EXEMPLAR 14:

Exterior Angles of Polygons

Objective: To explore the angle sum of the exterior angles of polygons

Key Stage: 3

Learning Unit: Angle related with Lines and Rectilinear Figures

Materials Required: Internet

Prerequisite Knowledge: The meaning of "exterior angle of polygon"

Description of the Activity:

1. The teacher distributes Worksheet 1 to students. Students need to go to the web site http://www.ies.co.jp/math/java/geo/logo/logo.html. The teacher briefly explains the web page and the problems in the worksheet (see figure below).
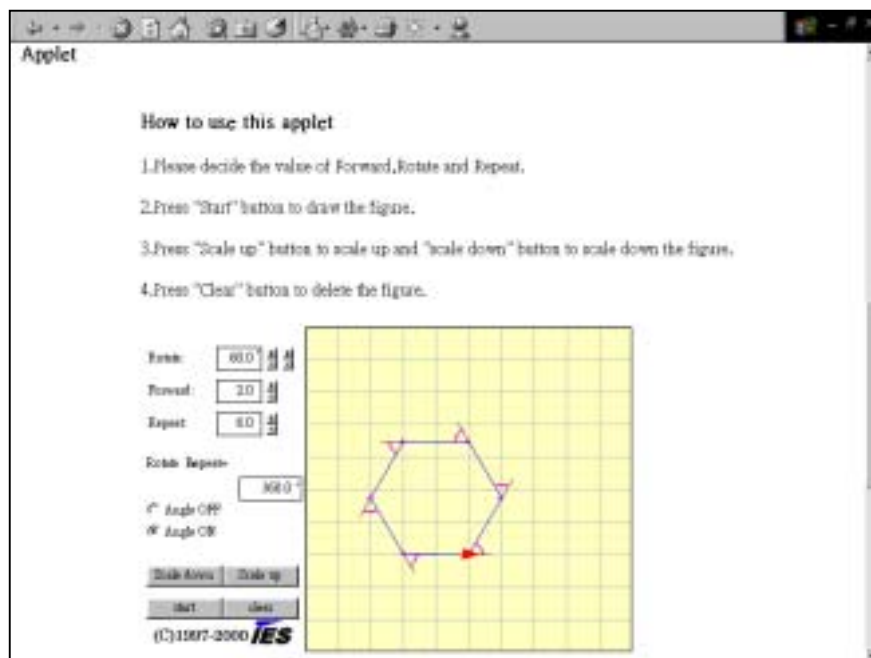
2. (a) Students have to create eight regular polygons by using the *Java Applet*. Those regular polygons are equilateral triangle, square, regular pentagon, regular hexagon, regular octagon, regular nonagon, regular decagon. They have to guess the angle rotated in creating a desired regular polygon.

   (b) Students have to record the angle rotated -"Rotate", number of repetition - "Repeat" and the total angle rotated - "Rotate Repeat" in Table 1 in Worksheet 1.

   (c) Students then use the records in Table 1 of Worksheet 1 to complete Table 2 of the same worksheet.

   (d) Students need to make a conjecture on the sum of exterior angles of a regular polygon and generalize the conjecture to any polygon.

3. The teacher invites some students to present their findings and conjectures to the whole class. There is no need to give proofs at this moment. The teacher can then use Worksheet 2 to give a "visual proof" to students.

