Detailed Content	Time Ratio	Notes on Teaching
		1. If $f'(x) = 0$ for all x in an interval, then f(x) is constant in that interval. 2. If $f'(x) = g'(x)$ for all x in an interval, then f(x) and g(x) differ in that interval by a constant. 3. Prove that if $\frac{a_0}{n+1}+\frac{a_1}{n}+\ldots+\frac{a_{n-1}}{2}+a_n=0$ then the equation $a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n=0$ has at least one root between 0 and 1
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Unit B4: Application of Differentiation

Objective: (1) To learn and to use the L' Hospital's Rule.
(2) To learn the applications of differentiation.

Detailed Content	Time Ratio	Notes on Teaching
4.1 The L' Hospital's Rule	4	Limits having the following indeterminate forms should be introduced: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty$, $\infty-\infty$, 0^0, ∞^0, 1^∞ Accompanying examples illustrating the type mentioned are highly desirable. The L' Hospital's Rule $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$ for the indeterminate form $\frac{0}{0}$ and $\frac{\infty}{\infty}$ should be taught in the first place. The examples that follow may be considered: (1) $\lim_{x\to\frac{1}{2}}\frac{\cos^2\pi x}{e^{2x}-2ex}$ (2) $\lim_{x\to a^+}\frac{\ell n\sin(x-a)}{\ell n\tan(x-a)}$ Teachers should emphasize that $\frac{f'(x)}{g'(x)}$ should be simplified before taking limit and the process can be repeated until $\lim_{x\to a}\frac{f^{(m)}(x)}{g^{(m)}(x)}$ is obtained in a non-indeterminate form. As for the other indeterminate forms, examples should be worked out showing that they can be expressed in the determinate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$ so that the rule may be applied. The following examples may be considered: (1) $\lim_{x\to 0}(\frac{1}{x}-\cot x)$ (2) $\lim_{x\to 0^+}x^x$ (3) $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$ The proof of the L' Hospital's Rule is not required.

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Detailed Content	Time Ratio	Notes on Teaching
4.2 Rate of change	3	The meaning of $\frac{dy}{dx}$ as the rate of change of y with respect to x should be introduced and thoroughly discussed with reference to some common quantities like velocity and acceleration etc. Examples for consideration: (1) A snowball is melting with its volume decreasing at a constant rate of x cm^3/s. When its radius is a cm, find (a) the rate of change of its radius; (b) the rate of change of the surface area. (2) The displacement x of a moving particle measured from a fixed point at time t is given by $x = a\sin t + b\cos t$. (a) Find its velocity and acceleration at time t and describe the motion of the particle. (b) Show that the velocity at time t can be expressed as $\sqrt{a^2+b^2-x^2}$.
4.3 Monotonic functions	2	To begin with, teachers may state an intuitively obvious result that if $f'(a) > 0$ then $f(x) < f(a)$ for values of x less than a but sufficiently close to a, and $f(x) > f(a)$ for values of x greater than a but sufficiently close to a. From a geometrical point of view, the result can easily lead to the statement that f(x) is strictly increasing at x = a. (N.B. It is assumed that the function under consideration is continuous and differentiable.) Similar description should be provided for f(x) strictly decreasing at x = a. Following this, the idea of monotonic increasing may be presented as follows: if $f'(x) > 0$ for every x of the interval, then f(x) is a monotonic increasing function throughout the interval. Teachers should help the students to derive the following important result: if $f'(x) > 0$ throughout the interval, (a, b), f(x) is continuous at x = a and $f(a) \geq 0$, then f(x) is positive throughout the interval. A parallel treatment for monotonic decreasing function is expected and this may be done by the students.

Detailed Content	Time Ratio	Notes on Teaching
		This result is of special relevance in proving inequalities like (i) $(1 + x)^{\alpha} \leq 1 + \alpha x$ for $0 < \alpha < 1$ and $x \geq -1$ (ii) $(1 + x)^{\alpha} \geq 1 + \alpha x$ for $\alpha < 0$ or $\alpha > 1$ and $x \geq -1$ (iii) $\sin x > x - \frac{x^3}{3!}$ for $x > 0$
4.4 Maxima and minima	5	The geometrical interpretation of derivative as the gradient of a curve should be explained and emphasized following the introduction of the definition of the gradient of a curve. In this connection, the visualization of a curve being increasing or decreasing can be once again reinforced. Consequenting upon the mastery of this knowledge, students may then be led to acquire the ability of identifying points of local maximum and local minimum (i.e. the turning points of the curve.) They should be helped to appreciate the conditions for the occurrence of local extrema, like the following version: For a function f(x) (a) find a such that $f'(a) = 0$ and (b) test the sign of $f''(a)$ or test for change of sign of $f'(x)$ in a neighbourhood of a. Teachers should remind students of the following points: (i) Local or relative extrema are not necessarily the global or absolute extrema; (ii) turning points may occur at points where the derivatives do not exist; e.g. $y = x^{2/3}$ and $y = \|x\|$; (iii) Stationary points are points whose derivatives are zero; (iv) $f'(a) = 0$ is NOT sufficient to conclude that at x = a there is a local extremum; e.g. $x^3 \sin(\frac{1}{x})$ and x^3. Examples illustrating the foregoing skills and remarks should be worked out and discussed thoroughly with the students prior to the discussion on point of inflection. The procedures commonly adopted is as follows: (a) find a such that $f''(a) = 0$ (b) test $f''(x)$ for change of sign in a neighbourhood of a.

