Detailed Content	Time Ratio	Notes on Teaching
9.2 Existence and uniqueness of solution	5	Students should be able to know the conditions for the existence and uniqueness of solution for a system of linear equations in two or three unknowns. For a system of linear equations in two unknowns: $a_1x + b_1y = d_1$ $a_2x + b_2y = d_2$ (i) If $\det\begin{pmatrix}a_1 & b_1\\ a_2 & b_2\end{pmatrix} \neq 0$, the system has unique solution. Geometrically, the equations represent a pair of intersecting straight lines. (ii) If $\det\begin{pmatrix}a_1 & b_1\\ a_2 & b_2\end{pmatrix} = 0$ and $\det\begin{pmatrix}d_1 & b_1\\ d_2 & b_2\end{pmatrix} \neq 0$, the system has no solution. Geometrically, the equations represent a pair of parallel (but not coincident) straight lines. (iii) If $\det\begin{pmatrix}a_1 & b_1\\ a_2 & b_2\end{pmatrix} = 0$ and $\det\begin{pmatrix}d_1 & b_1\\ d_2 & b_2\end{pmatrix} = 0$, the system has infinite number of solutions. Geometrically, the equations represent a pair of coincident straight lines. For systems of equation in three unknowns, examples like the following should be mentioned. The corresponding geometrical meaning may be discussed if the students have grasped some ideas of three dimensional coordinate geometry. (i) In solving the equations $\begin{cases}2x + y - z = 7\\ 5x - 4y + 7z = 1\\ 7x - 3y + 6z = 8\end{cases}$, it is obvious that the third one is redundant. Teachers may discuss with the students on the method to obtain the solution $x = \dfrac{29 - 3\lambda}{13}, y = \dfrac{33 + 19\lambda}{13}, z = \lambda$ where λ is arbitrary. (ii) Solving equations like $\begin{cases}x + y + z = 3\\ 2x - 3y + 2z = 1\\ 3x - 2y + 3z = 7\end{cases}$ which are inconsistent. Following this manner, the conditions for the existence and uniqueness of solution for a system of equations in three unknowns may be given in more abstract terms.
	10	

42

Unit A10: Complex Numbers

Objective: (1) To learn the properties of complex numbers, their geometrical representations and applications.
(2) To learn the De Moivre's Theorem and its applications in finding the nth roots of complex numbers, in solving polynomial equations and proving trigonometric identities.

Detailed Content	Time Ratio	Notes on Teaching
10.1 Definition of complex numbers and their arithmetic operations	3	A short introduction of the symbol i should be given. The number z = x + yi, where x, y are real, is called a complex number and x and y are known respectively as its real (Re z) and imaginary (Im z) parts. When x = 0, y $\neq$ 0, z = yi is said to be purely imaginary and when y = 0, z = x is real. Students may be asked what definition should be adopted for the equality of complex numbers, however there is no ordering property for complex numbers. The sum, difference, product and quotient of two complex numbers should be defined.
10.2 Argand diagram, argument and conjugate	6	Students are expected to know the definitions of the terms modulus $\|z\|$, argument arg z, principal (value of) argument (or amplitude) and conjugate $\bar{z}$ of a complex number z. The complex number z = r(cosθ + isinθ), in the modulus — argument form (polar form), can be written as z = rcisθ. Students are expected to know the following properties of complex numbers: (i) $\|z_1 z_2\| = \|z_1\|\|z_2\|$ (ii) arg $z_1 z_2$ = arg z_1 + argz_2 +2kπ where k is an integer (iii) $\left\|\dfrac{z_1}{z_2}\right\| = \dfrac{\|z_1\|}{\|z_2\|}$ (iv) $\arg\dfrac{z_1}{z_2} = \arg z_1 - \arg z_2 + 2k\pi$ where k is an integer and $z_2 \neq 0$. Properties about conjugate complex numbers should be taught: 1. $\bar{\bar{z}} = z$ 2. $\bar{z}$ = 0 iff z=0 3. A complex number is self-conjugate (conjugate to itself) iff it is real. 4. $z \cdot \bar{z} = \|z\|^2$

43

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Detailed Content	Time Ratio	Notes on Teaching
		5. $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ 6. $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$ 7. $\overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$ 8. $\overline{(\frac{z_1}{z_2})} = \frac{\overline{z_1}}{\overline{z_2}}$, $z_2 \neq 0$ From these properties students can easily prove that if α is a root of a polynomial equation with real coefficients, then $\overline{\alpha}$ is also a root. i.e. in a polynomial equation with real coefficients, roots which are not real occur in conjugate pairs. Students are expected to know the following inequalities: (i) $\lvert \mathrm{Re}\, z \rvert \le \lvert z \rvert$ (ii) $\lvert \mathrm{Im}\, z \rvert \le \lvert z \rvert$ (iii) $\lvert z_1 + z_2 \rvert \le \lvert z_1 \rvert + \lvert z_2 \rvert$ (Triangle inequality) Students can also be asked to prove in a similar way that $\lvert z_1 - z_2 \rvert \ge \lvert z_1 \rvert - \lvert z_2 \rvert$ and $\lvert z_1 - z_2 \rvert \ge \lvert z_2 \rvert - \lvert z_1 \rvert$. The triangle inequality can easily be extended by induction to $\lvert z_1 + z_2 + \ldots + z_n \rvert \le \lvert z_1 \rvert + \lvert z_2 \rvert + \ldots + \lvert z_n \rvert$. The geometrical representation of complex numbers in an Argand diagram should be studied. Students should know the terms real axis and imaginary axis. The representation of a complex number in polar form and its geometrical meaning should also be taught. The notation $e^{i\theta}$ for $\mathrm{cis}\theta$ may be introduced so that $z = re^{i\theta}$. The notation is known as the exponential form or the Euler form of a complex number.
10.3 Simple applications in plane geometry	5	Students are expected to know the geometrical meaning of the triangle inequality. Various uses of complex number in plane geometry should be studied. The following are two examples: 1. In the Argand diagram, XYZ is an equilateral triangle whose circumcentre is at the origin. If X represents the complex number 1 + i, find the numbers represented by Y and Z. 2. If z_1, z_2 and z_3 are three distinct complex numbers denoting the vertices of an equilateral triangle, then $z_1^2 + z_2^2 + z_3^2 = z_2 z_3 + z_3 z_1 + z_1 z_2$.

