| Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|
| | | Other composition of transformations like the following may be mentioned:<br>*Example:*<br>The transformation having equations<br>$\begin{cases} x' = y + 2 \\ y' = -x + 4 \end{cases}$ whose representation in matrix form is<br>$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$<br>The transformation with matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is a clockwise rotation about the origin through 90°. Hence, it could be viewed as a rotation followed by a translation, however, with (3, 1) as the invariant point the transformation can be regarded as a clockwise rotation about (3, 1) through 90°. |
| | 21 | |



## Unit A9: System of Linear Equations in 2 or 3 Unknowns

**Objective:** (1) To solve a system of linear equations using Gaussian elimination.
(2) To recognize the existence and uniqueness of solution.

| Detailed Content | Time Ratio | Notes on Teaching |
|---|---|---|
| **9.1 Gaussian elimination and Echelon form** | 5 | A matrix which satisfies the following 2 properties is said to be in Echelon form:<br>(1) The 1st k rows are non-zero;<br>the other rows are zero.<br>(2) The 1st non-zero element in each non-zero row is 1, and it appears in a column to the right of the 1 st non-zero element of any preceding row.<br>*Example:*<br>The following 5 x 8 matrix is in Echelon form:<br>$\begin{pmatrix} 0 & 1 & * & * & * & * & * & * \\ 0 & 0 & 1 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$<br>Students should be able to solve a system of linear equations in two or three unknowns by using Gaussian elimination, which reduces a matrix in Echelon form by elementary operations on its rows.<br>*Example:*<br>Solve the system<br>$\begin{cases} x_1 + 2x_2 + x_3 = 2 \\ 3x_1 + x_2 - 2x_3 = 1 \\ 4x_1 - 3x_2 - x_3 = 3 \\ 2x_1 + 4x_2 + 2x_3 = 4 \end{cases}$<br>The augmented matrix<br>$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 3 & 1 & -2 & 1 \\ 4 & -3 & -1 & 3 \\ 2 & 4 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -5 & -5 & -5 \\ 0 & -11 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$<br>$\sim \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -11 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$<br>The original system of equations is equivalent to the system of equations<br>$\begin{cases} x_1 - x_3 = 0 \\ x_2 + x_3 = 1 \\ x_3 = 1 \end{cases}$<br>which gives $x_1 = 1$, $x_2 = 0$, $x_3 = 1$. |





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<tr><th>Detailed Content</th><th>Time Ratio</th><th>Notes on Teaching</th></tr>
<tr><td>9.2 Existence and uniqueness of solution</td><td>5</td><td>
Students should be able to know the conditions for the existence and uniqueness of solution for a system of linear equations in two or three unknowns.<br>
For a system of linear equations in two unknowns:<br>
$a_1x + b_1y = d_1$<br>
$a_2x + b_2y = d_2$<br>
(i) If $\det\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \neq 0$, the system has unique solution. Geometrically, the equations represent a pair of intersecting straight lines.<br>
(ii) If $\det\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = 0$ and $\det\begin{pmatrix} d_1 & b_1 \\ d_2 & b_2 \end{pmatrix} \neq 0$, the system has no solution. Geometrically, the equations represent a pair of parallel (but not coincident) straight lines.<br>
(iii) If $\det\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} = 0$ and $\det\begin{pmatrix} d_1 & b_1 \\ d_2 & b_2 \end{pmatrix} = 0$, the system has infinite number of solutions. Geometrically, the equations represent a pair of coincident straight lines.<br>
For systems of equation in three unknowns, examples like the following should be mentioned. The corresponding geometrical meaning may be discussed if the students have grasped some ideas of three dimensional coordinate geometry.<br>
(i) In solving the equations<br>
$\begin{cases} 2x + y - z = 7 \\ 5x - 4y + 7z = 1 \\ 7x - 3y + 6z = 8 \end{cases}$,<br>
it is obvious that the third one is redundant. Teachers may discuss with the students on the method to obtain the solution<br>
$x = \dfrac{29 - 3\lambda}{13}, y = \dfrac{33 + 19\lambda}{13}, z = \lambda$<br>
where $\lambda$ is arbitrary.<br>
(ii) Solving equations like<br>
$\begin{cases} x + y + z = 3 \\ 2x - 3y + 2z = 1 \\ 3x - 2y + 3z = 7 \end{cases}$ which are inconsistent.<br>
Following this manner, the conditions for the existence and uniqueness of solution for a system of equations in three unknowns may be given in more abstract terms.
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<tr><td></td><td>10</td><td></td></tr>
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## Unit A10: Complex Numbers

**Objective:** (1) To learn the properties of complex numbers, their geometrical representations and applications.
(2) To learn the De Moivre's Theorem and its applications in finding the nth roots of complex numbers, in solving polynomial equations and proving trigonometric identities.



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<tr><th>Detailed Content</th><th>Time Ratio</th><th>Notes on Teaching</th></tr>
<tr><td>10.1 Definition of complex numbers and their arithmetic operations<br><br>10.2 Argand diagram, argument and conjugate</td><td>3<br><br><br>6</td><td>
A short introduction of the symbol i should be given. The number z = x + yi, where x, y are real, is called a complex number and x and y are known respectively as its real (Re z) and imaginary (Im z) parts. When x = 0, y $\neq$ 0, z = yi is said to be purely imaginary and when y = 0, z = x is real.<br>
Students may be asked what definition should be adopted for the equality of complex numbers, however there is no ordering property for complex numbers.<br>
The sum, difference, product and quotient of two complex numbers should be defined.<br>
Students are expected to know the definitions of the terms modulus $|z|$, argument arg z, principal (value of) argument (or amplitude) and conjugate $\bar{z}$ of a complex number z.<br>
The complex number z = r(cos$\theta$ + isin$\theta$), in the modulus — argument form (polar form ), can be written as z = rcis$\theta$.<br>
Students are expected to know the following properties of complex numbers:<br>
(i) $|z_1 z_2| = |z_1||z_2|$<br>
(ii) $\arg z_1 z_2 = \arg z_1 + \arg z_2 + 2k\pi$ where k is an integer<br>
(iii) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$<br>
(iv) $\arg\dfrac{z_1}{z_2} = \arg z_1 - \arg z_2 + 2k\pi$ where k is an integer and $z_2 \neq 0$.<br>
Properties about conjugate complex numbers should be taught:<br>
1. $\bar{\bar{z}} = z$<br>
2. $\bar{z} = 0$ iff z=0<br>
3. A complex number is self-conjugate (conjugate to itself) iff it is real.<br>
4. $z \cdot \bar{z} = |z|^2$
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