Exemplar 6:

Construction and Properties of Polyhedra

Objectives: (1) To recognize and construct the regular polyhedra (Platonic solids)
(2) To discover some properties of polyhedra such as Euler's Formula and duality

Key Stage: 3

Learning Unit: Introduction to Geometry

Materials Required: (1) Plastic polyhedron set or plastic drinking straws with pins
(2) A set of all five Platonic solids, and other polyhedra such as Archimedean solids, pyramids, etc.

Prerequisite Knowledge: Experiences in constructing solids and knowledge of regular polygons

Description of the Activities:

Activity 1: Collection of polyhedra

1. The teacher asks students to collect examples (pictures and photographs) of different polyhedra found in buildings (e.g. modern high-rise buildings), art construction (e.g. pyramids), nature (e.g. crystals, etc).

2. Students are invited to present the pictures and photographs they collected in the class. The teacher guides students to discuss the idea of regular polyhedra and identifies which polyhedra are regular and which are not.

Activity 2: Construction of regular polyhedra

3. Students are arranged in groups of three or four. The teacher distributes each group with 2 sets of regular polygons with each set containing only one type of regular polygons. The teacher asks students to produce 2 regular polyhedra

separately from each set of polygons.

4. Students of different groups are invited to show the regular polyhedron they constructed. The teacher then introduces the name of each polyhedron.

5. The teacher discusses with students the type of polygon which can be used to construct regular polyhedra. Students count the total number of regular polyhedra and summarize that there are only 5 regular polyhedra. The term "Platonic solids" is introduced.

Activity 3: Properties of polyhedra

6. Referring to the regular polyhedra constructed in Activity 2 and other polyhedra collected in Activity 1, the teacher introduces the terms: face, edge and vertex of a polyhedron.

7. Students are asked to count the number of faces, edges and vertices of the polyhedra they constructed or collected. The teacher asks students to complete the table in the worksheet and asks them to find the pattern of the numbers.

Name	Number of faces, **F**	Number of vertices, **V**	Number of edges, **E**
Polyhedra collected in Activity 1			
Regular polyhedra constructed in Activity 2			
Regular Tetrahedron			
Cube (Regular Hexahedron)			
Regular Octahedron			

Regular Dodecahedron			
Regular Icosahedron			

8. The teacher asks students to present the patterns they find. These may include:
   - Euler's formula: F + V - E = 2,
   - Dual relationship of cube and regular octahedron
   - Dual relationship of regular dodecahedron and icosahedon

   The teacher guides students to observe the above properties or help students summarize the findings for less able classes.

9. The teacher explains further the properties of the dual polyhedra. For example, the vertices of a regular octahedron putting inside a cube touch the centres of the faces of the cube.

Worksheet: Properties on Polyhedra

1. Refer to the following polyhedra and the polyhedra collected, count the number of faces, vertices and edges of each polyhedron and complete the following table:

Name	Number of faces, **F**	Number of vertices, **V**	Number of edges, **E**
Polyhedra collected in Activity 1			
Regular polyhedra constructed in Activity 2			
Regular Tetrahedron			
Cube (Regular Hexahedron)			
Regular Octahedron			
Regular Dodecahedron			
Regular Icosahedron			

2. Observe the pattern of the number of edges, vertices and faces, discuss with your classmates the observations.

Notes for Teachers:

1. Regular polyhedra have the following features:
   (a) They look the same when viewed at any corner, edge, or centre of any face.
   (b) They have congruent regular polygon faces.
   (c) They have congruent edges, face edges, and corners.

   There are only 5 regular polyhedra (or called Platonic solids). They are regular tetrahedron, regular hexahedron (or cube), regular octahedron, regular dodecahedron, and regular icosahedron. (See Fig. 1)