4. The teacher distributes Worksheet 2 to students.

5. Students have to visit the web site
   http://www.ies.co.jp/math/java/geo/gaikaku/gaikaku.html to find the sum of
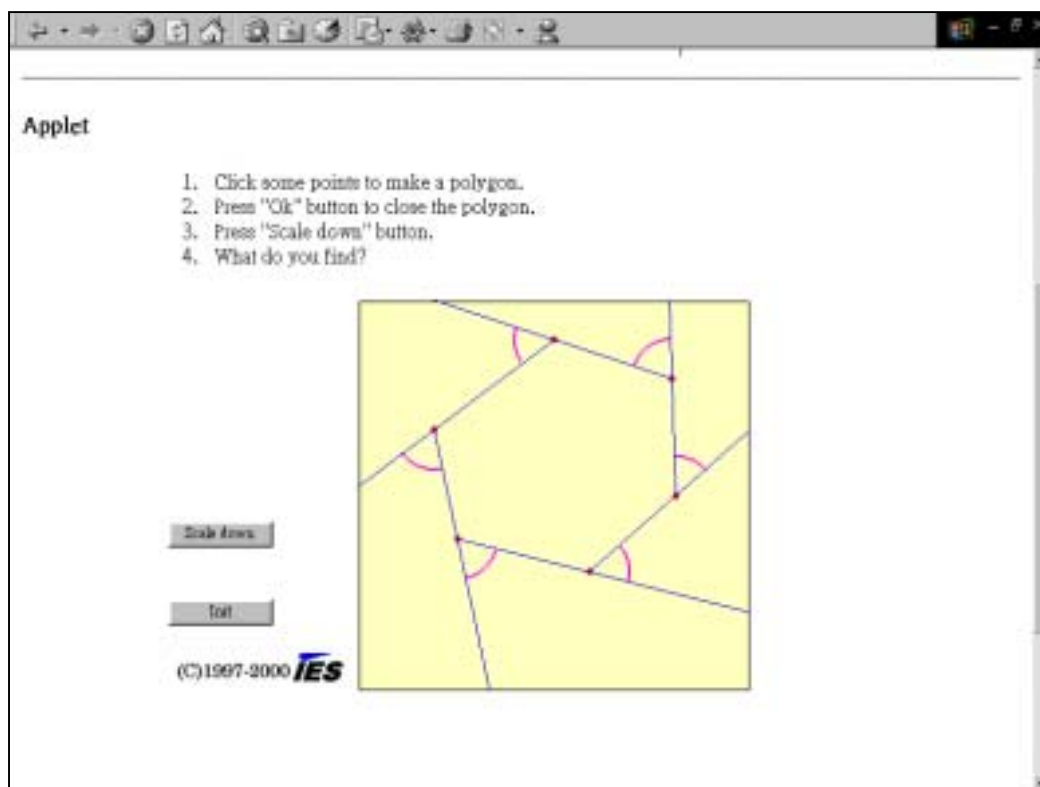
exterior angles of a polygon (see figure).

6. Firstly, they have to create their own polygons. Secondly, they have to press the "Scale down" button several times to make all the exterior angles come together to meet at a point and make up 360°. Finally they have to write down their findings in Worksheet 2.

7. The teacher may ask some students to explain this "visual proof" to the whole class. The teacher then summarizes the findings in Worksheets 1 and 2 as the sum of exterior angles of any convex polygons must be equal to 360°.

Worksheet 1: Polygon Creator

1. Go to the web site http://www.ies.co.jp/math/java/geo/logo/logo.html.

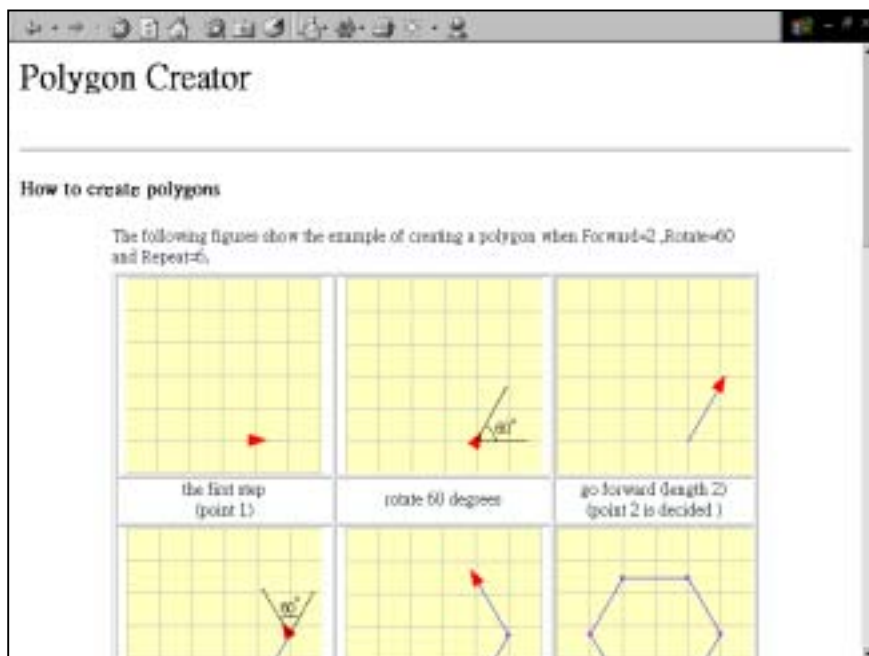2. You can find a *Java Applet* that can help you to create a polygon (see Fig.1).

