

Exemplar 13 :

The Expected Value of an Amount of Pocket Money

Objective : To understand the meaning of an expected value.

Learning Unit : Simple Idea of Probability

Key Stage : 3

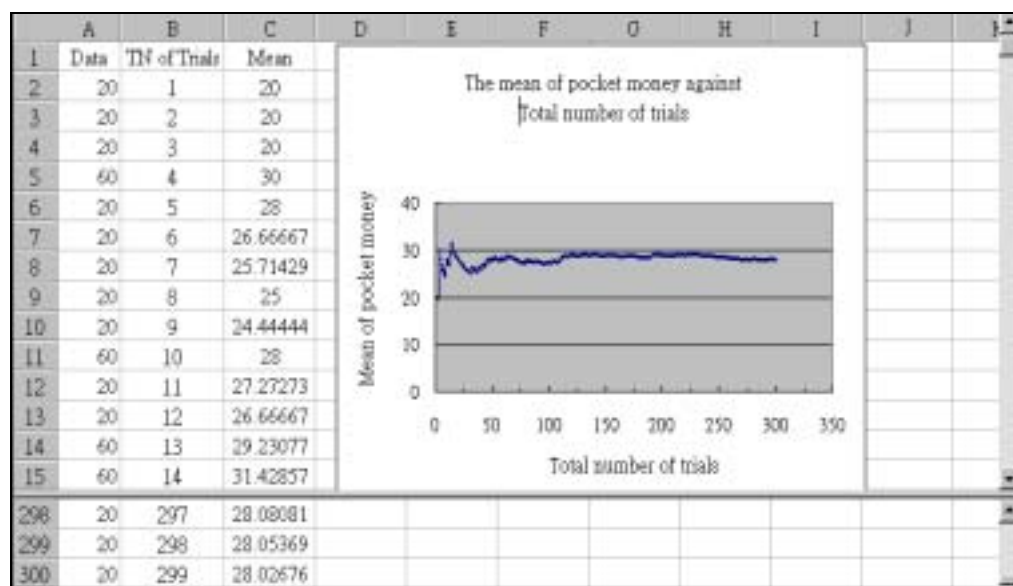
Materials Required : Spreadsheet programme – *Excel* or calculators,
the file dh13_e.xls

Prerequisite Knowledge : Mean, relationship between experimental probability and theoretical probability

Description of the Activity :

1. The teacher introduces the problem to students and distributes the worksheet to them.
2. Students are divided into groups of four for the activity.
3. The teacher asks students to open the file dh13_e.xls. The spin the pointer for 30 times and record the amount of pocket money obtained in each trial.
4. Students input all the data into the spreadsheet.
5. Students then use the spreadsheet programme to calculate the mean of pocket money for the 1st and 2nd trials, 1st, 2nd and 3rd trials and so on to include all the data from the students. If spreadsheet programme is not available, students can use calculators to do the calculation.

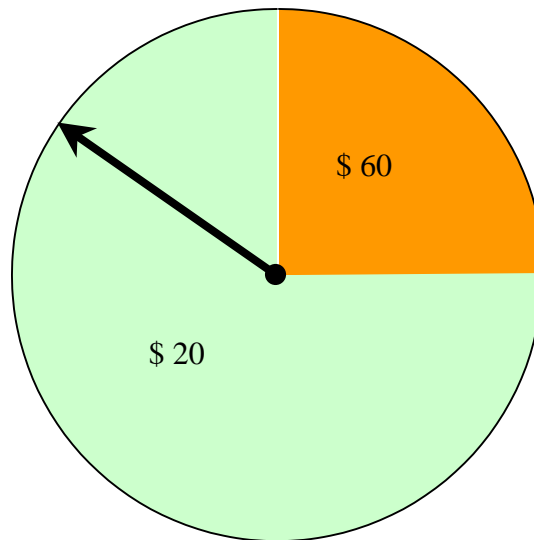
6. Students then plot the graph of the mean of the pocket money against the total number of trials. The following figure is an illustration.



7. The teacher asks students to guess how the mean would change in the graph when the total number of trials increases.
8. The teacher then introduces the meaning of an expected value as the mean of pocket money tends to a limit in the long run.
9. The definition of an expected value should then be introduced as an application of the weighted mean.

Worksheet

Jack usually receives \$30 as the pocket money from his father Donald everyday. One day Jack asks his father if he could have more pocket money. Donald makes the following suggestion. He uses a spinner to give Jack a choice. See the figure below.



Jack spins the pointer. If the pointer stops at the \$60 portion, Jack could get \$60. Otherwise, Jack could get \$20.

How much would you expect Jack to get in a month?

(Assume 30 days for convenience.)

1. Spin the pointer.

Record the amount of pocket money that you obtain in the table below.

Repeat the process for 30 times.

Group : _____

Table of records

Trial	Amount of pocket money	Trial	Amount of pocket money
1		16	
2		17	
3		18	
4		19	
5		20	
6		21	
7		22	
8		23	
9		24	
10		25	
11		26	
12		27	
13		28	
14		29	
15		30	
Total amount of pocket money			

How much would you expect Jack to get in a month? _____

What is the mean pocket money for Jack in this month? _____

Notes for Teachers :

1. The teacher can use other means such as dice instead of the computer spinner and modify the problem to suit his/her situation in the class.
2. Before students come to the formal definition of an expected value, they can visualize that the mean of pocket money tends to a certain limit in the long run with the help of a spreadsheet. The concept of an expected value can then be built up. This should be done before introducing the definition of an expected value.
3. The expected value can be introduced as an application of a weighted mean as follows.

Suppose in N trials, there are n_1 times to get \$60 and n_2 for \$20 (so $N = n_1 + n_2$).

The mean of pocket money can be found as follows.

$$\frac{60n_1 + 20n_2}{N} = 60 \times \frac{n_1}{N} + 20 \times \frac{n_2}{N}$$

$$\approx 60 \times P(\text{getting \$60}) + 20 \times P(\text{getting \$20}) \quad (\text{provided that } N \text{ is sufficiently large.})$$

In this exemplar, as $P(\text{getting \$60}) = \frac{1}{4}$ and $P(\text{getting \$20}) = \frac{3}{4}$, the mean of pocket money (in dollars) tends to $60 \times \frac{1}{4} + 20 \times \frac{3}{4} = 30$ which is called the expected value of the pocket money.

The teacher should remind students that the expected value is a theoretical value rather than an experimental finding.

So, Jack should expect to receive an amount of pocket money of $\$30 \times 30 = \900 in a month.