Unit A8: Matrices

Objective: (1) To learn the concept and operations of matrices.
(2) To learn the properties and operations of square matrices of order 2 and 3 and their determinants.
(3) To apply matrices to two dimensional geometry.

38

Detailed Content	Time Ratio	Notes on Teaching
8.1 Matrices and their operations	4	The general form of a matrix with m rows and n columns, namely an $m \times n$ matrix, should be introduced. Students should know the operations: addition, subtraction, multiplication and scalar multiplication of matrices and study their properties. The fact that, in general, $AB \neq BA$ holds for matrices A and B should be mentioned and explained. Terms like zero matrix, identity matrix and the transpose of a matrix should be introduced.
8.2 Square matrices of order 2 and 3	9	The definition of square matrices and their determinants should be defined. The concepts and uses of singular and non-singular matrices should also be made clear to students. Students should be able to evaluate determinants of square matrices and find the inverses for non-singular matrices. They are also expected to have knowledge of simple properties of inverses and determinants like: A. Properties of inverse (i) The inverse of a matrix is unique. (ii) A square matrix has inverse if and only if it is non-singular. (iii) If A-is non-singular, then AB = 0 implies B = 0. (iv) If A is non-singular, then AB = AC implies B = C. (v) If A, B are non-singular, λ is a non-zero scalar and n is a positive integer, then $AB, A^{-1}, A^t, \lambda A, A^n$ are non-singular and $(AB)^{-1} = B^{-1}A^{-1}$, $(A^{-1})^{-1} = A$, $(A^t)^{-1} = (A^{-1})^t$, $(\lambda A)^{-1} = \lambda^{-1}A^{-1}$, $(A^n)^{-1} = (A^{-1})^n$. B. Properties of determinant (i) If two rows (or columns) of a determinant are identical or proportional, the value of the determinant is zero. (ii) The interchange of two rows (or columns) changes the sign of the determinant without altering its numerical value.

39

Detailed Content	Time Ratio	Notes on Teaching
		(iii) The value of a determinant is unaltered by changing its rows into columns (or vise versa), i.e. $\det A^t = \det A$ or $\lvert A^t \rvert = \lvert A \rvert$ (iv) If every element in any row (column) is multiplied by the same number, then the value of the determinant is multiplied by that number. (v) The determinant of the product of two square matrices of the same order is equal to the product of the determinants of the matrices, i.e. $\det AB = \det A \cdot \det B$ or $\lvert AB \rvert = \lvert A \rvert \lvert B \rvert$
8.3 Applications to two dimensional geometry	8	Students should be familiar with matrix representation of a point and a vector and, furthermore, reflections, rotations, enlargements, shears, translations and their compositions. A few examples of such transformations, represented by 2 x 2 matrices are given below: (i) Reflection in the line $y = (\tan\theta)x$ is given by the matrix $A = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ (ii) Rotation through an angle ϕ about the origin is given by the matrix $B = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$ (iii) Enlargement about origin with scale factor $k \neq 0$ is given by the matrix $C = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ (iv) Shear parallel to x-axis with factor k is given by the matrix $D = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ For (iii) and (iv), the effect on shape.and area should be discussed. It should be made clear to students that under all these transformations of the plane, each point P(x, y) will be transformed to a new point P'(x', y') satisfying $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$ where T stands for a transformation. Elaboration on the composition of the transformations is essential to enable students to have a thorough understanding on matrix multiplication. *Example:* The effect of reflection in the line $y = (\tan\theta)x$ followed by a rotation through an angle θ about the origin is given by the product BA complying with the convention $BA\binom{x}{y} = \binom{x'}{y'}$ as mentioned above. It should be emphasized that the product is to be interpreted from right to left: first apply transformation A, then apply transformation B.

indicates the parts deleted

Detailed Content	Time Ratio	Notes on Teaching
		Other composition of transformations like the following may be mentioned: *Example:* The transformation having equations $\begin{cases} x' = y + 2 \\ y' = -x + 4 \end{cases}$ whose representation in matrix form is $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ The transformation with matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is a clockwise rotation about the origin through 90°. Hence, it could be viewed as a rotation followed by a translation, however, with (3, 1) as the invariant point the transformation can be regarded as a clockwise rotation about (3, 1) through 90°.
	21	

Unit A9: System of Linear Equations in 2 or 3 Unknowns

Objective: (1) To solve a system of linear equations using Gaussian elimination.
(2) To recognize the existence and uniqueness of solution.

Detailed Content	Time Ratio	Notes on Teaching
9.1 Gaussian elimination and Echelon form	5	A matrix which satisfies the following 2 properties is said to be in Echelon form: (1) The 1st k rows are non-zero; the other rows are zero. (2) The 1st non-zero element in each non-zero row is 1, and it appears in a column to the right of the 1 st non-zero element of any preceding row. *Example:* The following 5 x 8 matrix is in Echelon form: $\begin{pmatrix} 0 & 1 & * & * & * & * & * & * \\ 0 & 0 & 1 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ Students should be able to solve a system of linear equations in two or three unknowns by using Gaussian elimination, which reduces a matrix in Echelon form by elementary operations on its rows. *Example:* Solve the system $\begin{cases} x_1 + 2x_2 + x_3 = 2 \\ 3x_1 + x_2 - 2x_3 = 1 \\ 4x_1 - 3x_2 - x_3 = 3 \\ 2x_1 + 4x_2 + 2x_3 = 4 \end{cases}$ The augmented matrix $\begin{pmatrix} 1 & 2 & 1 & 2 \\ 3 & 1 & -2 & 1 \\ 4 & -3 & -1 & 3 \\ 2 & 4 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -5 & -5 & -5 \\ 0 & -11 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\sim \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -11 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ The original system of equations is equivalent to the system of equations $\begin{cases} x_1 - x_3 = 0 \\ x_2 + x_3 = 1 \\ x_3 = 1 \end{cases}$ which gives $x_1 = 1$, $x_2 = 0$, $x_3 = 1$.

indicates the parts deleted