



Exemplar 13: Angles Associated with Parallel lines and Intersecting Lines

Objective: To explore the angle properties associated with parallel lines and their transversals

Key Stage: 3

Learning Unit: Angles Related with Lines and Rectilinear Figures

Materials Required: Dynamic Geometry software such as *Geometer's Sketchpad* (later referred as *Sketchpad*) and files Corresp.gsp, Alterna.gsp and Interior.gsp.

Prerequisite Knowledge: Basic knowledge of parallel lines and angles.

Description of the Activity:

1. The teacher asks students to recall the meaning of parallel lines and introduces the meaning of corresponding angles, alternate angles and interior angles associated with parallel lines and their intersecting lines. The teacher can distribute Worksheet 1 to less able students to assess their understanding of the meaning of various angles. The teacher then states the objective of this learning activity. For other students, the teacher can state the objective of the learning activity and go straight to point 2.
2. The *Sketchpad* file, Corresp.gsp, is shown (Fig. 1). Worksheet 2 is also distributed to students. The teacher discusses with students whether the lines AB and CD are parallel. Students are expected to give reasons to their statements.

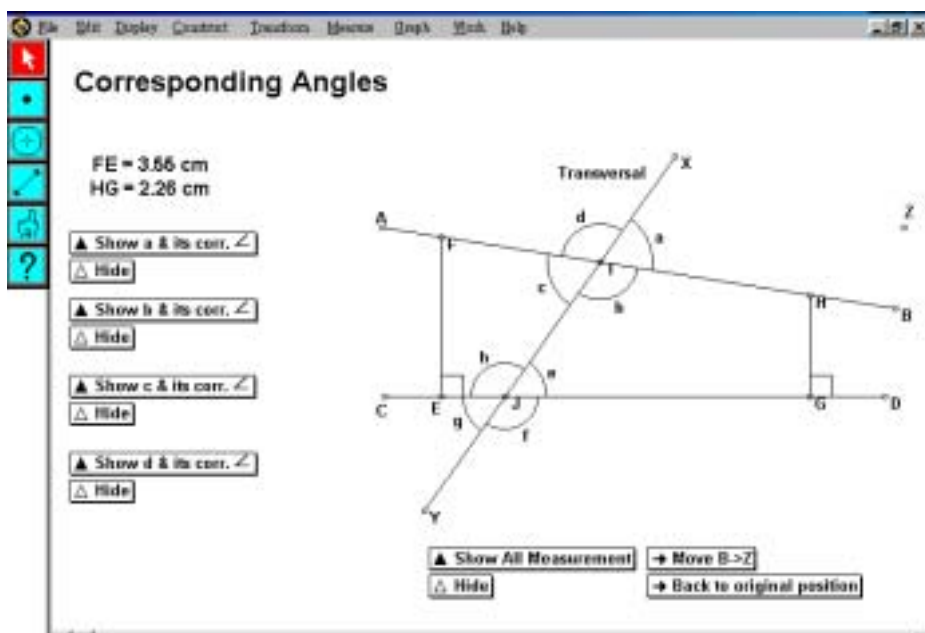


Fig. 1

3. While focusing on the file Corresp.gsp, the teacher asks students to name the corresponding angles of a , b , c and d respectively. The teacher then double clicks “▲ Show a & its corr. ∠” and other related buttons on angles to check students’ answers.
4. All the “△ Hide” buttons are double clicked leaving only angle a and its corresponding angle shown on the screen. Through dragging the point B to the point Z , i.e., to make FE equal to HG , students are requested to observe changes on the angles and to make their hypothesis (Fig. 2). The teacher discusses with students when a and its corresponding angle will be equal.

