Detailed Content	Time Ratio	Notes on Teaching
1.3 Convergence of a sequence and series	5	Further properties of convergent sequence like (i) convergent sequences are bounded (ii) a monotonic and bounded sequence is convergent should be introduced. Some typical convergent and divergent sequences should be discussed so as to illustrate the method in finding limits of sequences. The following examples may be considered: (A) Convergent sequences (i) $a_n = x^n$ with $\|x\| < 1$ (ii) $a_n = \sqrt[n]{n}$ (iii) $a_n = \frac{x^n}{n!}$ (B) Convergent series (i) $r + r^2 + r^3 \ldots$ with $\|r\| < 1$ (ii) $1+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\ldots$ (iii) $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots$ (iv) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ (C) Divergent series (i) $\sum\frac{1}{n}$ (ii) $\sum(1-\frac{1}{n})^n$ (iii) $\sum\frac{1}{\sqrt{n}}$ Some typical applications of the Sandwich Theorem should be included for illustration whereas convergence tests of series are not required.
	18	

Unit B2: Limit, Continuity and Differentiability

Objective: (1) To understand the intuitive concept of the limit of a function.
(2) To understand the intuitive concept of continuity and differentiability of a function.
(3) To recognize limit as a fundamental concept in calculus.

Detailed Content	Time Ratio	Notes on Teaching
2.1 Limit of a function	5	An intuitive understanding of the concept of limit of function is expected. As a matter of fact, the concept of the limit of a function y = f (x) at the point x = a can be related to the concept of the limit of a sequence. This is done by allowing the independent variable to run through a convergent sequence of numbers $\{x_n\}$ tending to the limit a (the abscissa sequence), and considering the ordinate sequence $\{f(x_n)\}$. Thus a more vivid visualization of the fact that $\{f(x_n)\}$ tends to a finite value ℓ as $\{x_n\}$ tends to a could be established i.e. $f(x) \to \ell$ when $x \to a$ or $\lim_{x\to a} f(x) = \ell$. Some teachers may perhaps prefer just to focus students' attention to the fact that the difference between f(x) and ℓ can be made arbitrarily small when x is sufficiently close to a so as to reinforce the idea that $f(x) \to \ell$ when $x \to a$. It must be pointed to students that, from the existence of the value f(a) of the function, one can certainly not conclude that the limit $\lim_{x\to a} f(x)$ must also exist and be equal to f(a), though this is very often the case. The following example may be considered: $f(x) = \begin{cases} 1 & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$ in which f(0) = 0 and $\lim_{x\to 0} f(x) = 1$ It may be important in the passage to the limit whether the independent variable approaches the value a in the sense of increasing values of x, that is, from the left, or in the sense of decreasing values of x, that is from the right. In these cases, the limits are referred to, respectively, as the left-hand limit, usually denoted by $\lim_{x\to a^-} f(x)$, and the right-hand limit $\lim_{x\to a^+} f(x)$. In this context, students could be led easily to appreciate that the function f(x) has a limit as $x \to a$ if and only if the left-hand and right-hand limits as $x \to a$ are equal. For a more comprehensive understanding of limit, teachers should touch upon the case when $x \to \infty$ by reiterating that the difference between f(x) and ℓ could be made arbitrarily small when x is sufficiently large. Symbolically, it is presented as $\lim_{x\to\infty} f(x) = \ell$.

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Detailed Content	Time Ratio	Notes on Teaching
		The following properties of the limit of a function should be included in discussion: (i) $\lim\limits_{x\to a}[f(x)+g(x)] = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$ (ii) $\lim\limits_{x\to a}[kf(x)] = k\lim\limits_{x\to a} f(x)$, k is independent of x (iii) $\lim\limits_{x\to a}[f(x)\cdot g(x)] = \lim\limits_{x\to a} f(x)\cdot \lim\limits_{x\to a} g(x)$ (iv) $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$ provided $\lim\limits_{x\to a} g(x) \neq 0$ (v) If $f(x) \le h(x) \le g(x)$ holds when x is close to a and $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = \ell$ then $\lim\limits_{x\to a} h(x) = \ell$ also. Some important limits, like the following, should be introduced: (1) $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$ (2) $\lim\limits_{x\to\infty}(1+\dfrac{1}{x})^x = e$ (3) $\lim\limits_{x\to\infty}(1+\dfrac{a}{x})^x = e^a$ (4) $\lim\limits_{x\to 0}\dfrac{e^x-1}{x} = 1$ (5) $\lim\limits_{x\to 0}\dfrac{a^x-1}{x} = \ell n\, a$; a > 0 (6) $\lim\limits_{x\to 0}\dfrac{\ell n(1+x)}{x} = 1$ (7) $\lim\limits_{x\to 1}\dfrac{\ell n\, x}{x-1} = 1$ Adequate practice in evaluating the limit of function should be provided for consolidation.

