Unit B7: Analytical Geometry

Objective: ~~(1) To learn polar coordinates as another system other than the rectangular coordinate system.~~
(2) To learn the conic sections.
(3) To study locus problems algebraically.
(4) To solve related problems.

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Detailed Content	Time Ratio	Notes on Teaching
7.1 Basic knowledge in coordinate geometry	5	Besides the knowledge in their secondary mathematics, students should acquire the following knowledge before they go on to the other topics in this unit: (1) external point of division; (2) area of rectilinear figure using $\frac{1}{2} \times \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_n & y_n \\ x_1 & y_1 \end{vmatrix}$ (3) angle between two lines using $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$; (4) the normal form of a straight line; (5) angle bisectors of two straight lines; (6) family of straight lines and (7) family of circles.
7.2 Sketching of curves in the polar coordinate system	4	Students should be able to make conversions between polar and rectangular coordinate systems. They should know how to change the equation of a curve in polar form into the corresponding rectangular form and vice versa. Polar to rectangular: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ Rectangular to polar: $\begin{cases} r = \sqrt{x^2 + y^2} \\ \tan\theta = \frac{y}{x} \end{cases}$ where $x \neq 0$ and θ is determined by the quadrant in which (x, y) lies Students should be able to plot curves with their polar equations given. They are the fundamentals to the topic "Applications of Integration". The following are some suggested simple curves in their polar forms that the students should be able to sketch:

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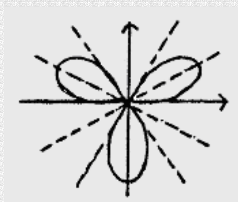
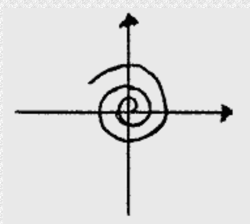
		(1) the straight line: θ = k where k is a positive constant; r$\cos\theta$ = a (vertical line). (2) the circle: r = k where k is a positive constant; r = $\sin\theta$. (3) the parabola: r(1 + $\cos\theta$) = k where k is a positive constant (4) the cardioid: r = a(1 − $\cos\theta$) where k is a positive constant; (5) the rose curve: r = a $\sin 3\theta$  (6) the spiral: r = θ 
7.3 Conic sections in rectangular coordinate system	7	Students should be able to distinguish the conic sections in the standard position, namely, $y^2 = 4ax$ (parabola) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (hyperbola) $xy = c^2$ (rectangular hyperbola)

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<tr><th>Detailed Content</th><th>Time Ratio</th><th>Notes on Teaching</th></tr>
<tr><td></td><td></td><td>The parametric representation of the conk sections should also be taught, namely
$y = r\sin\theta$; $x = r\cos\theta$ (circle)
$y = b\sin\theta$; $x = a\cos\theta$ (ellipse)
$y = b\tan\theta$; $x = a\sec\theta$ (hyperbola)
$y = \frac{c}{t}$; $x = ct$ (rectangular hyperbola)
$y = 2at$; $x = at^2$ (parabola)
The knowledge of asymptotes of a hyperbola is expected. The knowledge of the properties of conic sections such as eccentricity, focus and directrix may be taught but need not be emphasized.</td></tr>
<tr><td>7.4 Tangents and normals of conic sections</td><td>6</td><td>The knowledge of using various methods to find the equations of tangent to a circle is expected, For the simple circle $x^2 + y^2 = a^2$ the equation of tangent to the circle at (x_1, y_1) a point on the circle, is given by $x_1x + y_1y = a^2$ and the normal by $x_1y - y_1x = 0$. The derivation of these basic results should be provided to lead students' thinking and example like the following should also be worked out with due emphasis on the underlying methodology.
To find the equation of tangent at (x_1, y_1) on the circle
$x^2 + y^2 + 2gx + 2fy + c = 0$
(i) using the property that the tangent is always perpendicular to the radius of the circle;
(ii) by letting the equation of tangent be $y = mx + k$ and using the fact that the simultaneous equations
$\begin{cases} y = mx + k \\ x^2 + y^2 + 2gx + 2fy + c = 0 \end{cases}$
have equal roots
It should be noted that the method in (ii) can be applied to the case where (x_1, y_1) is not on the circle. Hence the result that the equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) on the circle given by $x_1x + y_1y + g(x + x_1) + f(y + y_1) + c = 0$ can be obtained and upon generalization, the following results can be obtained:
(a) Tangent to the parabola $y^2 = 4ax$ at (x_1, y_1) on the curve is $y_1y = 2a(x + x_1)$</td></tr>
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<tr><td></td><td></td><td>(c) Tangent to the hyperbola $\frac{x^2}{a^2} - \frac{x^2}{b^2} = 1$ at (x_1, y_1) on the curve is $\frac{x_1x}{a^2} - \frac{y_1y}{b^2} = 1$
(d) Tangent to the rectangular hyperbola $xy = c^2$ at (x_1, y_1) on the curve is $y_1x + x_1y = 2c^2$
Once the equation of tangent is obtained, students should have no difficulty to obtain the equation of the normal.
Furthermore, the corresponding results when the conic sections concerned are presented in parametric form should also be discussed. Teachers may ask the students to do the derivation for themselves:
(a) For the parabola $\begin{cases} x = at^2 \\ y = 2at \end{cases}$; the tangent is $y = \frac{x}{t} + at$
(b) For the ellipse $\begin{cases} x = a\cos\theta \\ y = b\sin\theta \end{cases}$; the tangent is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$
(c) For the hyperbola $\begin{cases} x = a\sec\theta \\ y = b\tan\theta \end{cases}$; the tangent is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$
(d) For the hyperbola $\begin{cases} x = ct \\ y = \frac{c}{t} \end{cases}$; the tangent is $x + t^2y = 2ct$.
To consolidate students' mastery of the concept as well as the manipulative technique, some basic ideas concerning the chord of contact should be discussed.</td></tr>
<tr><td>7.5 Locus problems in rectangular coordinate system</td><td>5</td><td>Cases in which a certain set of points satisfying certain constraints and can be represented by equations in the rectangular coordinate system should be studied,
e.g. (1) The locus of a movable point with fix distance from a fixed point is a circle.
(2) The locus of a movable point which is equidistant from a fixed point and a fixed line is a parabola.
(3) The locus of a point on the rim of a circle when the circle is rolled on a straight line represent a cycloid.</td></tr>
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Detailed Content	Time Ratio	Notes on Teaching
7.6 Tangents and normals of plane curves	4	After students have learnt differential calculus, they should be able to apply differentiation to find the equations of tangent and normal of a plane curve in rectangular coordinate plane. Using differentiation formulae and the chain rule, students can find the equations of tangents and normals of curves whose functions are implicitly defined or in parametric form. In this connection, teachers are advised to determine the teaching sequence of this unit, as a prologue to the harder application of differential calculus or as an epilogue of the basic application of differential calculus, according to the ability of the students.
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