



Exemplar 14 :

A Game of Dice – “Dice Difference”

- Objectives :**
- (1) To investigate probabilities in real-life activities
 - (2) To consolidate the relation between empirical and theoretical probabilities

Learning Unit : Simple Idea of Probability

Key Stage : 3

Materials Required : Dice, transparencies and overhead projector

Prerequisite Knowledge : Meaning of probability, calculations of empirical and theoretical probabilities

Description of the Activity :

1. The teacher group students in pairs and assigns a number to each group. One of them is called “Mr. L” and the other is “Mr. H”.
2. The teacher distributes the Worksheets 1 & 2 and two dice to each pair of students for the game.
3. The teacher ask students to perform the following game according to the following rules :
 - (a) Throw two dice 25 times.
 - (b) In each throw, subtract the smaller number from the larger number. The answer is called “Dice Difference”.
 - (c) If the “Dice Difference” is 0, 1 or 2, then Mr. L wins. If the “Dice Difference” is 3, 4 or 5, Mr. H wins.
4. Students enter their results in the Worksheet 1.

5. After each pair of students has finished throwing the dice for 25 times, the teacher collected all the data from the students and enter all of them in a table for investigation. The teacher uses a transparency to show the data collected. An example of the table is shown below.

Table of Whole Class

Group number	Number of times Mr. L wins	Number of times Mr. H wins
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
Total		

6. The teacher then discuss the following questions with students:
 - (a) What is the empirical probability that Mr. L wins in each group? (The teacher can add one more column to the table and write down the answers.)
 - (b) Is there any pattern found in (a)?
 - (c) Does the game seem to be fair for both players? Why?

At this moment, the teacher does not make any comments on students' findings.
7. The teacher asks students to use the data obtained in the Table of Whole Class to complete Worksheet 2.
8. After students have completed the table in Worksheet 2, the teacher discusses the following questions with students:
 - (a) What will the empirical probability that Mr. L wins tend to in the long run?
 - (b) What will the empirical probability that Mr. H wins tend to in the long run?
 - (c) Is the game fair to both players? What conclusion can be drawn?
9. By using the grid paper or any other method, students list out the sample space of the outcomes of throwing two dice and then determine the numbers of favorable outcomes (i.e. Mr. L or Mr. H wins).
10. Students are guided to draw conclusions from the theoretical probability that each player wins and determine whether the game is fair or not.
11. If students think the game is not fair, they can modify the rules to make it fair.
12. They should give justifications for their suggestions.

Worksheet 1

Rules of the game

1. Throw two dice 25 times.
2. In each throw, subtract the smaller number from the larger number. The answer is called “Dice Difference”.
3. If the “Dice Difference” is 0, 1 or 2, then Mr. L wins. If the “Dice Difference” is 3, 4 or 5, Mr. H wins.
4. Put a tick (✓) in the appropriate box for the result of each throw.

No. of Throw	Mr. L wins	Mr. H wins
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		
Total no. of wins		

Who is the winner? _____

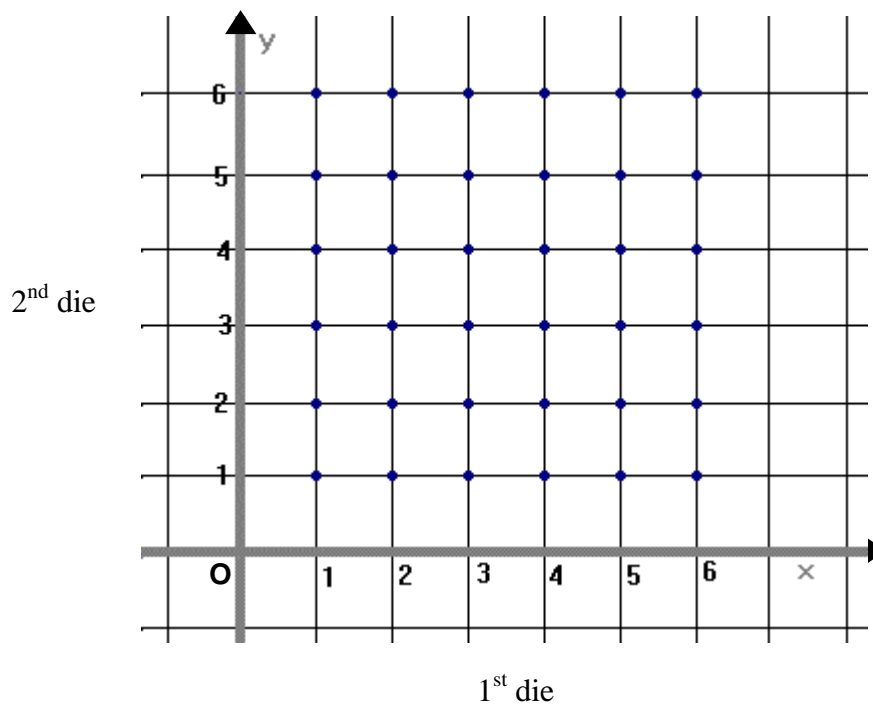
Worksheet 2

Use the results from the Table of Whole Class from your teacher's transparency to complete the following table.

Group number less than or equal to	Accumulated number of times Mr. L wins	Number of rounds played	Empirical probability that Mr. L wins
1		25	
2		50	
3		75	
4		100	
5		125	
6		150	
7		175	
8		200	
9		225	
10		250	
11		275	
12		300	
13		325	
14		350	
15		375	
16		400	
17		425	
18		450	
19		475	
20		500	

Notes for Teachers :

- Throughout the game, students will experience that the empirical probability will get close to the theoretical probability as the number of trials increases.
- Students can use (a) a grid paper; (b) ordered pairs, or (c) the tabulation method to show all possible outcomes when two dice are thrown.
 - Grid paper



- Ordered pairs

	1 st die					
	1	2	3	4	5	6
1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

(c) Tabulation method

Dice Difference	Favorable outcomes in ordered pairs
0	(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)
1	(1,2) (2,3) (3,4) (4,5) (5,6) (2,1) (3,2) (4,3) (5,4) (6,5)
2	(1,3) (2,4) (3,5) (4,6) (3,1) (4,2) (5,3) (6,4)
3	(1,4) (2,5) (3,6) (4,1) (5,2) (6,3)
4	(1,5) (2,6) (5,1) (6,2)
5	(1,6) (6,1)

3. Study the following table.

Dice Difference	0	1	2	3	4	5
Total number of favorable outcomes	6	10	8	6	4	2

This game is not fair since the theoretical probability of Mr. L wins is $\frac{6+10+8}{36} = \frac{2}{3}$ and that of Mr. H is $\frac{6+4+2}{36} = \frac{1}{3}$.

4. Suggestion for modifying the rules to make the game more fair:

Replace Mr. L and Mr. H by Mr. E and Mr. O. Mr. E wins if the Dice Difference is 0, 2 or 4 and Mr. O wins if the Dice Difference is 1, 3 or 5.

Justification:

$$P(\text{Mr. E wins}) = \frac{6+8+4}{36} = \frac{1}{2}$$

$$P(\text{Mr. O wins}) = \frac{10+6+2}{36} = \frac{1}{2}$$

Accept other modifications from students provided that the two players have equal chance of winning.