Exemplar 9:

Reflectional and Rotational Symmetry in a Plane

Objective: To explore the relation between reflectional symmetry and rotational symmetry of 2-D figures

Key Stage: 3

Learning Unit: Transformation and Symmetry

Materials Required: (1) Pre-cut polygons in coloured paper
(2) Computer for demonstration and the computer files in 2D_Sym

Prerequisite Knowledge: Basic understanding of the meaning of reflectional and rotational symmetries

Description of the Activity:

1. The teacher reminds students the idea of reflectional symmetry of 2-D figures. For some students who can't remember the idea, the teacher can use some daily life examples to recall the idea, such as the strip patterns in tiger, butterfly, leaf veins. (Fig. 1a to Fig.1c) (The teacher can use the computer files in 2D_Sym to animate the reflections).

Fig. 1a

Fig. 1b

Fig. 1c

2. Students are then asked to investigate the reflectional symmetry in polygons.

They are divided into groups and are given the Worksheet and various pre-cut polygons. Students are asked to draw as many as possible axes of reflection for each figure by folding the paper or other methods. They are requested to write their answers in the 1st column of the table in the worksheet.

3. The teacher checks students' answers and demonstrates the axes of reflection for each figure with the given computer file 2D_Symmetry.exe (Fig. 2).

Fig. 2

4. Besides reflectional symmetry, the teacher asks students to find the number of folds of rotational symmetry for each polygon and fill in the 2nd column of the table in the worksheet.

5. The teacher invites students to present their answers. The teacher may also use the computer file 2D_symmetry to animate the rotation (Fig. 3).

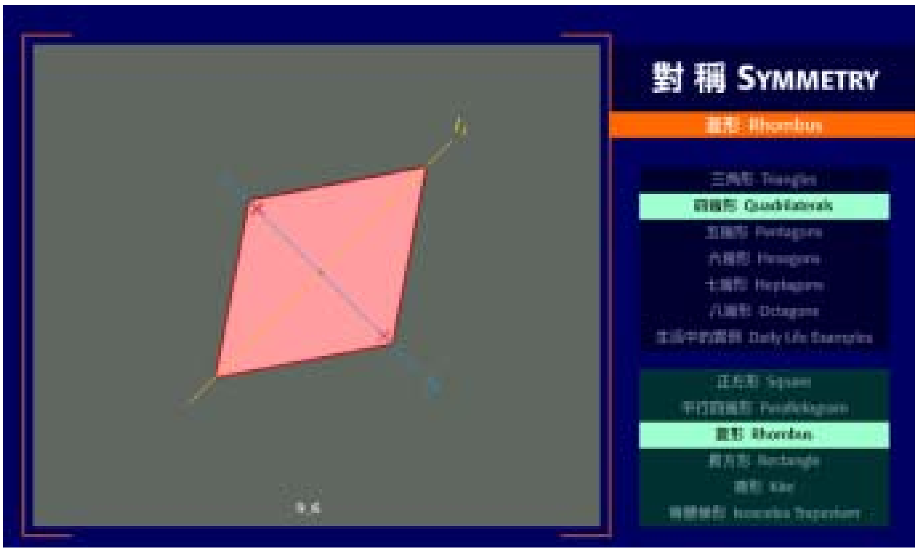

Fig. 3

6. Students are guided to summarize the following conjectures:
   (a) For regular n-sided polygons, there are n axes of reflection.
   (b) If n is an odd number, the axes of reflection of a regular n-gon will be the angle bisectors. If n is an even number, the axes of reflection of a regular n-gon will be either angle bisectors or medians.
   (c) A regular n-gon is a n-fold rotational symmetry.

Fig. 4

7. The teacher may further guide students to observe that:
   (a) If there are more than one axis of reflection, the axes must intersect at one point.
   (b) The intersecting point will be the centre of rotation.

   (See Fig. 3 and Fig. 4)

   From this observation, students are guided to discuss the conjecture "Figures with two or more different axes of reflection will have a rotational symmetry". The teacher challenges students whether the converse of the conjecture is correct and asks them to give counter-examples.