Detailed Content	Time Ratio	Notes on Teaching
		Examples on the loci of points moving on the Argand plane should be studied. Two simple examples are given below: (i) To find the locus of the point z such that $\lvert z - a \rvert = k$, where a is a complex number and k is a positive constant. (ii) To find the locus of the point z which moves such that $\left\lvert \frac{z-a}{z-b} \right\rvert = k$, where a, b are complex numbers, for various values of the positive constant k.
10.4 De Moivre's theorem		
10.4a De Moivre's theorem for rational indices	3	Students should learn how to prove the theorem $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ when n is a positive integer with the assumption that $(\cos\theta + i\sin\theta)^0 = 1$. When n is a negative integer, by putting $n = -m$ where m is a positive integer, students should be able to prove that the theorem is also true for negative integer n. However for the case $n = \frac{p}{q}$, where p, q are integers and $q \neq 0$, the proof may be provided afterwards till the students have acquired the knowledge of the n^{th} roots of a complex number.
10.4b Applications to trigonometric identities	3	By De Moivre's theorem, $\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n$ where n is positive integer, expressions for $\cos n\theta$ and $\sin n\theta$ can be obtained in terms of the powers of $\cos\theta$ and $\sin\theta$. By considering $z = \cos\theta + i\sin\theta$, then expressions $\begin{cases} z + \frac{1}{z} = 2\cos\theta \\ z - \frac{1}{z} = 2i\sin\theta \end{cases}$ and $\begin{cases} z^n + \frac{1}{z^n} = 2\cos n\theta \\ z^n - \frac{1}{z^n} = 2i\sin n\theta \end{cases}$

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Detailed Content	Time Ratio	Notes on Teaching
		can be used to express powers of $\cos\theta$ and $\sin\theta$ in terms of sines and cosines of multiples of θ. For example, students should be able to express $\cos^4\theta\sin^3\theta$ as a sum of sines of multiples of θ and $\cos^3\theta\sin^4\theta$ as a sum of cosines of multiples of θ.
10.4c n^{th} roots of a complex number and their geometrical interpretation	~~5~~ 4	Students should learn the meaning of the n^{th} roots of a complex number. The n^{th} roots of unity should be studied in detail. Several examples can be discussed in class: 1. To find the fifth roots of –1. 2. To solve the equation $z^4+z^3+z^2+z+1=0$. 3. To find the cube roots of 1 + i. 4. Factorize $z^{2n}-2z^n\cos n\theta+1$ into real quadratic factors.
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Unit B1: Sequence, Series and their Limits

Objective: (1) To learn the concept of sequence and series.
(2) To understand the intuitive concept of the limit of sequence and series.
(3) To understand the behaviour of infinite sequence and series.

Detailed Content	Time Ratio	Notes on Teaching
1.1 Sequence and series	6	Clear concepts of sequence and series should be provided. The following suggested versions may be adopted: If a_n is a function of n which is defined for all positive integral values of n, its values a_1, a_2, a_3, …, a_n, … are said to form a sequence. The sequence is finite or infinite according to the numbers of terms of it being finite or infinite. Furthermore a_1 + a_2 + … + a_n + … is said to form a series. Likewise, it is finite or infinite according to the numbers of terms contained. The notation $S_n=\sum_{r=1}^{n}a_r$ or $\sum_{1}^{n}a_r$ is commonly used. Some simple rules concerning the operations of sequences and series may be introduced. For the sake of convenience, denote the sequences a_1, a_2, a_3, … and b_1, b_2, b_3, … by $\{a_i\}$ and $\{b_i\}$, then (i) $\{a_i\}\pm\{b_i\}=\{a_i\pm b_i\}$ (ii) $\lambda\{a_i\}=\{\lambda a_i\}$, viz, the idea of termwise operations may be touched upon. Regarding series, the following methods of summation should be discussed. (1) Mathematical induction: already dealth with in Unit A3. (2) Method of difference: teachers should amplify in the expressing the rth term of the series as the difference of f(r + 1) and f(r) where f(x) is a function of x. i.e. if $a_r = f(r+1) - f(r)$ then $\sum_{1}^{n}a_r=\sum_{1}^{n}(f(r+1)-f(r))$ $= f(n+1) - f(1)$. Some typical examples are $\sum_{1}^{n}\frac{1}{r(r+1)}$ and $\sum_{1}^{n}r(r+1)$.

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