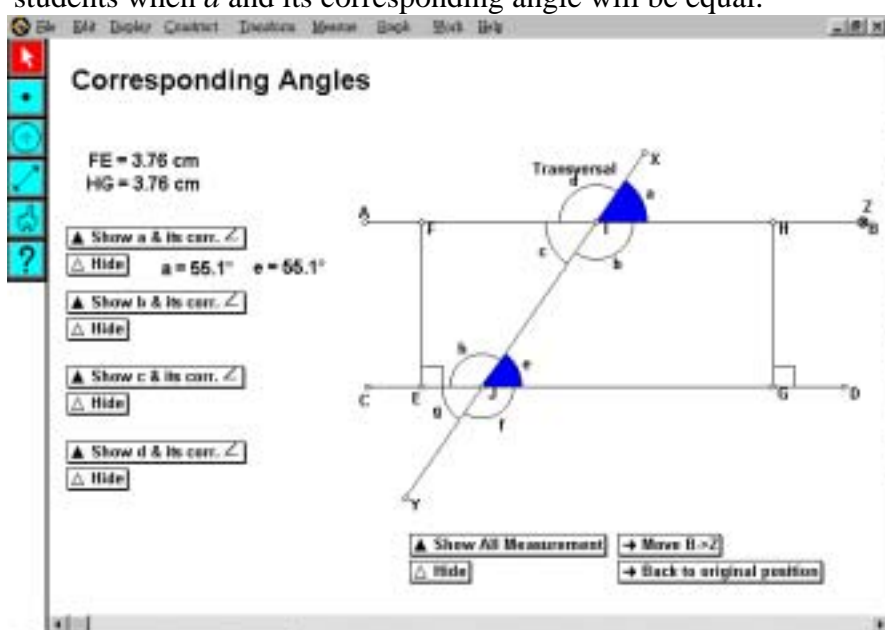


Fig. 2

5. Students are asked whether different sets of corresponding angles will be equal. The teacher double clicks “▲ Show *b* & its corr. ∠” and other related buttons on angles and “▲ Show All Measurement” button. Through dragging the point *B* and making the line *AB* parallel to *CD* again, students are asked to observe the changes on the values of corresponding angles (Fig. 3). Then, the teacher asks students to summarize the finding that “Each pair of corresponding angles in this pair of parallel lines are equal”.

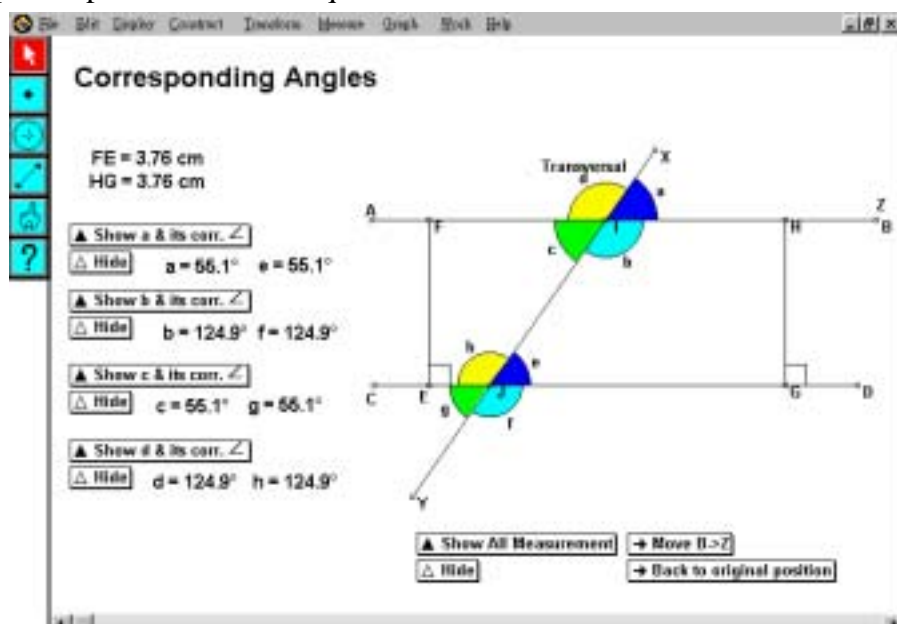


Fig. 3

6. The teacher asks students whether the finding in point 5 still holds in different transversals. Students are asked what happens to the relation when altering the direction of the transversal *XY*. The teacher can then check students' hypotheses through dragging the point *X* (Fig. 4).

