaluate $\sum_{r=1}^{n}\dfrac{1}{r(r+1)}$
6.3 Polynomial equations with real coefficients in one variable	6	For the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ with α, β as roots, students should be familiar with the relations $\alpha+\beta=-\dfrac{b}{a}$, $\alpha\beta=\dfrac{c}{a}$ For the general polynomial equation of degree n, $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 = 0$ or $\sum_{k=0}^{n} a_kx^k = 0$, the following theorem gives the relations between coefficients and roots: If $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the roots of the polynomial equation $f(x) = 0$, then the sum s_k of all possible products of the α_j's taken k at a time (k = 1 , 2, . . . , n) is equal to $(-1)^k\dfrac{a_{n-k}}{a_n}$ i.e. if $f(x)=a_n(x-\alpha_1)(x-\alpha_2)\ldots(x-\alpha_n)$, then $s_1=\alpha_1+\alpha_2+\ldots+\alpha_n=-\dfrac{a_{n-1}}{a_n}$, $s_2=\alpha_1\alpha_2+\alpha_2\alpha_3+\ldots+\alpha_{n-1}\alpha_n=\dfrac{a_{n-2}}{a_n}$, $s_3=\sum\alpha_1\alpha_2\alpha_3=-\dfrac{a_{n-3}}{a_n}$, ………………………… …………………………….. $s_n=\alpha_1\alpha_2\ldots\alpha_n=(-1)^n\dfrac{a_0}{a_n}$.

Detailed Content	Time Ratio	Notes on Teaching
		The following properties should be studied in detail: (i) The number of distinct zeros of a non-zero polynomial is less than or equal to the degree of the polynomial. (ii) If the polynomial equation with integral coefficients has a rational root of the form $\dfrac{p}{q}$ where p and q are coprime integers, then p is an exact divisor of the constant term and q is an exact divisor of the leading coefficient of the polynomial. (iii) The condition for repeated (multiple) roots: For the polynomial equation $f(x) = 0$ to have $x = \alpha$ as a repeated root, it is necessary and sufficient that $f(\alpha) = 0$ and α is also a root of the equation $f'(x) = 0$ where $f'(x)$ denotes the derivative of $f(x)$. The more general form will be: For any positive integer k, α is a root of multiplicity k + 1 of the equation $f(x) = 0$ if and only if $f(\alpha) = 0$ and α is a root of multiplicity k of $f'(x) = 0$. and α is a root of multiplicity k + 1 of the equation $f(x) = 0$ if and only if α is a common root of $f(x) = 0$, $f'(x) = 0$ $\vdots$ $f^{(k)}(x)=0$ but not of $f^{(k+1)}(x)=0$ N.B. Complex roots occurring in conjugate pairs will be treated in the study of complex number in Unit A10.
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