Detailed Content	Time Ratio	Notes on Teaching
		can be used to express powers of $\cos\theta$ and $\sin\theta$ in terms of sines and cosines of multiples of θ. For example, students should be able to express $\cos^4\theta\sin^3\theta$ as a sum of sines of multiples of θ and $\cos^3\theta\sin^4\theta$ as a sum of cosines of multiples of θ.
10.4c n^{th} roots of a complex number and their geometrical interpretation	~~5~~ 4	Students should learn the meaning of the n^{th} roots of a complex number. The n^{th} roots of unity should be studied in detail. Several examples can be discussed in class: 1. To find the fifth roots of –1. 2. To solve the equation $z^4 + z^3 + z^2 + z + 1 = 0$. 3. To find the cube roots of 1 + i. 4. Factorize $z^{2n} - 2z^n \cos n\theta + 1$ into real quadratic factors.
	~~25~~ 24	

Unit B1: Sequence, Series and their Limits

Objective: (1) To learn the concept of sequence and series.
(2) To understand the intuitive concept of the limit of sequence and series.
(3) To understand the behaviour of infinite sequence and series.

Detailed Content	Time Ratio	Notes on Teaching
1.1 Sequence and series	6	Clear concepts of sequence and series should be provided. The following suggested versions may be adopted: If a_n is a function of n which is defined for all positive integral values of n, its values a_1, a_2, a_3, …, a_n, … are said to form a sequence. The sequence is finite or infinite according to the numbers of terms of it being finite or infinite. Furthermore $a_1 + a_2 + \ldots + a_n + \ldots$ is said to form a series. Likewise, it is finite or infinite according to the numbers of terms contained. The notation $S_n = \sum_{r=1}^{n} a_r$ or $\sum_{1}^{n} a_r$ is commonly used. Some simple rules concerning the operations of sequences and series may be introduced. For the sake of convenience, denote the sequences a_1, a_2, a_3, … and b_1, b_2, b_3, … by $\{a_i\}$ and $\{b_i\}$, then (i) $\{a_i\} \pm \{b_i\} = \{a_i \pm b_i\}$ (ii) $\lambda\{a_i\} = \{\lambda a_i\}$, viz, the idea of termwise operations may be touched upon. Regarding series, the following methods of summation should be discussed. (1) Mathematical induction: already dealth with in Unit A3. (2) Method of difference: teachers should amplify in the expressing the rth term of the series as the difference of f(r + 1) and f(r) where f(x) is a function of x. i.e. if $a_r = f(r + 1) - f(r)$ then $\sum_{1}^{n} a_r = \sum_{1}^{n} (f(r+1) - f(r))$ $= f(n + 1) - f(1)$. Some typical examples are $\sum_{1}^{n} \frac{1}{r(r+1)}$ and $\sum_{1}^{n} r(r+1)$.

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Detailed Content	Time Ratio	Notes on Teaching
		For series whose terms are presented in a recurrence of the form $a_r - a_{r-1} = f(r)$ or $a_r = Aa_{r-1} + Ba_{r-2}$ some basic methods should be introduced, especially for the latter one the following approach may be discussed: Suppose α, β are the roots of the auxiliary equation $\lambda^2 = A\lambda + B$, (i) if $\alpha \neq \beta$, $a_r = k_1\alpha^r + k_2\beta^r$; (ii) if $\alpha = \beta$, $a_r = (k_1 + rk_2)\alpha^r$ where k_1 and k_2 are constants to be determined.
1.2 Limit of a sequence and series	7	The concept of the limit of a sequence should be taught with an intuitive approach. The following version may be considered: Let $a_1, a_2, \ldots, a_n, \ldots$ be a sequence. If for all sufficiently large values of n, the difference between a_n and a constant ℓ is as small as we please, we say that $a_n \to \ell$ when $n \to \infty$ or $\lim_{n\to\infty} a_n = \ell$. Teachers should emphasize on the following points: (i) ℓ is called the limit of the sequence; (ii) the limit ℓ, if exists, is unique; (iii) the sequence is said to converge to ℓ or the sequence is convergent with limit ℓ; (iv) if a sequence does not converge to any limit, it is said to be divergent. Ample examples illustrating convergence and divergence should be provided. (1) $\lim_{n\to\infty} a^{\frac{1}{n}} = 1$ for a>0. (2) The sequence $a_n = \frac{\sin \frac{1}{2} n\pi}{n}$ which converges to the limit 0. (3) The divergent sequence $a_n = \{1 + (-1)^n\}\sqrt{n}$. (4) The divergent oscillatory sequence $a_n = (-1)^n(1 + \frac{1}{n})$. N.B. Sequences could be classified as convergent, divergent (to $+\infty$ or $-\infty$) or oscillatory (does not converge nor diverge (to $+\infty$ or $-\infty$)). Some common properties of convergent sequence should be included in the discussion with students:

