EXEMPLAR 4:

Accumulated Errors

Objective: To be aware of the accumulated errors arisen in using formulas

Key Stage: 3

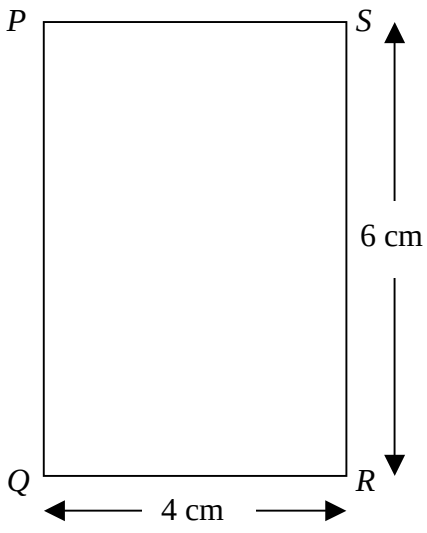
Learning Unit: Simple Idea of Areas and Volumes

Materials Required: Dynamic geometry software such as *Geometer's Sketchpad* (later referred as *Sketchpad*), ruler (graduated in cm only) and the file error01.gsp

Prerequisite Knowledge: Meaning of errors

Description of the Activity:

1. The teacher asks students to draw a rectangle *PQRS* with *SR* = 6 cm and *QR* = 4 cm on a piece of paper. Students use rulers (graduated in cm only) to draw the rectangle (see figure below).



2. The teacher poses the question "What is the area of the rectangle?" to students.

3. Students are expected to give the answer 24 cm^2. The teacher queries students whether the actual area of the figure is exactly 24 cm^2. It is expected that students may raise the question that there may be some errors either in drawing the figure or in measuring the length/width.

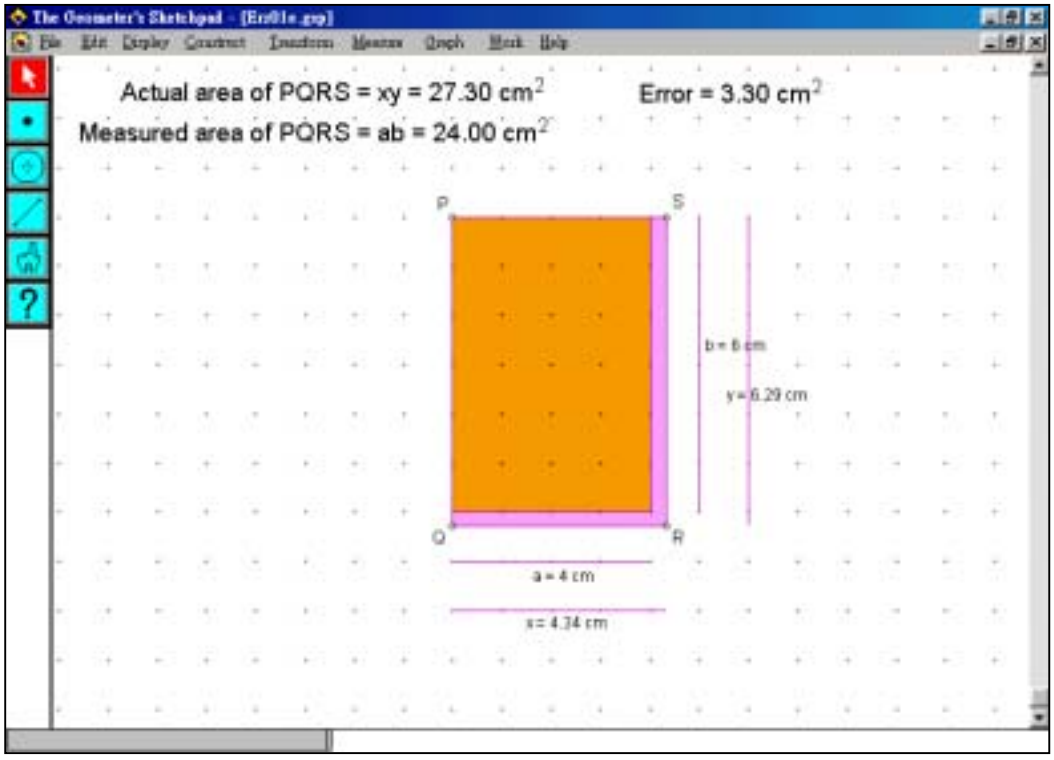
4. The teacher goes on asking students the question "How do you find the accumulated error of the area of the rectangle?". The teacher may further explain the meaning of accumulated error. (The term "accumulated error" refers to the indirect error due to the operations on the measured length and/or width.)

5. The teacher makes a brief revision on the “error in measurement”. As the measurement of length/width is correct to the nearest cm, the maximum error is 0.5 cm. The teacher also makes revision on the range that the length/width lies in. Students are expected to understand that *QR* lies between 3.5 cm and 4.5 cm whereas *SR* lies between 5.5 cm and 6.5 cm.

6. The teacher asks students whether the accumulated error of area is 0.5cm×0.5cm = 0.25cm^2. Students are invited to present their opinions to the whole class. It is suggested that the teacher makes no comments at this moment.

7. The teacher distributes Worksheet to students and briefly explains the activity.

8. Students make use of Worksheet and the *Sketchpad* file error01.gsp by dragging the vertex *R* of the rectangle in the file to investigate the problems of error (see the figure below).