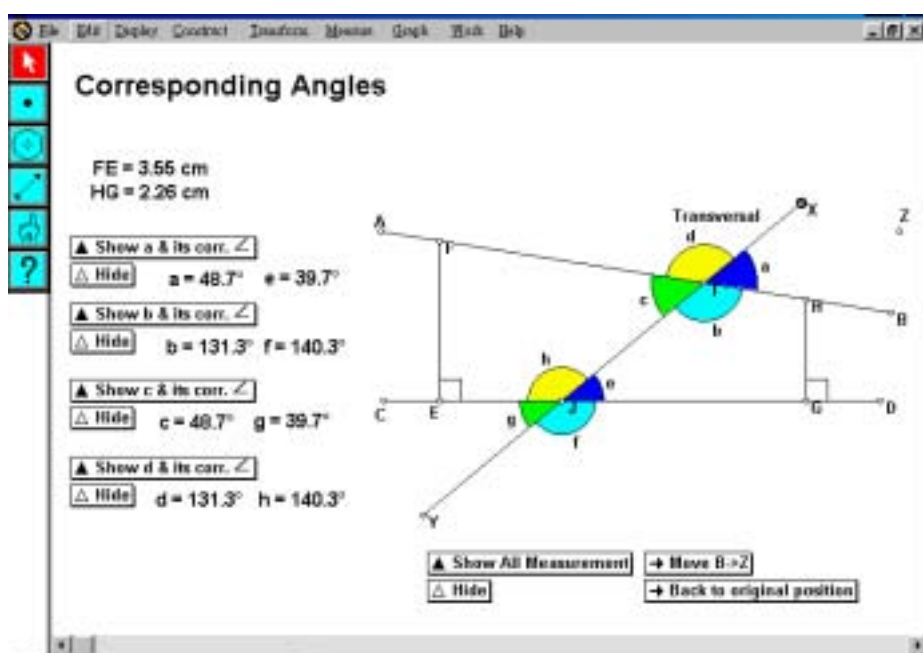


Fig. 4

- The teacher drags the point C to form different sets of parallel lines. Students are asked whether the previous finding is true (Fig. 5). The teacher then concludes that “once 2 lines are parallel, their pairs of corresponding angles are equal”.

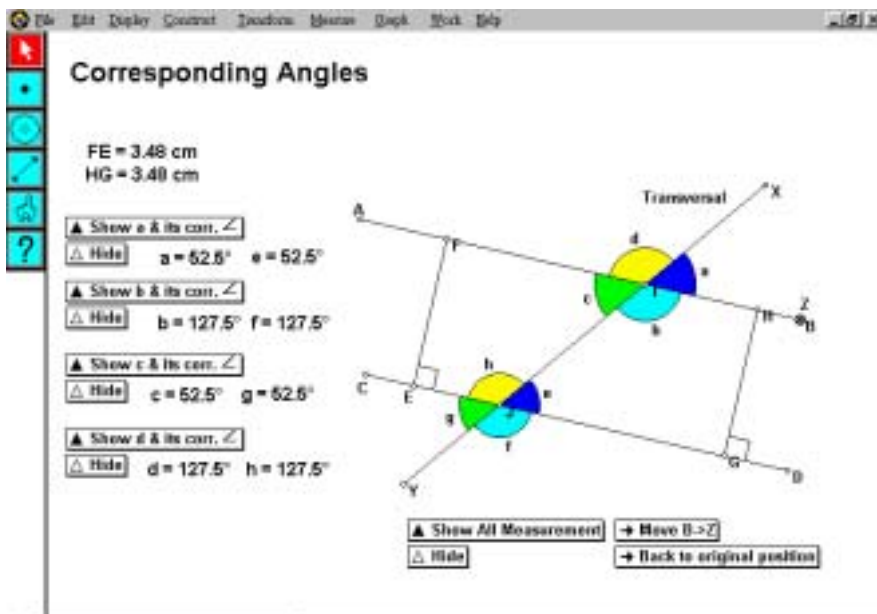


Fig. 5

- Students are distributed Worksheet 3, *Sketchpad* files *Alterna.gsp* (Fig. 6) and *Interior.gsp* (Fig. 7). They are asked to work on the files and to explore whether similar finding is true for alternative angles or interior angles associated with parallel lines.