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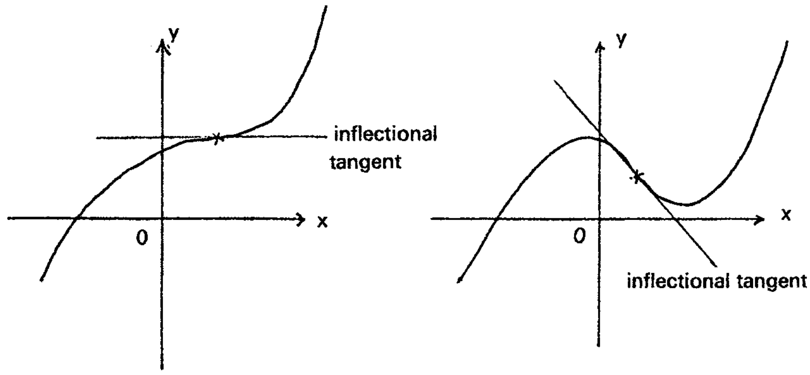
Detailed Content	Time Ratio	Notes on Teaching

Teachers should remind students of the following points:

(i) at points of inflection, the derivative may not be equal to zero;

(ii) $f''(a) = 0$ is not sufficient to conclude the occurrence of an inflection point at $x = a$, e.g. x^4.

Hence, diagrams showing the different orientation of the inflectional tangents are very helpful.



As final touching up, teachers may elaborate briefly on the idea of absolute extrema in relation to the domain of the function being extended or shrinked.

Detailed Content	Time Ratio	Notes on Teaching
4.5 Curve Sketching	6	Prior to full embarkation on this topic, students should be taught how to determine the vertical, horizontal and oblique asymptotes to a curve whenever they exist. It is recommended that illustrating examples should go with the explanation. For example, the curve of $y = \frac{x^3}{x^2 - 1}$.

Students should be alerted to look for points of discontinuity where vertical asymptotes are likely to exist. They should also be led to study the behaviour of the function at infinity, viz,

Detailed Content	Time Ratio	Notes on Teaching

$\frac{x^3}{x^2-1} = x + \frac{x}{x^2-1} \to x$ as x is sufficiently large and so they can realize that y = x represents an asymptote.

To accomplish this topic, teachers are advised to help students look for, extract and organize every single bit of glue and information so as to sketch the curve in a more systematic way. The following points are noteworthy:

(1) Symmetry about the axes: inspect the equation to detect any symmetry using rules

(a) if no odd powers of y appear the curve is symmetrical about the x-axis.

(b) if no odd powers *of x* appear the curve is symmetrical about the y-axis.

e.g. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is symmetric about both axes.

(2) Limitation on the range *of* values of x and y.

e.g. (*a*) For $y^2 = 4x$, x must be non-negative whi le all values of y are permissible.

(*b*) For $x^2y^2 = a^2(x^2 - y^2)$, upon re-writing $y^2 = \frac{a^2x^2}{x^2 + a^2}$, thus all values *of* x are permissible whereas upon another presentation as $x^2 = \frac{a^2y^2}{a^2 - y^2}$, it is obvious that $|y| < a$. Actually, the curve is included between the asymptotes $y = \pm a$.

(3) Intercepts with the axes or any obvious points on the curve.

e.g. For $y = \frac{x(x+2)}{x-2}$, the curve intercepts the x-axis at -2 and 0 and there is no intercept made with the y-axis except at the origin.

(4) Points of maximum, minimum and inflection.

(5) Asympotoes to the curve

To encompass the various facets, examples should be worked out for students' heeding, however for trigonometric functions, the attention to the period of the curve is desirable. Regarding curve given by parametric equations, no specific rules can be taken heed of and it is advisable to obtain the corresponding Cartesian representation prior to sketching it.

