



Exemplar 8 :

Exploring the Effects of the Change of Data on the Measures of Central Tendency

Objectives : To explore and make conjectures on the effects on the measures of central tendency of data upon:

- (1) adding a common constant to the whole set of data
- (2) multiplying the whole set of data by a common constant
- (3) removing a datum from the set of data
- (4) inserting a zero into the set of data

Learning Unit : Measures of Central Tendency

Key Stage : 3

Materials Required : *Excel* and the file dh08_e.xls

Prerequisite Knowledge : Meaning of various measures of central tendency

Description of the Activity :

1. Students open the spreadsheet file dh08_e.xls for the exploring activities on the worksheet. The figure below illustrates the spreadsheet in the file dh08_e.xls.

	A	B	C	D	E	F	G	H
1		original data	add a constant	multiply by a constant	remove a datum	insert a zero		
2		2	10	1				
3		4						
4		8						
5		8						
6		23						
7		6						
8		16						
9		7						
10		5						
11		8						
12								
13								
14	Mean	8.90	0.00	0.00	0.00	0.00		
15	Median	7.5	#NUM!	#NUM!	#NUM!	#NUM!		
16	Mode	8	#N/A	#N/A	#N/A	#N/A		
17								
18								

2. There are four exploring activities in the worksheet. They are:

Activity 1 To explore the change of the measures of central tendency when a constant is added to or subtracted from each datum in the data set

Activity 2 To explore the change of the measures of central tendency when each datum in the data set is multiplied by a constant

Activity 3 To explore the change of the measures of central tendency when one or more data is removed from the set of data

Activity 4 To explore the change of the measures of central tendency when a zero is inserted into the set of data

3. Students follow the instructions in each activity and answer the questions. Discussions among the students could be carried out to generate conjectures.
4. The teacher guides the students to finalize conclusions.

Worksheet: *Exploring the Effects of the Change of Data on the Measures of Central Tendency*

Open the *Excel* file dh08_e.xls for the following activities. Input some data in cells B2 to B11. In choosing a set of data, it is preferable to have a single mode for the following exploring activities.

The following example of data set containing **positive integers** is used as an illustration.

	A	B	C	D	E	F	G	H
1		original data	add a constant + 10	multiply by a constant x 5	remove a datum	insert a zero		
2		2						
3		4						
4		8						
5		8						
6		23						
7		6						
8		18						
9		7						
10		5						
11		8						
12								
13								
14	Mean	8.90	0.00	0.00	0.00	0.00		
15	Median	7.5	#NUM!	#NUM!	#NUM!	#NUM!		
16	Mode	8	#N/A	#N/A	#N/A	#N/A		
17								
18								

Activity 1: To explore the change of the measures of central tendency when a constant is added to or subtracted from a set of the data



	A	B	C
1		original data	add a constant + 00
2		2	
3		4	
4		8	
5		8	
6		23	
7		6	
8		18	
9		7	
10		5	
11		8	
12			
13			
14	Mean	8.90	0.00
15	Median	7.5	#NUM!
16	Mode	8	#N/A
17			
18			
19			
20			

- Copy the original data set to cells C2 to C11. Hold down the arrow key  in cell C1 and select a number to be added to the original data set. Press  button in cell C1. The new data set will be displayed in cells C2 to C11.

Observe the change of the 3 measures of central tendency and note the relations between

- the original mean and the new mean,
 - the original median and the new median, and
 - the original mode and the new mode.
- Repeat the above steps for a different set of data of positive integers.
 - Write down your conjecture on the relations between these new measures of central tendency and the old ones.

CONJECTURE :

Activity 2: To explore the change of the measures of central tendency when each datum in the data set is multiplied by a constant



	A	B	C	D
1		original data	add a constant + 10	multiply by a constant x 5
2		2		
3		4		
4		8		
5		8		
6		23		
7		6		
8		18		
9		7		
10		5		
11		8		
12				
13				
14	Mean	8.90	0.00	0.00
15	Median	7.5	#NUM!	#NUM!
16	Mode	8	#N/A	#N/A
17				
18				
19				
20				

- Copy the original data set to cells D2 to D11. Hold down the arrow key  in cell D1 and select a number to be multiplied to the original data set. Press  button in cell D1. The new data set will be displayed in cells D2 to D11.

Observe the change of the 3 measures of central tendency and note the relations between

- the original mean and the new mean,
 - the original median and the new median, and
 - the original mode and the new mode.
- Repeat the above steps for a different set of data of positive integers.
 - Write down your conjecture on the relations between these new measures of central tendency and the old ones.

CONJECTURE :

Activity 3: To explore the change of the measures of central tendency when one or more data is removed from the set of data



	A	B	C	D	E
		original data	add a constant	multiply by a constant	remove a datum
1			+ 10	× 5	
2		2			
3		4			
4		8			
5		8			
6		23			
7		6			
8		18			
9		7			
10		5			
11		8			
12					
13					
14	Mean	8.90	0.00	0.00	0.00
15	Median	7.5	#NUM!	#NUM!	#NUM!
16	Mode	8	#N/A	#N/A	#N/A

- Copy the original data set to cells E2 to E11. Remove the largest datum from the data set. Select the cell of the datum to be deleted (the cell B6 in this case), press **Backspace** or **Del** and then press **Enter**. The new data set will be displayed in cells E2 to E11.
Observe the change of the 3 measures of central tendency and note the relations between
 - the original mean and the new mean,
 - the original median and the new median, and
 - the original mode and the new mode.
 - Repeat the above activity by removing the smallest datum.
 - Remove a datum other than the largest and the smallest data. Observe the change of the 3 measures of central tendency.
- Repeat the above steps for a different set of data of positive integers.
- Write down your conjecture on the relations between these new measures of central tendency and the old ones.