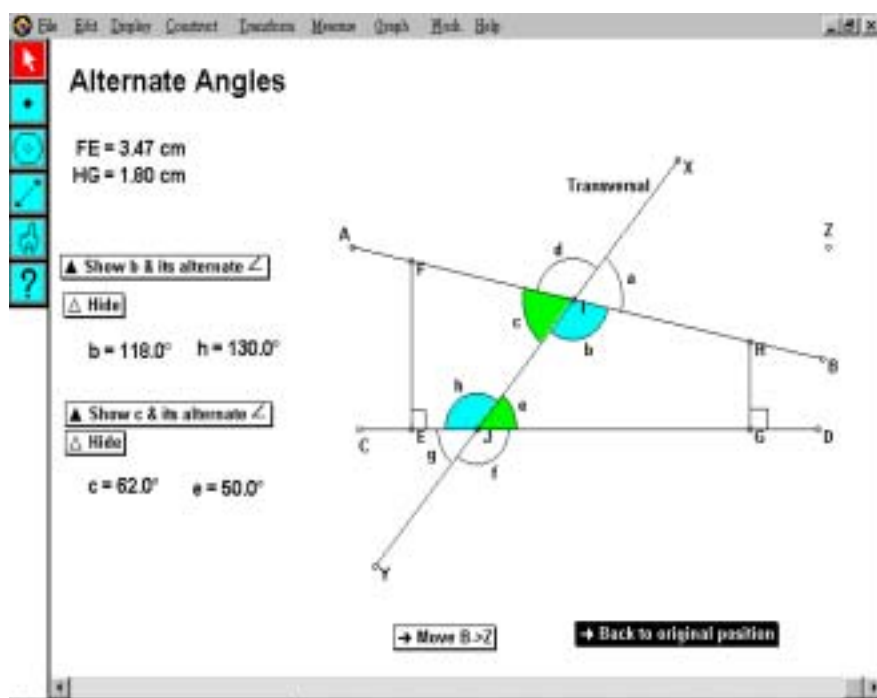


Fig. 6

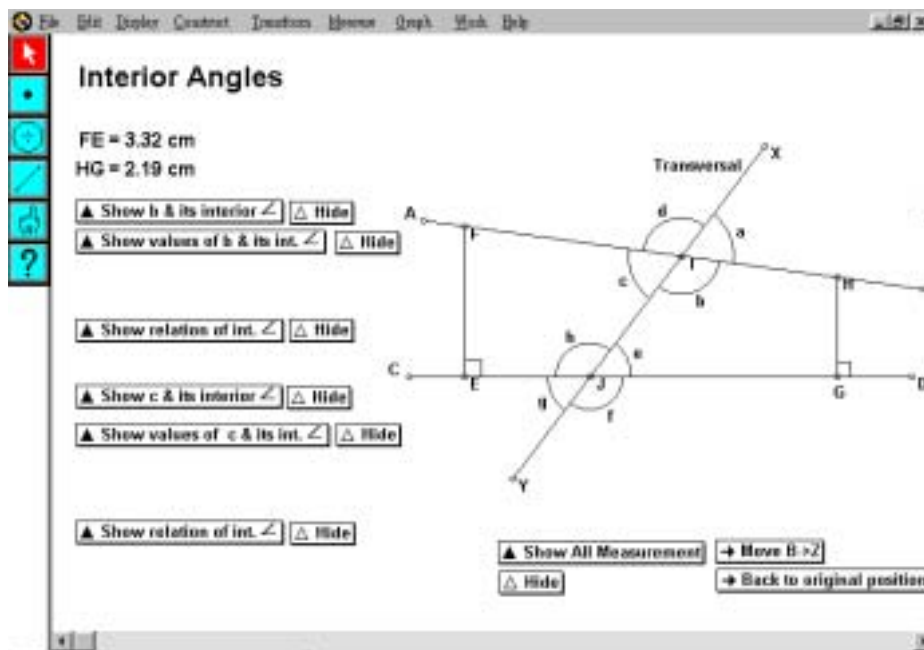


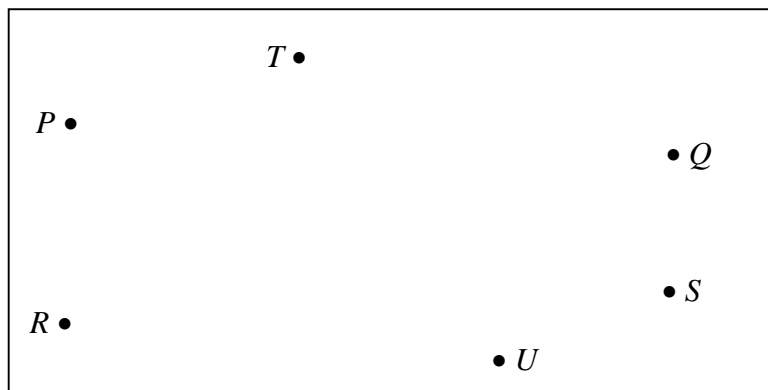
Fig. 7

9. Students are invited to conclude their findings that:
- No matter how the inclination of the lines and the transversals,
- corresponding angles of parallel lines are the same;
 - alternate angles of parallel lines are the same;
 - interior angles of parallel lines are supplementary.

Worksheet 1: Angles Associated with Parallel Lines and Transversal

Identifying corresponding angles, alternate angles and interior angles