9. Students are guided to find that when *R* is dragged to the position at which *PQRS* has maximum area (i.e. 6.5 cm × 4.5 cm), the error is 5.250 cm^2. When *R* is dragged to a position at which *PQRS* has minimum area (i.e. 5.5 cm × 3.5 cm), the error is 4.750 cm^2. Thus, the value of the (maximum) accumulated error is 5.25 cm^2.

10. Students are guided to find out that the maximum accumulated error changes when the width *a* and the length *b* of the original rectangle changes within the respective ranges specified in point 5 above. (This is illustrated by point 5 of Worksheet.)

11. Students are asked to present their findings to the class. They should be guided to conclude that

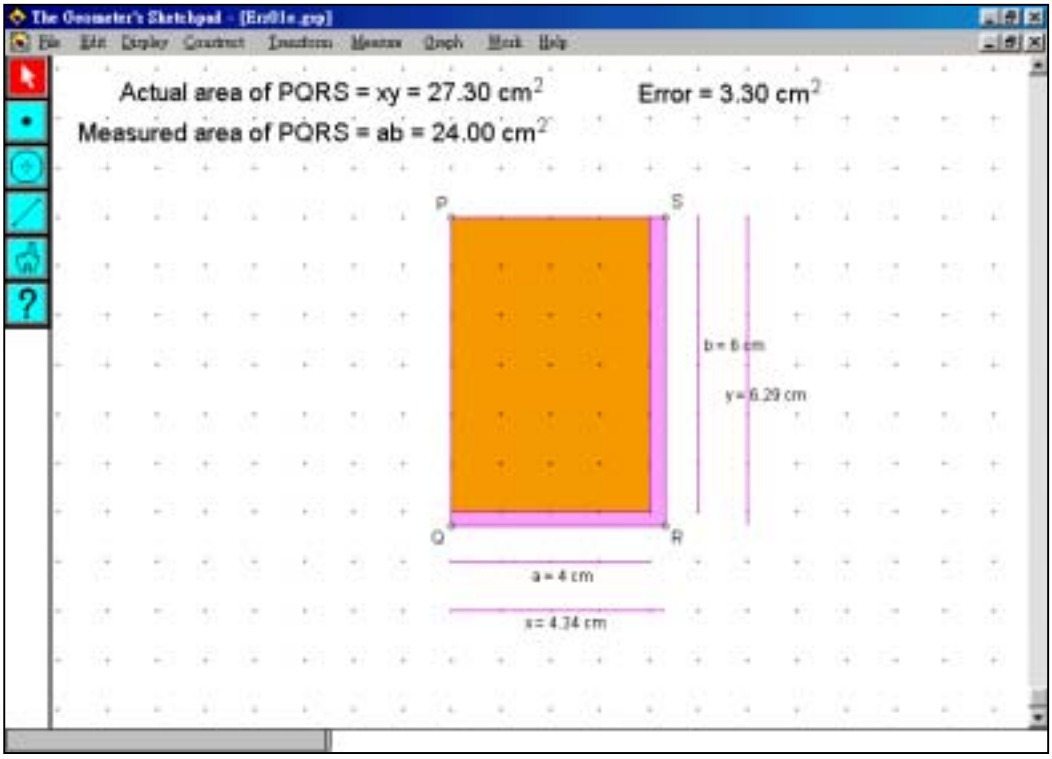
    - although the errors in measuring the length and width are 0.5 cm, the error of the area is, in general, not 0.5 cm × 0.5 cm (= 0.25cm^2);
    - the accumulated error is not the direct product of the respective errors in measuring the length and the width.

    It is important to emphasize the way to find the (maximum) accumulated error to students in this exemplar.

Worksheet : Accumulated Errors

1. Open the *Sketchpad* file error01.gsp.

2. You can find a rectangle with width a = 4cm and length b = 6cm. $PQRS$ is another rectangle with actual width $QR = x$ and actual length $SR = y$ (see the figure below).



3. Drag the vertex R to observe the change. Record the results in the following table.

Width (x cm)	Length (y cm)	Area of $PQRS$ (xy cm^2)	Error, in cm^2

4. Find the maximum accumulated error of the area of the rectangle.

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5. Drag the point *P* to other position to change the width *a* and the length *b* of the original rectangle. Write down the ranges of *a* and *b*. Does the maximum accumulated error of the area of the rectangle change with respect to question 4?

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Notes for Teachers:

1. The objective of this exemplar is to help students get an intuitive idea on the accumulated error arisen by using the formula "Area of a rectangle = length × width". Students should also know that the accumulated error is not the direct product of the error of the length and the error of the width.

2. Teachers may remind students that the term "error" and "maximum accumulated error" are respectively equivalent to "absolute error" and "maximum absolute error" if students have learned the learning unit "Approximation and Errors" before.

3. Students are reminded that the decimal places for *x* and *y* should be 1 rather than 2. The display in the file of 2 decimal places for *x* and *y* is due to the requirement of the software in order to show the 2 decimal places of the product of *x* and *y*.

4. In this activity, it is assumed that the length and the width of the rectangle are measured with a ruler and correct to the nearest cm. Hence, the *Sketchpad* file is written to restrict the location of the point P in integral values such that the numerical values of *a* and *b* are integral. Teacher may select **Graph**|**Hide Grid** to hide the grid points in the window. In this case, dragging the vertex P may result in non-integral values of *a* and *b* for the rectangle.

5. The *Sketchpad* file is written to confine the value of *a* and *b* within the ranges of the length and width specified in point 5 of the Description of the Activity when dragging the point *R*. When *R* is dragged so that the figure has maximum area, the error is 5.250 cm^2. When *R* is dragged so that the figure has minimum area, the error is 4.750 cm^2. Thus, the maximum accumulated error is found and it is 5.25 cm^2.