CONJECTURE :

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Activity 4: To explore the change of the measures of central tendency

when a zero is inserted into the set of data



	A	B	C	D	E	F
		original data	add a constant	multiply by a constant	remove a datum	insert a zero
1						
2		2				
3		4				
4		8				
5		8				
6		23				
7		6				
8		18				
9		7				
10		5				
11		8				
12						
13						
14	Mean	8.90	0.00	0.00	0.00	0.00
15	Median	7.5	#NUM!	#NUM!	#NUM!	#NUM!
16	Mode	8	#N/A	#N/A	#N/A	#N/A
17						
18						

- Copy the original data set to cells F2 to F11. Enter '0' (zero) in cell F12. The new data set containing one more datum than the original data set will be displayed in cells F2 to F12.

Observe the change of the 3 measures of central tendency and note the relations between

- the original mean and the new mean,
 - the original median and the new median, and
 - the original mode and the new mode.
- Repeat the above steps for a different set of data of positive integers.
 - Write down your conjecture on the relations between these new measures of central tendency and the old ones.

CONJECTURE :

Notes for Teachers:

1. The worksheet in this exemplar is designed to be used with *Excel*. However, if computers are not accessible, calculators can be used.
2. As actual calculations of mean, mode and median are not involved in the activity, the teacher should make sure that students know the meaning of these measures of central tendency before the activity starts.
3. The teacher may use the exemplar as a class practice. However, students are encouraged to explore by themselves.
4. Built-in buttons and pull-down menus of numbers are provided in the spreadsheet for convenience of operation.
5. As the MODE function in *Excel* may cause ambiguities for calculations on multi-modal data sets, the teacher should remind students to choose the data with a single mode as the original data.
6. The teacher may ask students to choose different values of the common constant so as to widen the scope of data for investigation.
7. For more able students, formal proofs may be introduced after students' investigations. For less able students, more examples may be introduced for illustration.
8. Suggested answers for the activities:

Activity 1 If a positive constant is added to the data set, the new mean, the new median and the new mode will be increased by the same amount of the constant. If the constant added is a negative value, the three measures of central tendency will be decreased by the same amount of the constant.

Activity 2 If a positive constant greater than 1 is multiplied to the data set, the new mean, the new median and the new mode will be increased by the same multiple of the constant. If the constant multiplied is positive and less than 1, the three measures of central tendency will be decreased by the same ratio of the constant.

Activity 3 If a datum is removed from the data set, the values of the new mean, the new median and the new mode depend on the value of the datum removed.

If the datum removed is smaller than the original mean, the new mean will increase. If the datum removed is larger than the original mean, the new mean will decrease. If the value of the datum removed is equal to the mean, the new mean will be equal to the original mean.

The median may or may not change. Factors affecting the change include whether the datum removed is itself a median, whether there are many data having the same value as the median, whether the datum removed is bigger or smaller than the median.

If the datum removed is not a mode, the new mode is the same as the original mode. If the datum removed is a mode, the new mode may remain the same or the new data set could become bi-modal, tri-modal, etc, or even does not exist.

Activity 4 If '0' is added to the data set, the value of the new mean will be decreased.

If the median of the original data set is a unique datum, the new median will be decreased. If the original data set has more than one datum with the same value as the median, then the new median may or may not change.

The mode would remain the same because '0' is not the most frequent number in the original data set.

9. For more able students, the teacher can ask them to construct data sets containing non-positive integers in addition to positive integers for investigation.
10. The suggested answers provided in Point 8 refer to data sets, which contain only positive integers, as provided in this exemplar. However, if the data set contains numbers other than positive integers, the results may not be the same. The teacher should draw students' attention to this point. Some of the cases giving different results are provided below. The teacher should note that these examples are not exhaustive and are for illustrative purposes only.

- Activity 1 The same results apply as stated in Point 8
- Activity 2 If the data set is $\{0, 0, 0, 0, 1\}$, after multiplying a positive constant, say 3, to the data set, it becomes $\{0, 0, 0, 0, 3\}$. The mean is now increased but the median and the mode remain the same. If the data set is multiplied by a negative constant, say -2, the data set becomes $\{0, 0, 0, 0, -2\}$. The mean is now decreased but the median and the mode remain the same.
- Activity 3 If the data set is $\{0, 0, 0, 0, 1\}$, after a datum is removed, say 0, it becomes $\{0, 0, 0, 1\}$. The mean is now increased because the value of the datum removed is smaller than the old mean. If the datum removed is 1, the data set becomes $\{0, 0, 0, 0\}$. The mean is now decreased because the value of the datum removed is larger than the old mean. The median and the mode remain the same in these two cases. However, the change of the median or the mode depends on many factors, e.g. whether the datum removed is the median or the mode, whether the data set is bimodal, whether many data having the same value as the median.
- Activity 4 When 0 is added to the data set, the mean can be increased or decreased. For example, $\{4, 4, 6, 6\}$ becomes $\{0, 4, 4, 6, 6\}$, the mean is decreased from 5 to 4. For the data set $\{-4, -4, -6, -6\}$ changes to $\{0, -4, -4, -6, -6\}$, the mean is increased from -5 to -4. For the median, it depends on whether the original median has 'shifted away' from the position of the median after including 0 in the data set. The mode may change, remain the same or the data set may become bi-modal or the data set changes from bi-modal to uni-modal. For example, $\{1, 1\}$ becomes $\{0, 1, 1\}$, then the mode remains the same. If $\{0, 1, 1\}$ becomes $\{0, 0, 1, 1\}$, the data set becomes bi-modal. If $\{0, 0, 1, 2, 2\}$ becomes $\{0, 0, 0, 1, 2, 2\}$, the data set is changed from bi-modal to one with a single mode.