

EXEMPLAR 12:

Minimal Conditions in Fixing a Triangle

Objective: To explore and recognize the minimal conditions in fixing a triangle.

Key Stage: 3

Learning Unit: Congruence and Similarity

Materials Required: Dynamic Geometry software such as *Cabri Geometry* and the *Cabri* files provided

Prerequisite Knowledge: Basic concepts about angles

Description of the Activity:

1. The teacher asks students "If one angle or one side is given, can we draw a triangle satisfying the given condition? Is the triangle drawn unique?". Students can suggest some answers to the questions.
2. The teacher poses the problem "What is the minimal conditions to fix a triangle?" to students.
3. The teacher then explains the meaning of "conditions" in this problem as either the number of sides or angles of a triangle. The teacher also explains the meaning of the term "minimal conditions" to students.
4. The teacher asks students to suggest some ideas to solve the problem.
5. The teacher begins the discussion on considering the condition on one given side or one given angle first. That is, if one side or one angle is given, can we fix the triangle formed?
6. As it is easy to answer the problem, the teacher goes on by reconsidering the problem with two given conditions.
7. The teacher divides students into groups of four and distributes Worksheet 1 to each student.
8. The teacher asks students to classify the conditions into different cases when only two conditions are given.

9. The teacher invites some students to present their classification to the whole class. At this moment, other students can comment on them.
10. The teacher concludes that there are only four different cases as stated in Worksheet 2.
11. The teacher distributes Worksheet 2 to students. The teacher selects a *Cabri* file to demonstrate the use of the file. Students are asked to do their own investigation whether the triangle can be fixed with the given 2 conditions using the corresponding *Cabri* files. They need to write their findings on the worksheet.
12. A representative from each group is invited to present his/her findings. From these, the teacher guides students to conclude that two conditions cannot fix a triangle.
13. The teacher distributes Worksheet 3 to students. Students are asked to classify the given conditions into different cases. Group discussion is suggested in order to let students share their points of view with others.
14. After discussion, the teacher invites some students to present their classification to the whole class. At this moment, other students can comment on them.
15. The teacher concludes that there are only six different cases as stated in Worksheet 4.
16. The teacher distributes Worksheet 4 to each student. Students are asked to do their own investigation and complete the worksheet by using the corresponding *Cabri* files.
17. The teacher asks a representative from each group to present their findings.
18. Students answer the posed problem and draw the final conclusion that the cases SSS, SAS, ASA and AAS can fix a triangle. As a result, the minimal number of condition in fixing a triangle is three.

Worksheet 1: Analyze the problem with 2 given conditions

A triangle consists of three sides and three angles. If 2 conditions are given, how many different cases can you find? Discuss with your group members. List all the cases from your discussion below.

Worksheet 2: Can you fix a triangle from 2 given conditions?

1. Table 1 states different cases and their corresponding files on 2 given conditions of a triangle.

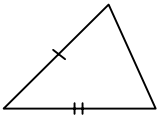
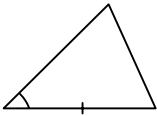
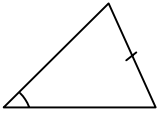
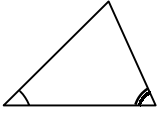
Case	Representing figure	File
2 sides		2conSS.fig
1 side and an adjacent angle		2conSA.fig
1 side and an opposite angle		2conA_S.fig
2 angles		2conAA.fig

Table 1

2. The following demonstrates the investigation on one of the cases in Table 1. Open the file 2conSS.fig. You will find two line segments drawn on the upper part of the window as the 2 given conditions and the corresponding triangle formed in the lower part of the window (see Fig. 1).