Fig. 1

3. The web page shows an example of creating a regular hexagon. Read the instruction carefully.

4. In order to construct a regular polygon, you should input the angle rotated in the "Rotate" field, the length of a side in the "Forward" field and the number of sides in the "Repeat" field. Remember to choose "Angle ON" to view the angle rotated. Press the "start" button to do the construction (see Fig.2).
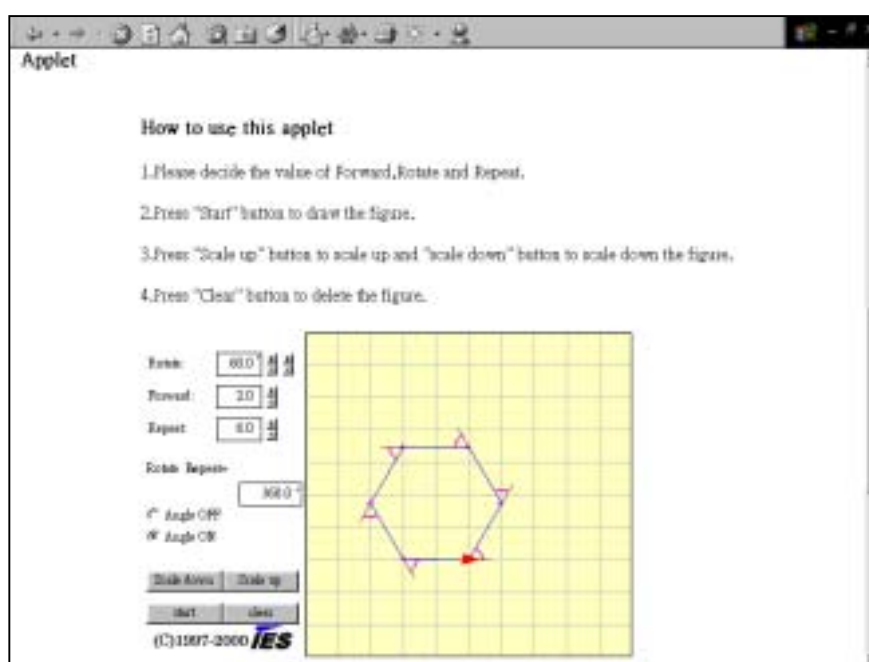

Fig. 2

5. Use the *Java Applet* to create the regular polygons shown in Table 1. Also record the corresponding data in Table 1.

Regular Polygon	"Rotate"	"Repeat"	"Rotate Repeat"
Equilateral triangle			
Square			
Regular Pentagon			
Regular Hexagon			
Regular Octagon			
Regular Nonagon			
Regular Decagon			

Table 1

6. Use Table 1 to complete Table 2.

| Regular Polygon	Size of an exterior angle	Number of exterior angles	Sum of exterior angles
Equilateral triangle			
Square			
Regular Pentagon			
Regular Hexagon			
Regular Octagon			
Regular Nonagon			
Regular Decagon			