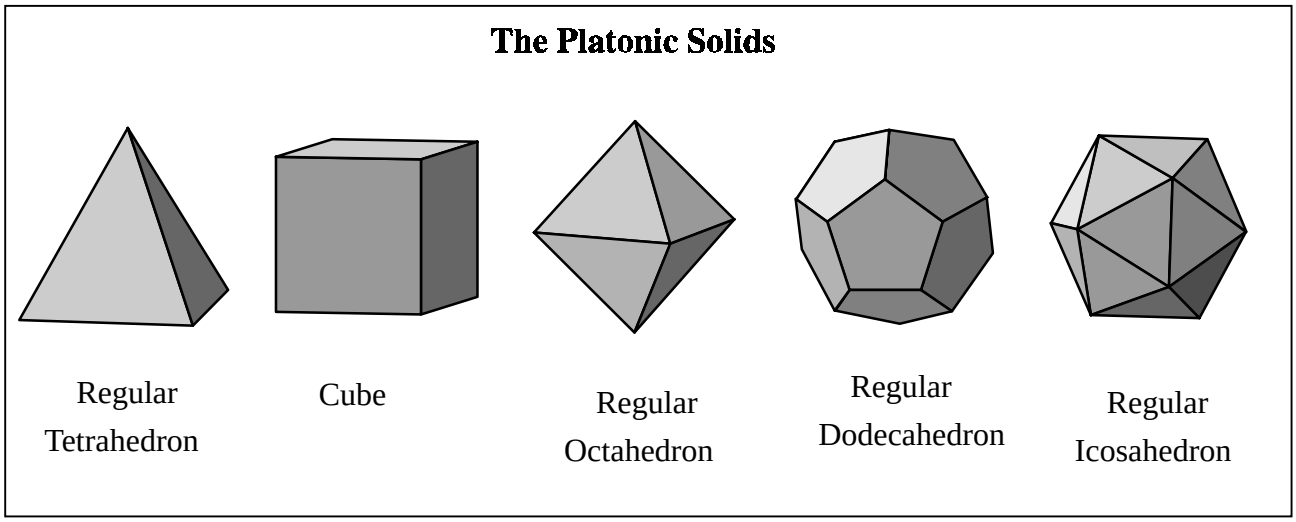

Fig. 1

2. The objective of the activity is to allow students to have hands on experience with manipulation of polyhedra in order to foster their spatial ability. Sufficient time should be allowed for students to play around the solids. The conclusion drawn from the activity is not, comparatively, the main emphasis in this activity.

3. Some students in primary schools may have experiences in constructing regular polyhedra. However, it will be interesting and fun to allow students to construct polyhedron in the class. Alternatively, teachers can demonstrate the construction of tetrahedron in the class and ask students to construct other regular polyhedra as homework. Polyhedra can be made from the nets shown in Appendix A, or plastic polyhedron set, or even with drinking straw with pins and covering with coloured paper. The photos of polyhedra found in the environment can also be downloaded from the CD-ROM 「空間探究圖片集」 of the learning package 「空間探究」 produced by the Education Department in 2000.

4. It is suggested to distribute one set of equilateral triangles to each group so as to minimize students' negative reaction towards no regular polyhedra constructed. Further, it is more likely that students will not construct dodecahedron or icosahedron. Hence, the teacher may prepare some models of them. If students do not (or cannot) construct these 2 solids, the teacher can show the prepared models. Drawing of regular polygons can be found in Appendix B.

5. For the discussion on the number of Platonic solids, the teacher can prepare some heptagons and octagons to give students **intuitive impressions** on why they cannot be used to form a regular polyhedron. A video produced by Key Curriculum Press on *Platonic Solids* may also be used to demonstrate the reasons for having only 5 Platonic solids.

6. Further exploration on the reasons of having only 5 regular polyhedra can be made in the learning unit "Angles Related with Lines and Rectilinear Figures" after students acquire the knowledge of the interior angles of regular polygons. Students may follow the arguments as follows:
   (a) only equilateral triangles, squares and regular pentagons can be used to produce Platonic solids.
   (b) with equilateral triangles, at each vertex of the polyhedra formed, there can only be either 3, 4 or 5 triangles joined together which resulted in tetrahedron, octahedron, and icosahedron respectively.
   (c) with squares, at each vertex of the polyhedra formed, there can only be 3 squares joining together, which resulted in the cube.
   (d) with regular pentagons, at each vertex of the polyhedra formed, there can only be 3 pentagons joining together which results in the dodecahedron.

   As a result, students should be able to draw the conclusion that there are only 5 regular polyhedra.

7. For more able students, the teacher may ask them to study some readings on the name of Platonic solids and the discussion made on Euclid's book *Element*.

8. Concerning with the properties of polyhedra, further discussion on the short-cut of finding the number of vertices and edges can be carried out.

   (e.g. for a dodecahedron, E = no. of faces × no. of edges per face ÷ 2,
   V = no. of faces × no. of vertices per face ÷ 3)

9. For the discussion on dual relationship, it is difficult for some students to discover that the numbers of edges for some pairs of regular polyhedra are the same and for these pairs, the numbers of faces and vertices are interchanged. Hence, the teacher may give hints to students and then introduce the term “dual polyhedra”. The explanation for such a rule can be a project for students. Students are encouraged to explain their arguments in class to foster their communication skills and their ability to express visual images. Besides, teacher can use some pictures to demonstrate that the edges of a octahedron connect the centres of the faces of a cube (See Fig. 2 & Fig. 3).