Some typical curves illustrating the above steps should be sketched for students' reference. The following may be considered:

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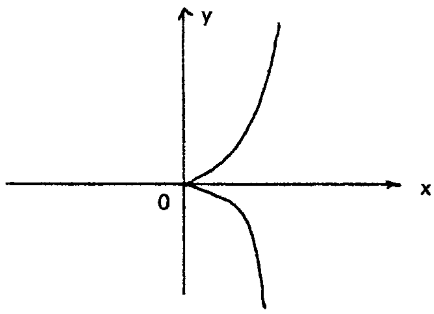
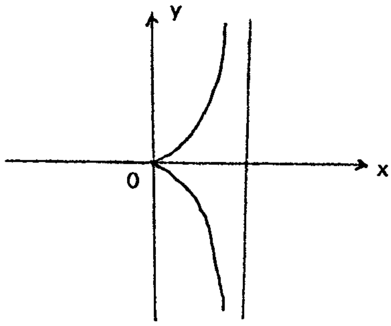
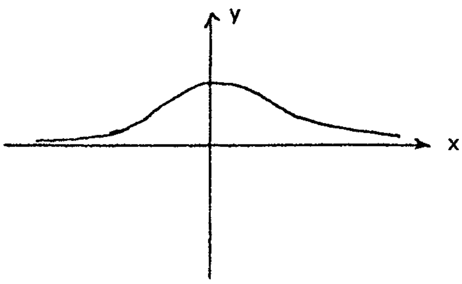
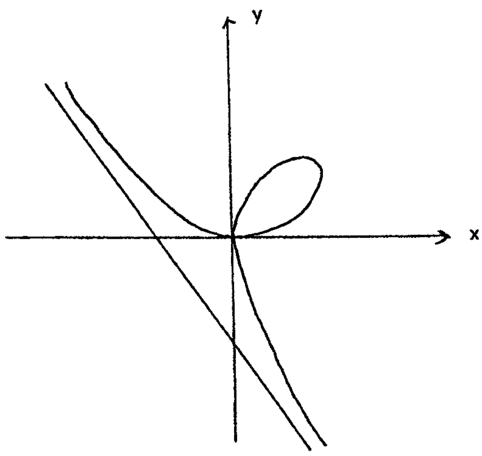
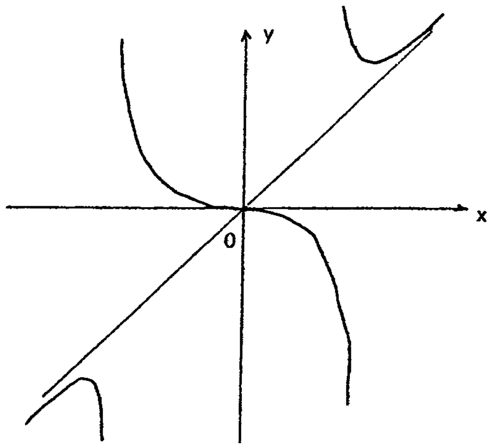
Detailed Content	Time Ratio	Notes on Teaching
		1. $y^2 = ax^3$  2. $y^2 (a - x) = x^3$ 

Detailed Content	Time Ratio	Notes on Teaching
		3. $y = \frac{1}{x^2 + a^2}$  4. $x^3 - 3axy + y^3 = 0$ 

Detailed Content	Time Ratio	Notes on Teaching
		5. $y = \dfrac{x^3}{x^2 - 1}$ 
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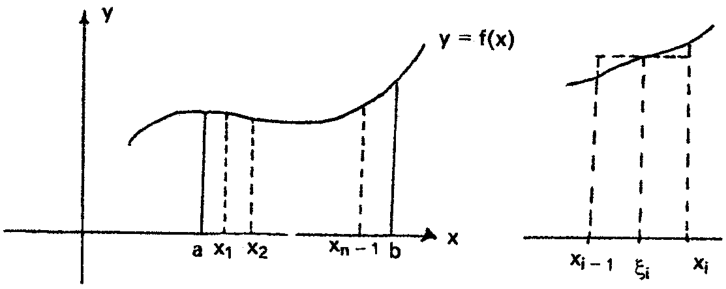
Unit B5: Integration

Objective: (1) To understand the notion of integral as limit of a sum.
(2) To learn some properties of integrals.
(3) To understand the Fundamental Theorem of Integral Calculus.
(4) To apply the Fundamental Theorem of Integral Calculus in the evaluation of integrals.
(5) To learn the methods of integration.
(6) To acquire the first notion of improper integral.

Detailed Content	Time Ratio	Notes on Teaching
5.1 The Riemann definition of integration	5	The theory of the definite integral can be presented in two distinct ways, according as we adopt the geometrical approach or the analytical approach. In the former, the idea of area is presumed, while in the latter the notion of the definite integral as the limit of an algebraic sum without any appeal to geometry is employed. Teachers should determine their choices and sequences of teaching according to the needs of their students. Teachers may start with a function f(x) ≥ 0 for easy understanding and the following simplified version of an intuitive approach is for reference: Let the function f(x) ≥ 0 in the interval [a, b] and therein let the graph of y = f(x) be finite and continuous. 

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