- (a) Draw 3 lines by joining P, Q ; R, S ; and T, U in the following box. Use p, q, r, \dots to label all the angles formed by these 3 lines.

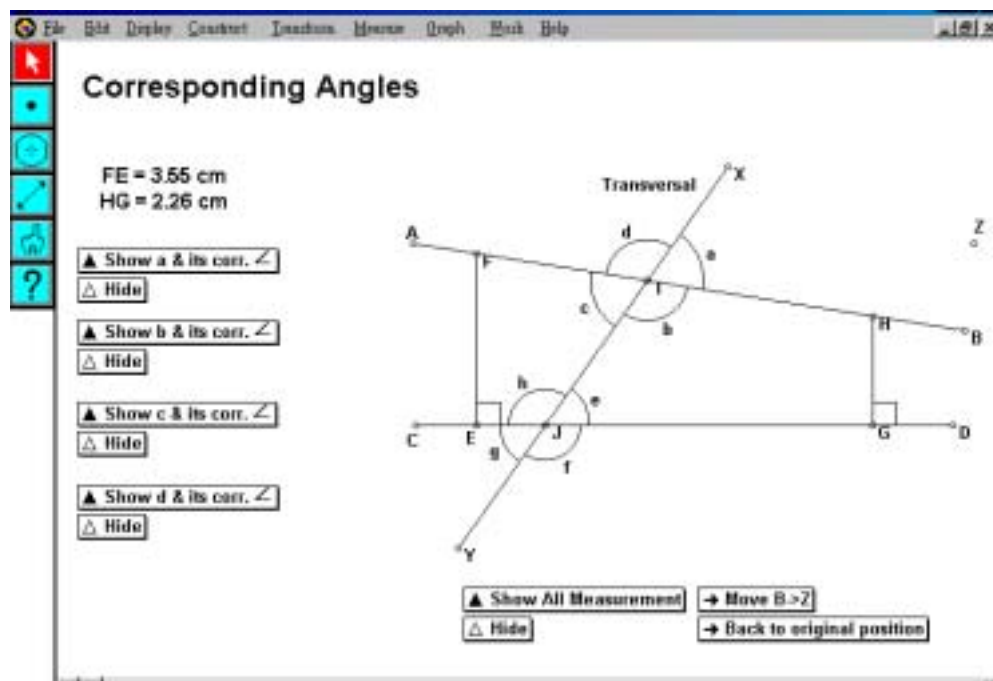


- (b) Which line is the transversal?
 _____ is the transversal.
- (c) Write down all pairs of corresponding angles in the figure.
 _____ & _____ ,
 _____ & _____ ,
 _____ & _____ ,
 _____ & _____ are 4 pairs of corresponding angles.
- (d) Write down all pairs of alternate angles in the figure.
 _____ & _____ ,
 _____ & _____ are 2 pairs of alternate angles.
- (e) Write down all pairs of interior angles in the figure.
 _____ & _____ ,
 _____ & _____ are 2 pairs of interior angles.