Worksheet: Rotational and Reflectional Symmetry of Polygons

With the given polygons, find the number of axes of reflection and the number of folds of the rotational symmetry:

	Polygons	No. of axes of reflection	No. of folds of rotational symmetry
(a)	Right-angled triangles		
	Isosceles triangles		
	Scalene triangles		
	Equilateral triangles		
(b)	Parallelograms*		
	Isosceles Trapeziums*		
	Rhombuses*		
	Rectangles*		
	Kites*		
	Squares		
(c)	Equilateral pentagons*		
	Equiangular pentagons*		
	Regular pentagons		
(d)	Equilateral hexagons*		
	Equiangular hexagons*		
	Regular hexagons		
(e)	Equilateral octagons*		
	Equiangular octagons*		
	Regular octagons		
(f)	Regular heptagons		

* Assuming that the polygons have no other special features.

Notes for Teachers:

1. The teacher can use Appendix B to prepare polygons.

2. The answers to the Worksheet are as follow:

	Polygons	No. of axes of reflection	No. of folds of rotational symmetry
(a)	Right-angled triangles	0	--
	Isosceles triangles	1	--
	Scalene triangles	0	--
	Equilateral triangles	3	3
(b)	Parallelograms*	0	2
	Isosceles Trapeziums*	1	--
	Rhombuses*	2	2
	Rectangles*	2	2
	Kites*	1	--
	Squares	4	4
(c)	Equilateral pentagons*	0	--
	Equiangular pentagons*	0	--
	Regular pentagons	5	5
(d)	Equilateral hexagons*	0	--
	Equiangular hexagons*	0	--
	Regular hexagons	6	6
(e)	Equilateral octagons*	0	--
	Equiangular octagons*	0	--
	Regular octagons	8	8
(f)	Regular heptagons	7	7

3. If students are very familiar with the activity, the teacher can skip point 1 and asks students to fill in the column directly. In finding the number of axes of

reflection, the teacher may ask students other strategies such as using mirror, or using reflection function in dynamic geometry software such as *Sketchpad*. The file 2D_symmetry.exe provided is only an alternative.

4. In using the 2D_symmetry.exe, it should be noted that some lines drawn on the figure in the file are not axes of reflection (Fig. 5). These lines are drawn to provide a counter-example for axes of reflection. The teacher can click the line l_1 in the reflection box to demonstrate why these lines cannot be named as axes of reflection. Students should be reminded that these figures do not have axes of reflection for any other lines drawn on the figure. For the case of figures without properties of rotational symmetry, the assumed centre is added on the figure to demonstrate why the figure does not overlap. The teacher should remind student that the figure will not overlap even other point is chosen to be the centre of rotation.