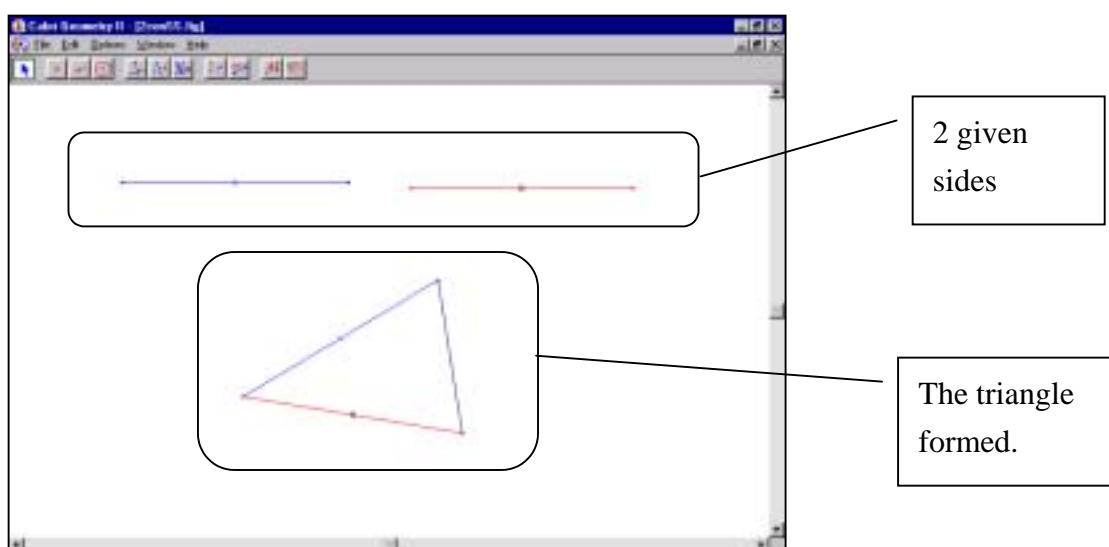


Fig. 1

3. Drag each vertex of the triangle to see if the given conditions can fix the triangle drawn.
4. Drag the end points of a line segment to change the length of the side of the given condition. Note the change of the triangle drawn from the given conditions. Repeat point 3.
5. Record your results in Table 2.
6. Guess an additional condition that can fix a triangle and write down in Table 2.
7. Repeat the same steps for the investigation of other files and write down your result in Table 2.

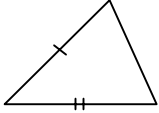
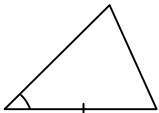
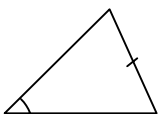
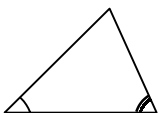
Case	Representing figure	Can you fix the triangle?	Write down an additional condition that can fix a triangle if necessary.
2 sides			
1 side and an adjacent angle			
1 side and an opposite angle			
2 angles			

Table 2

Worksheet 4: Can you fix a triangle from 3 given conditions?

1. Table 1 states different cases and their corresponding files on 3 given conditions of a triangle.

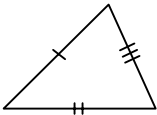
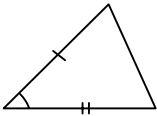
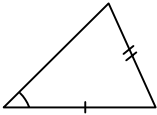
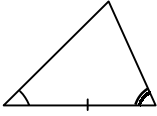
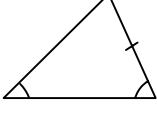
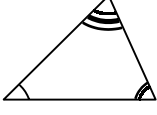
Case	Representing figure	File
3 sides		3conSSS.fig
2 sides and a included angle		3conSAS.fig
2 sides and a non-included angle		3conSSA.fig
2 angles and an included side		3conASA.fig
2 angles and a non-included side		3conAAS.fig
3 angles		3conAAA.fig

Table 1

- The following demonstrates the investigation of one of the cases in Table 1. Open the file 3conASA.fig. You will find two angles and one line segment drawn on the upper part of the window as the 3 given conditions and the corresponding triangle formed in the lower right hand corner of the window (see Fig. 1).