Worksheet 2: Corresponding Angles Associated with Parallel Lines and Transversal

Exploration on the relation of corresponding angles

Refer to the following figure of the given dynamic geometry file, Corresp.gsp.



- Are the lines AB and CD parallel? Why?

- Double click the button “▲ Show a & its corr. ∠”. Drag the point B to rotate the line AB in various directions. In the process of rotation, pay attention to the values of this pair of corresponding angles. Finally, drag the point B to Z until the line AB has equal width with the line CD .
- Then copy the FE and HG from the screen.
 - $FE = HG =$ _____ cm
 - Are the lines AB and CD parallel now? Why?

4. Show the measurement of all pairs of corresponding angles by double clicking the button “▲Show All Measurement” on the screen.
5. Copy the values of all pairs of corresponding angles at this instant.
 $a = \underline{\hspace{1cm}}, e = \underline{\hspace{1cm}};$ $b = \underline{\hspace{1cm}}, f = \underline{\hspace{1cm}};$
 $c = \underline{\hspace{1cm}}, g = \underline{\hspace{1cm}};$ $d = \underline{\hspace{1cm}}, h = \underline{\hspace{1cm}}.$

Can you discover any relation about corresponding angles?

6. Move the transversal by dragging the point X to any two positions so that 2 sets of angles are obtained. Write down the values of all pairs of corresponding angles:

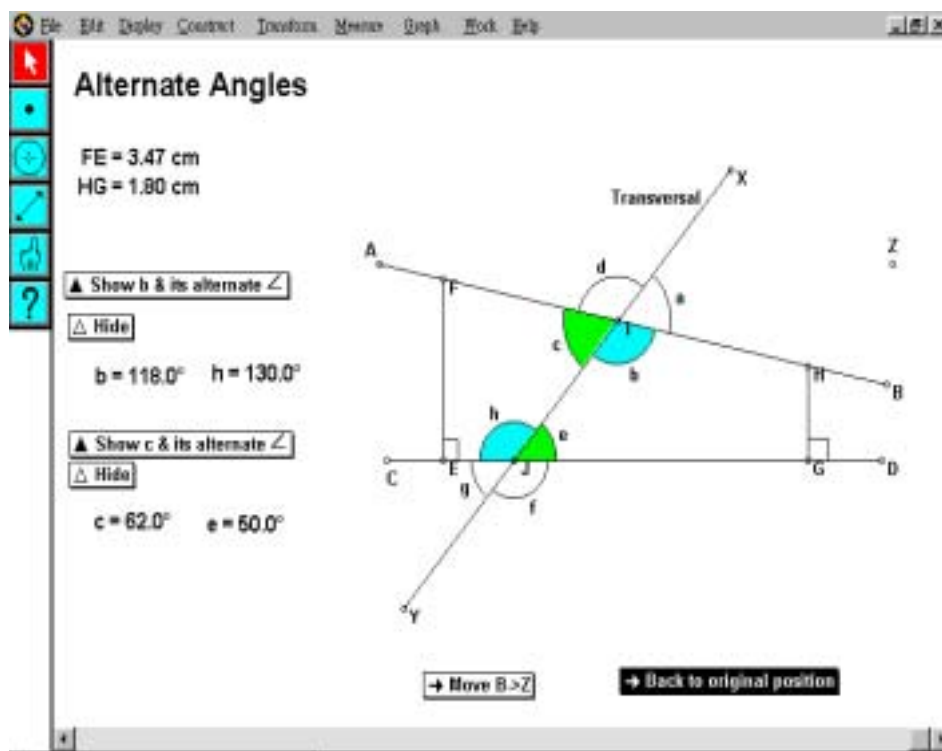
1 st set of data	2 nd set of data
$a = \underline{\hspace{1cm}}, e = \underline{\hspace{1cm}}$	$a = \underline{\hspace{1cm}}, e = \underline{\hspace{1cm}}$
$b = \underline{\hspace{1cm}}, f = \underline{\hspace{1cm}}$	$b = \underline{\hspace{1cm}}, f = \underline{\hspace{1cm}}$
$c = \underline{\hspace{1cm}}, g = \underline{\hspace{1cm}}$	$c = \underline{\hspace{1cm}}, g = \underline{\hspace{1cm}}$
$d = \underline{\hspace{1cm}}, h = \underline{\hspace{1cm}}$	$d = \underline{\hspace{1cm}}, h = \underline{\hspace{1cm}}$