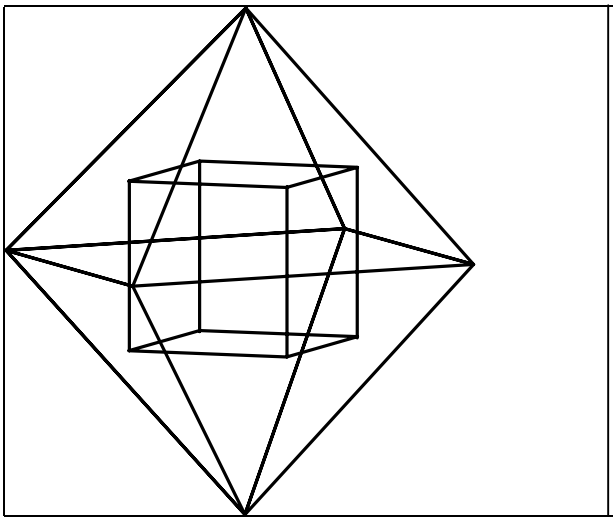

Fig. 2

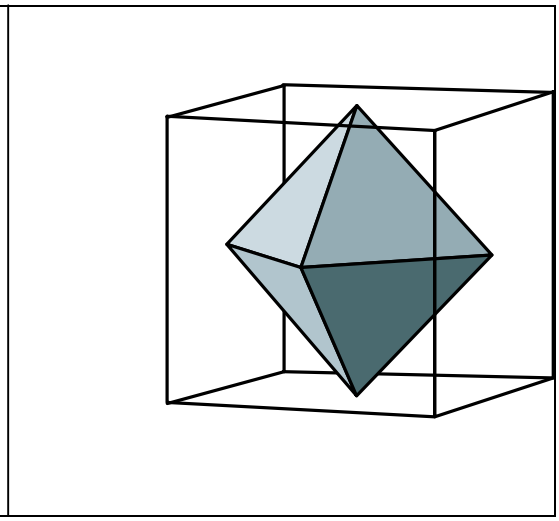

Fig. 3

References:

Books:

1. Glidden, P. & Fry, E. (1993). Two proofs that only five regular polyhedra exist. In *Mathematics Teacher*, pp. 657-61.
2. Hopley, R.B. (1994). Nested Platonic Solids: A class project in solid geometry. In *Mathematics Teacher*, pp. 312-8.
3. Pedersen, J. & Hilton, P. (1988). *Build Your Own Polyhedra.* Menlo Park, CA: Addison-Wesley Innovative Learning Publications.
4. Pugh, A. (1990). *Polyhedra: A visual Approach.* Palo Alto, CA: Dale Seymour Publication.
5. Seymour, Dale. (1994). *Polyhedra Blocks.* Palo Alto, CA: Dale Seymour Publication.
6. M. Senechal & G. Fleck ed. (1988). *Shaping Space —A Polyhedral Approach* USA: Birkhauser Boston (This book includes lots of diagrams on polyhedra and discussions applications of polyhedra in science and in daily life.)
7. Seymour, Dale. (1995). *Advanced Polyhedra Blocks.* Palo Alto, CA: Dale Seymour Publication.

8. 藍紀正、朱恩寬譯。(1992)。《歐幾里得 — 幾何原本》。中華台灣：九章出版社翻譯。(This book includes the proof on the number of regular polyhedra。)
9. 黃毅英。(1997)。《邁向大眾數學的數學教育》。台灣：九章出版社。 (This books includes lots of illustrations on 3-D figures.)

Web sites:

1. Building Polyhedra Project
   - http://www.frontiernet.net/~decoates/poly.html
2. Platonic solids
   - http://www.teleport.com/~tpgettys/platonic. html
   - http://www.construct.net/projects/euclid/index.html
   - http://www-personal.umich.edu /~horsey/match/proof.html
   - http://www.mhri.edu.au/~pdb/geometry/platonic/
3. Compounds of Regular Solids
   - http://www.teleport.com/~tpgettys/stellate.shtml

Teaching aids:

The teaching kit “Platonic Solids” produced by Key Curriculum Press includes videos, pre-cut polygons for construction of regular polyhedra with the use of rubber bands to tie them up.