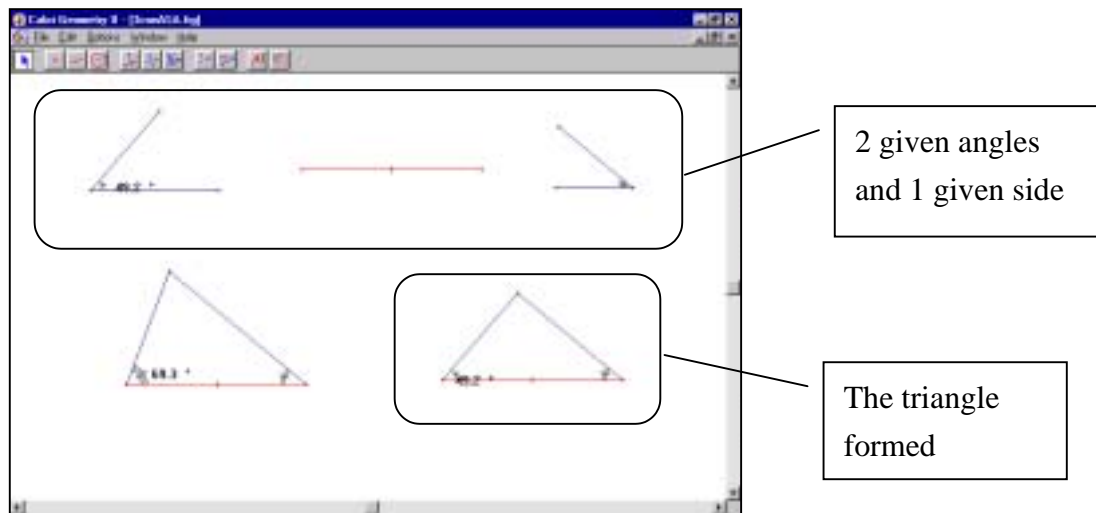


Fig. 1

- You may drag the end points of the angle or the line segment in the box at the top to change the size of the given angle or the length of the side of the 3 given conditions. Note the change in the triangle formed from the given conditions.
- At the lower left-hand corner of the window you will find a triangle drawn by using two of the given conditions only. (The two given conditions are one fixed side and one fixed angle.) Drag the vertex of this triangle to see if you can form a triangle exactly equal to the triangle at the lower right-hand corner. You can drag a vertex of the triangle at the lower left-hand corner to move the triangle you drew and place it at the top of the lower right-hand corner of the triangle to see if the two triangles are identical to each other (see Fig.2).

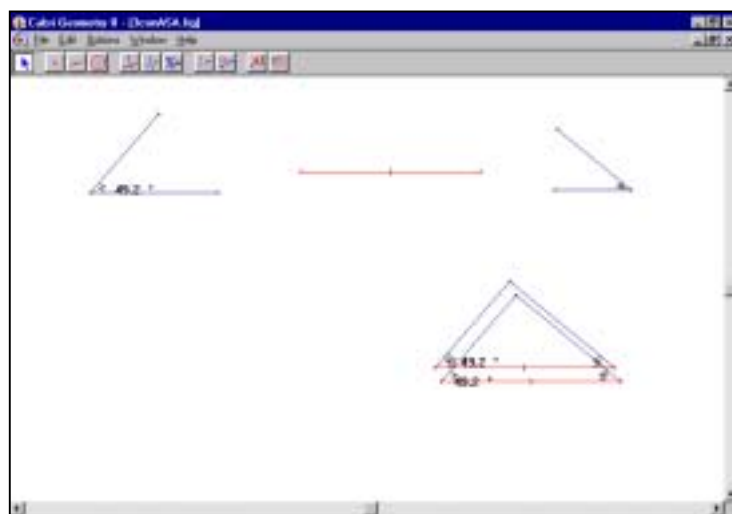


Fig. 2

5. Move the triangle you drew to the lower left-hand corner again. Drag the vertex of this triangle to see if you can form another triangle. Record your results in Table 2.
6. Guess an additional condition that can fix a triangle if necessary.
7. Repeat the same steps for the investigation of other files and record your results in Table 2.