Table 2

7. What is the relation between the size of exterior angles and the number of sides of the regular polygons?

    _____

    _____

8. What is the sum of exterior angles of a regular polygon? Write down your conjecture below.

    _____

    _____

9. Does the conjecture in question 8 hold for **any convex polygons**?

    _____

    _____

Worksheet 2: Sum of Exterior Angles of Polygon

1. Go to the web site http://www.ies.co.jp/math/java/geo/gaikaku/gaikaku.html.

2. You can find a *Java Applet* that shows the sum of angles of a polygon (see the figure below).
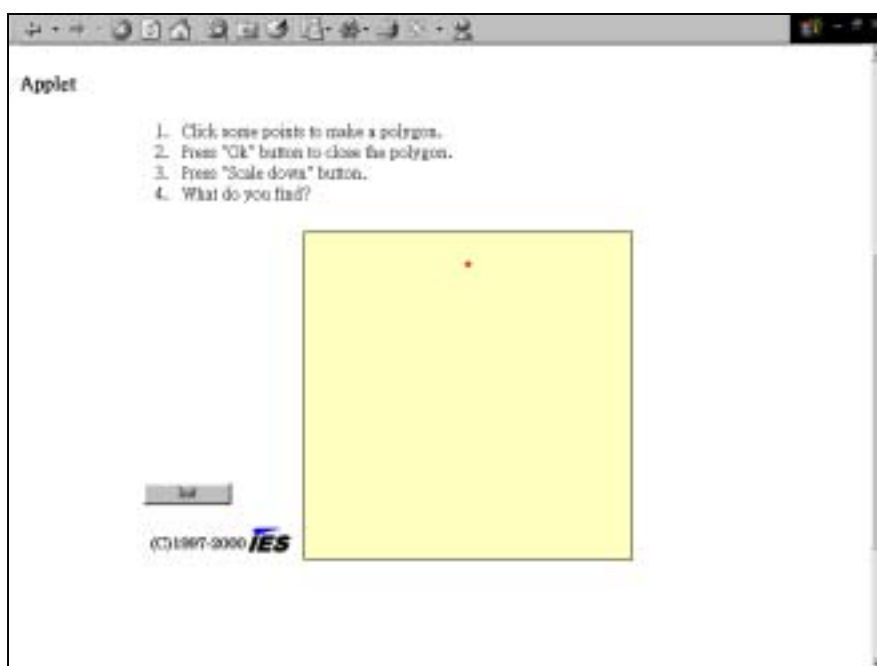
3. Follow the instruction to make a polygon. Press "Scale down" button several times. Observe the change. Press the "Init" button. Try another polygon. What is the sum of exterior angles of a polygon?

    _____

    _____

Notes for Teachers:

1. The objective of Worksheet 1 is to let students explore the way to construct a regular polygon. They have to guess the angle rotated (i.e. the size of an exterior angle of a polygon) in order to have a successful construction. The method of trial and error is encouraged to use in this exploration. After the exploration, students will know the size of an exterior angle of a regular polygon and the sum of the exterior angles of a regular polygon. Moreover, they should be able to derive a formula to calculate the size of an exterior angle of a regular polygon as dividing 360° by the number of sides of a regular polygon.

2. The teacher may go through the *Java Applet* "Polygon Creator" stated in Worksheet 1 with students to construct a regular hexagon. Then students may be asked to construct other regular polygons independently.

3. The *Java Applet* in Worksheet 2 can serve as a "visual proof" of the theorem "Sum of exterior angles of polygon". The proof is expected to be performed in the learning unit "Simple Introduction to Deductive Geometry".

4. Regular heptagon is not considered in Worksheet 2 because the size of its exterior angle is not an integral value. The teacher may tell this point to students after the discussion on Worksheet 1.

5. Some students may ask the problems of the rule for non-convex polygons. It should be noted that there is no similar definition of exterior angles for these re-entrant polygons. The rule, thus, does not hold in these polygons.