Detailed Content	Time Ratio	Notes on Teaching
2.2 Continuity of a function	4	Continuity should be defined on the basis of the limit of function with an intuitive approach; the $\varepsilon - \delta$ approach may not be desirable. The following suggested version may be considered: A function f(x) is continuous at x = a if $\lim\limits_{x\to a} f(x)$ exists and is equal to f(a). A function is continuous in an interval if it is continuous at every point of the interval. Some common functions like (i) $f(x) = x^2$ which is continuous in every interval; (ii) $f(x) = \dfrac{1}{x-1}$ which is not continuous in the whole interval $0 \le x \le 5$ should be discussed as a prelude to introduce the concept of point of discontinuity. It should be noted that just informal treatment on this concept is expected, however, teachers are advised to provide students with a good spectrum of examples as a form of reinforcement. Furthermore, the fact that the sum, difference and product of two functions continuous at x = a are likewise continuous at this point. Their quotient is continuous provided that the denominator is not zero at x = a. Teachers may quote a lot of everywhere continuous functions to initiate students' further study on this topic: (i) polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$ (ii) exponential function $f(x) = a^x$; a > 0 (iii) logarithmic function $f(x) = \log_a x$; a > 0, $a \neq 1$ (iv) trigonometric functions like sinx, cosx. Concerning the continuity of composite function, teachers may consider the suggested version : Let y = f[g(x)] be a composite function, when inner function g(x) is continuous at x = a and whose outer function y = f(t) is continuous at t = g(a), then the composite function y = f[g(x)] is continuous at x = a. Also teachers may highlight the fact that every continuous function of a continuous function is again continuous. Teachers should point out that functions which are continuous in an interval form a class of functions with noteworthy properties, like the following, and the discussion of them is expected but formal proof of them is not desirable.

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Detailed Content	Time Ratio	Notes on Teaching
		(i) If a function f(x) is continuous in a closed interval [a, b] with f(a) = A and f(b) = B where $A \neq B$, then f(x) takes every value between A and B at least once. (The Intermediate Value Theorem). (ii) A function that is continuous in a closed interval is bounded there. (iii) A function that is continuous in a closed interval attains maximum and minimum values in the interval. (Properties (ii) and (iii) are known as Weierstrass Theorem).
2.3 Differentiability of a function	4	Regarding the differentiability of a function f(x) at the point x = a the following version may be considered. A function f(x) is said to be differentiable at the point x = a if and only if the limit $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ or $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ exists Teachers may also at the same time put forth the idea that if a function is differentiable at a certain point, it is also continuous there and that continuity is a necessary condition for differentiability but not a sufficient one. Moreover, the definition of the derivative of a function at x = a, being the value of the above limit, can be taught very smoothly following students' acceptance of the idea of differentiability. The common notations for the derivative of f(x) at x = a like $f'(a), \frac{d}{dx} f(x) \Big\vert_{x=a}$ and $\frac{dy}{dx} \Big\vert_{x=a}$ should be mentioned. Teachers may also touch upon the differentiability of a function in the whole interval in the context that the .derivative of the function exists for all points in that interval. Furthermore, teachers should elaborate on the property that to each value x in the interval, there corresponds the derivative f'(x) of the function at the point x; thus f'(x) is again a function of x and is called the derived function of f(x). At this stage, ample examples should be worked out to reinforce students' mastery of the concept and skills concerning differentiation. In particular, examples to find the derivative of different typical functions from the first principles are of particular importance. In this connection, adequate practices are indispensable. The following examples are typical:

Detailed Content	Time Ratio	Notes on Teaching
		(1) Find the derivative of the functions from the first principles (i) x^2 at x = 1 (ii) e^x at x = 0 (iii) sinx at $x = \pi/4$ (2) Differentiate, from the first principles, the following functions (i) $f(x) = x^n$ where n is a positive integer (ii) $f(x) = e^x$
	13	

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