Write down any conjecture(s), including the necessary condition(s) observed.

Worksheet 3: Angles Associated with Parallel Lines and Transversal

Part I: Exploration on the relation of alternate angles

Refer to the following figure of the dynamic geometry software file, Alterna.gsp.



1. Double click the button “▲ Show b & its alternate ∠”.
2. Drag the point B to rotate the line AB in various directions. In the process of rotation, pay attention to the values of all pairs of alternate angles. Finally, drag the point B to Z until the line AB is parallel to the line CD.
3. Show the measurement of all pairs of alternate angles. Copy the values of all pairs of alternate angles at this instant.

$b = \underline{\hspace{2cm}}$, its alternate angle = $\underline{\hspace{2cm}}$ = $\underline{\hspace{2cm}}$.

$c = \underline{\hspace{2cm}}$, its alternate angle = $\underline{\hspace{2cm}}$ = $\underline{\hspace{2cm}}$.

Do you discover any relation about alternate angles?

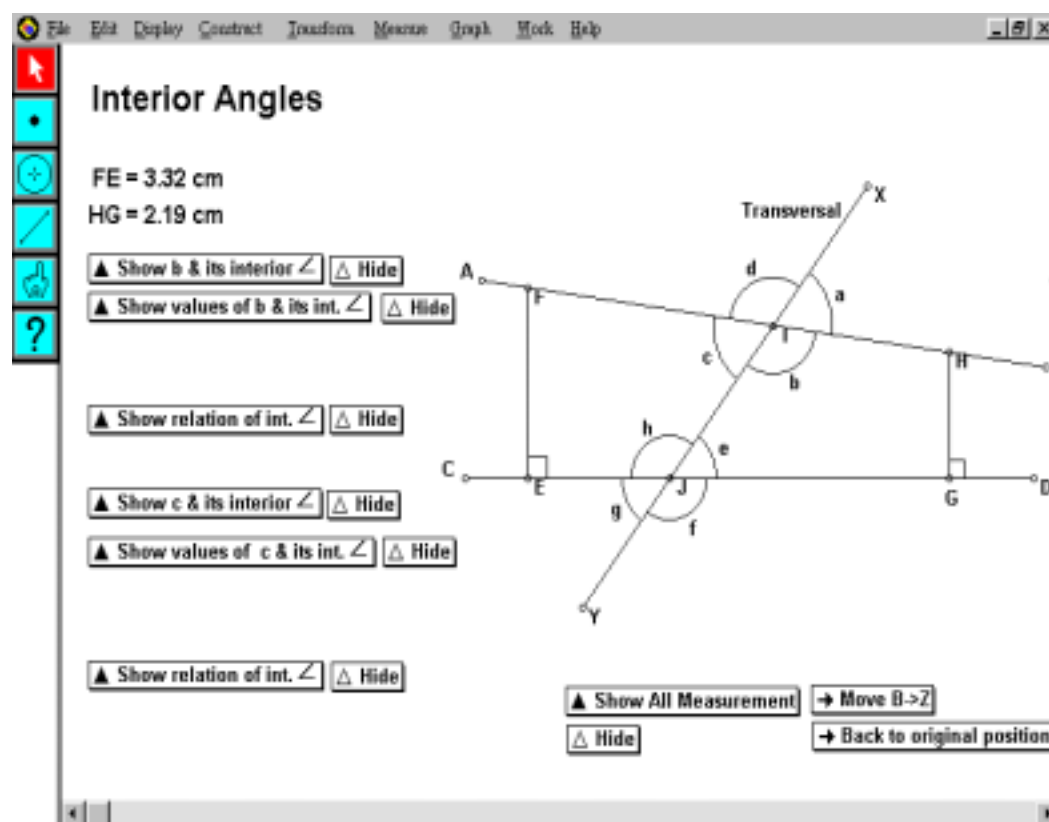
4. Move the transversal by dragging the point X so that 3 sets of angles are obtained. Write down the values of all pairs of alternate angles:

	b	Its alternate \sphericalangle	c	Its alternate \sphericalangle
1 st set of angles				
2 nd set of angles				
3 rd set of angles				

Write down any conjecture(s), including the necessary condition(s), observed.

Part II: Exploration on the Relation of Interior Angles

Refer to the following figure of the dynamic geometry software file, Interior.gsp.



1. Double click the button “▲ Show b & its interior \angle ”.
2. Drag the point B to rotate the line AB in various directions. In the process of rotation, pay attention to the values of all pairs of interior angles. Finally, drag the point B to Z until the line AB is parallel to the line CD .
3. Show the measurement of all pairs of interior angles. Copy the values of all pairs of interior angles at this instant. Do you discover any relation about interior angles?

$b = \underline{\hspace{2cm}}$, its interior angles , $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

$c = \underline{\hspace{2cm}}$, its interior angle , $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

4. Move the transversal by dragging the point X so that 3 sets of angles are obtained.
Write down the values of all pairs of alternate angles:

	b	Its interior \sphericalangle	c	Its interior \sphericalangle
1 st set of angles				
2 nd set of angles				
3 rd set of angles				

5. Write down any conjecture(s), including the necessary condition(s), observed.

To view the answer, double click the button "**▲Show relation of interior \sphericalangle** " on the diagram.

6. Summarize the findings of the relations of corresponding angles, alternate angles, or interior angles.

Notes for Teachers:

1. These activities can be used for students' self-exploration on the relationship of all types of angles associated with parallel lines. Nevertheless, students should be given appropriate guidance on controlling the computers and sufficient time for making the conjectures on the relations of angles should be allowed.
2. For point 1 of the Activity, some students may use "2 line segments will not intersect" as the definition of parallel lines. The teacher can guide students to see the meaning of "will not intersect" should be "will not intersect when prolonged the line segments infinitely". This meaning can be further elaborated as "having same distance apart". For the less able students, Worksheet 1 could be distributed for consolidating their understanding of angles associated with parallel lines.
3. For the buttons, the teacher needs to explain briefly the meaning of the short names, "corr." and "int." in diagrams. "**▲Move B → Z**" button is inserted in the files to help students who find it difficult to drag the point B to the point Z . When this button is double clicked, the point B will move to Z gradually and automatically. Hence, students can focus their attention on the changes of the values of angles.
4. For points 5 to 7 of the Activity, students should observe that the relations are not only true for the fixed parallel lines and transversal but also true on various pairs of parallel lines and transversals. Therefore, dragging the points C or X to change the directions of parallel lines and transversal is a very important step in the activities. For more able students, the teacher can also skip this guided discovery steps as mentioned in points 5 to 7. Instead, students can work on Worksheets 2 and 3 for their own exploration.
5. Due to the limitation of the software, some minor errors of measurements are found. For example, when EF is not exactly the same as HG , the pairs of angles are equal as well. It is important to alert students that the activities only provide intuitive experience to visualize the conjecture and remind them the limitation of this approach. The teacher may then guide students to see the importance of deductive approach on the relations of angles in parallel lines. These would be discussed later in the learning unit "Simple Introduction to Deductive Geometry".