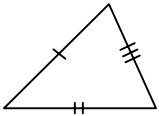
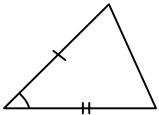
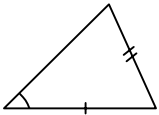
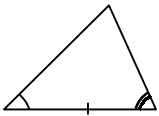
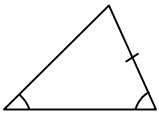
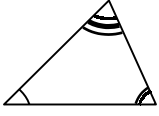
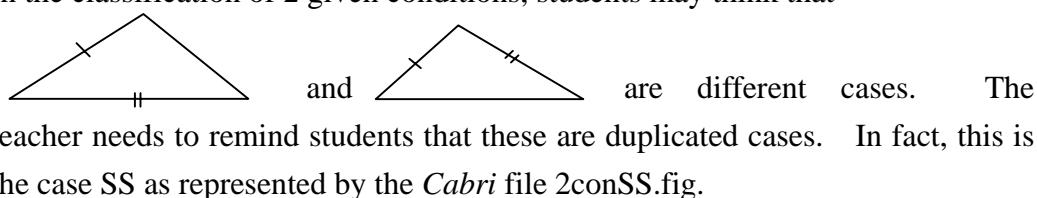
Case	Representing figure	Can you fix the triangle?	Write down an additional condition that can fix a triangle if necessary.
3 sides			
2 sides and a included angle			
2 sides and a non-included angle			
2 angles and an included side			
2 angles and a non-included side			
3 angles			

Table 2

Notes for Teachers:

1. This exemplar aims at introducing the concept of minimal conditions in fixing a triangle. A triangle consists of three sides and three angles. If all the sides and angles are given, a triangle can be formed provided that it obeys the triangle inequality and the sum of the angles should be 180° . This exemplar assumes that a triangle can be formed under the given conditions. The problem is whether the triangle formed is unique. The treatment for the triangle inequality is in the non-foundation part of the learning unit "Simple Introduction to Deductive Geometry".
2. The "conditions" in this exemplar refers to "the number of sides or the number of angles given". In Worksheet 1, there are various ways in classifying and analyzing the given conditions. This exemplar considers the cases from two given conditions to three given conditions. Firstly, students have to analyze the problem and classify the conditions into different cases. Students may experience that no unique triangle can be formed with only two given conditions (two sides; two angles; or one side and one angle). On the other hand, a unique triangle may be formed in some cases of three given conditions.
3. The teacher should explain clearly to students about the meaning of minimal conditions. As 2 given conditions cannot fix a triangle, the number of minimal conditions should be greater than 2. In the cases of 3 given conditions, the cases for SSS, SAS, ASA, AAS can fix a triangle whereas the cases SSA and AAA cannot. We can say that the number of minimal conditions to fix a triangle is 3.
4. In some of the above cases, a triangle cannot be formed. For example, in the case of SSS of 3 given conditions, a triangle cannot be formed if students drag a side longer than the sum of the other two. The teacher may raise a problem on why the triangle cannot be formed and relates to the concept of triangle inequality.
5. In the classification of 2 given conditions, students may think that



6. The case for RHS in 3 given conditions is not mentioned here to avoid confusion.

7. In Worksheet 1, the teacher is expected to lead students to arrive at the result of 4 cases mentioned in Table 1 of Worksheet 2. So teachers should not distribute Worksheet 2 to students until the final discussion in Worksheet 1 has been done. The same principle applies to Worksheet 4.
8. In the fourth column of Table 2 in Worksheet 2, students are expected to write down only their guesses on an additional condition that can fix a triangle. They need not examine this additional condition as they can find out the correctness of the additional condition by the result of Table 2 in Worksheet 4. For example, in the case of 2 sides in Table 2 of Worksheet 2, the additional condition to fix a triangle is "an included angle" or "the third side". The supporting reason can be found in the cases of SAS and SSS of 3 given conditions in Table 2 of Worksheet 4.