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Detailed Content	Time Ratio	Notes on Teaching
		Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ and $b_1, b_2, b_3, \ldots, b_n, \ldots$ be convergent sequences with limits be a and b respectively, the following sequence are also convergent: (i) $\lambda a_1, \lambda a_2, \lambda a_3, \ldots$ converges λa, where λ is a constant. (ii) $a_1 + b_1, a_2 + b_2, a_3 + b_3, \ldots$ converges to a + b. (iii) $a_1b_1, a_2b_2, a_3b_3, \ldots$ converges to ab. (iv) $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \ldots$ converges to $\frac{a}{b}$ provided $b \neq 0$. Finally, students should be led to appreciate the following results that (i) for the convergent sequence $a_1, a_2, a_3, \ldots$ with limit a, $\lim_{n\to\infty} a_{n+k} = \lim_{n\to\infty} a_n = a$, where k is a positive integer. (ii) for the two convergent sequences $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$ with the same limit ℓ and if a sequence $c_1, c_2, c_3, \ldots$ such that $a_i \leq c_i, \leq b_i$ when i > k for some positive integer k, then $c_1, c_2, c_3, \ldots$ also converges and to the same limit ℓ. This property is commonly known as the Sandwich Theorem. Teachers may also touch upon the meaning of monotonic sequence and bounded sequence to broaden students' understanding. As for infinite series, a parallel treatment could be provided as follows: (1) Concept of convergence The series $u_1 + u_2 + u_3 + \ldots$ is convergent if $\lim_{n\to\infty} \sum_1^n u_i = S$ exists and the series is said to be convergent to the limit. (Sometimes S may be called the sum of the series.) If S_n represents $u_1 + u_2 + \ldots + u_n$, then the result may be stated as $S_n \to S$ as $n \to \infty$ or $\lim_{n\to\infty} S_n = S$. ($S_n = u_1 + u_2 + \ldots + u_n$ is commonly known as the n^{th} partial sum). And, in a more or less the same situation, divergent series and/ or oscillatory series may be introduced subject to teachers' preference. (2) Properties of convergent series $u_1 + u_2 + u_3 + \ldots$ with limit S and $v_1 + v_2 + v_3 + \ldots$ with limit S' then (a) $\lambda u_1 + \lambda u_2 + \lambda u_3 + \ldots$ converges to λS where λ is a constant. (b) $(u_1 + v_1) + (u_2 + v_2) + (u_3 + v_3)$ converges to S + S'. (c) If $u_1 + u_2 + u_3 + \ldots$ Is convergent, then $\lim_{n\to\infty} u_n = 0$

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Detailed Content	Time Ratio	Notes on Teaching
1.3 Convergence of a sequence and series	5	Further properties of convergent sequence like (i) convergent sequences are bounded (ii) a monotonic and bounded sequence is convergent should be introduced. Some typical convergent and divergent sequences should be discussed so as to illustrate the method in finding limits of sequences. The following examples may be considered: (A) Convergent sequences (i) $a_n = x^n$ with $\|x\| < 1$ (ii) $a_n = \sqrt[n]{n}$ (iii) $a_n = \frac{x^n}{n!}$ (B) Convergent series (i) $r + r^2 + r^3 \ldots$ with $\|r\| < 1$ (ii) $1+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\ldots$ (iii) $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots$ (iv) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ (C) Divergent series (i) $\sum\frac{1}{n}$ (ii) $\sum(1-\frac{1}{n})^n$ (iii) $\sum\frac{1}{\sqrt{n}}$ Some typical applications of the Sandwich Theorem should be included for illustration whereas convergence tests of series are not required.
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Unit B2: Limit, Continuity and Differentiability

Objective: (1) To understand the intuitive concept of the limit of a function.
(2) To understand the intuitive concept of continuity and differentiability of a function.
(3) To recognize limit as a fundamental concept in calculus.

Detailed Content	Time Ratio	Notes on Teaching
2.1 Limit of a function	5	An intuitive understanding of the concept of limit of function is expected. As a matter of fact, the concept of the limit of a function $y = f(x)$ at the point $x = a$ can be related to the concept of the limit of a sequence. This is done by allowing the independent variable to run through a convergent sequence of numbers $\{x_n\}$ tending to the limit a (the abscissa sequence), and considering the ordinate sequence $\{f(x_n)\}$. Thus a more vivid visualization of the fact that $\{f(x_n)\}$ tends to a finite value ℓ as $\{x_n\}$ tends to a could be established i.e. $f(x) \to \ell$ when $x \to a$ or $\lim_{x\to a} f(x) = \ell$. Some teachers may perhaps prefer just to focus students' attention to the fact that the difference between f(x) and ℓ can be made arbitrarily small when x is sufficiently close to a so as to reinforce the idea that $f(x) \to \ell$ when $x \to a$. It must be pointed to students that, from the existence of the value f(a) of the function, one can certainly not conclude that the limit $\lim_{x\to a} f(x)$ must also exist and be equal to f(a), though this is very often the case. The following example may be considered: $f(x) = \begin{cases} 1 & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$ in which f(0) = 0 and $\lim_{x\to 0} f(x) = 1$ It may be important in the passage to the limit whether the independent variable approaches the value a in the sense of increasing values of x, that is, from the left, or in the sense of decreasing values of x, that is from the right. In these cases, the limits are referred to, respectively, as the left-hand limit, usually denoted by $\lim_{x\to a^-} f(x)$, and the right-hand limit $\lim_{x\to a^+} f(x)$. In this context, students could be led easily to appreciate that the function f(x) has a limit as $x \to a$ if and only if the left-hand and right-hand limits as $x \to a$ are equal. For a more comprehensive understanding of limit, teachers should touch upon the case when $x \to \infty$ by reiterating that the difference between f(x) and ℓ could be made arbitrarily small when x is sufficiently large. Symbolically, it is presented as $\lim_{x\to\infty} f(x) = \ell$.

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