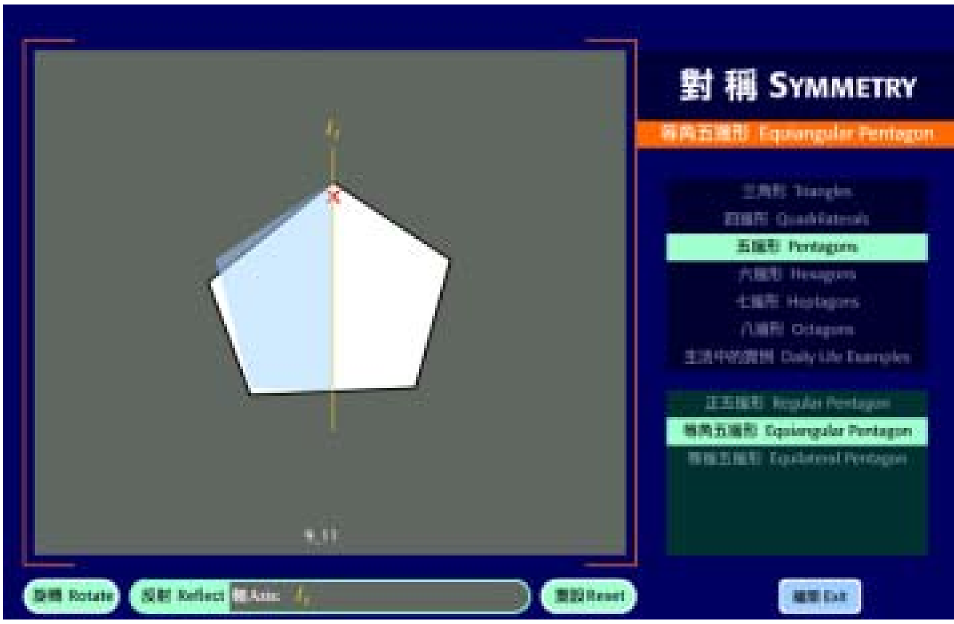

Fig. 5

5. Further, the main focus of the activity is to let students find out the relationship between reflectional symmetry and rotational symmetry. Sufficient time should be allowed for students to discuss the related conjecture.

6. For checking the converse of the conjecture “Figures with two or more different axes of reflection will have rotational symmetry”, the teacher can use alphabets like “N, Z” or parallelogram, etc. as counter-examples. These 2 letters have rotational symmetry but no reflectional symmetry.

7. Besides polygons, the teacher can give some letters to students to find their axes

of reflection as an enrichment activity. These can be:

(a) A, M, T, U, V, W, Y
(b) B, C, D, E, K
(c) H, I, O, X
(d) N, S, Z
(e) F, G, J, L, P, Q, R

Some of the letters can be cut from Annex of Exemplar 8.

8. The answer for each group of the above enrichment activity is as follows:
   (a) reflectional symmetry with a vertical axis (e.g. A)
   (b) reflectional symmetry with a horizontal axis (e.g. E)
   (c) reflectional symmetry with BOTH vertical and horizontal axes (at least two reflectional axes and with rotational symmetries also) (e.g. H)
   (d) rotational symmetry only (e.g. Z)
   (e) no reflectional symmetry and rotational symmetry (e.g. P)

9. For some students who are very interesting in finding the properties of reflectional symmetry of polygons, the teacher can invite them to check the validity of the conjecture

   "The number of axis of reflection of a polygon must always divide the number of sides of the polygon" or in other words "An n-gon has k axes of reflection only if k divides n".

   This proof can be found in P.364 of the Journal *The Mathematical Gazette* Vol. 77 No. 480 (1993).

References:

Books:

1. Bezuszka, S. Kenney, M. & Silvey, L. (1977). *Tessellations*: *The Geometry of Patterns*. Oak Lawn, IL: Creative Publications.
2. Crowe, D. (1986). *Symmetry, Rigid Motions, and Patterns*. Arlington, MA: COMAP.
3. Haber, H. & Kane, G. (1986). Is Nature Super Symmetric?. In *Scientific American*, 1986 June: pp. 52-75.
4. Tarasov, L. (1986). *This Amazingly Symmetrical World*. Moscow: Mir Publishers.
5. Thomas, D. (1980). Mirror Images. In *Scientific American*. 1980 December: pp. 206-30.
6. Weyl, H. (1980). *Symmetry*. Princeton, NJ: Princeton University Press.
7. Wilgus, W. & Pizzuto, L. (1997). *Exploring the Basics of Geometry with Cabri*.

Texas Instruments.

8. Wiltshire, A. (1989). *Symmetry Patterns*. Norfolk, England: Tarquin Publications.
9. Wyatt, K.W. et. al. (1998). *Geometry Activities for Middle School Studies with the Geometer's Sketchpad*. Key Curriculum Press.
10. 黃毅英 (1997)。《邁向大眾數學的數學教育》。台灣：九章出版社。
11. 項武義 (1994)。《幾何學的源起與發展》。台灣：九章出版社。

CD-ROMs

S*hape* in the CD-ROM **Fun with Learning 1998**

Web-sites:

1. Symmetry and Pattern: The Art of Oriental Carpets at:
   http://forum.swarthmore.edu/geometry/rugs/
   The site is a very informative site on concept building and examples of symmetry. The various border and field patterns are shown and explained in details. Computer software and online tutorials on tessellation (with Claris Works, HyperCard, Hyperstudios, PC PaintBrush, Logowriter, and others) are available at this site.
2. http://www.bbc.co.uk/schools/gcsebitesize/maths/shape/symmetryrev1.shtml/
   This site provides activities on the idea of symmetry with short exercises, examples and animation.
3. http://www.learn.co.uk/