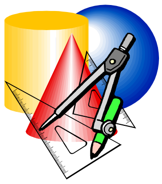


Exemplar 15:

Tessellation in a Plane

Objectives: (1) To understand the idea of tessellation in a plane
(2) To explore polygons that tessellate
(3) To identify what type of regular polygons can tessellate and how polygons can be used together to tessellate

Key Stage: 3

Learning Unit: Angles related with Lines and Rectilinear Figures

Materials Required: Small polygons made from cardboard, sheets of paper, an overhead projector, transparencies, markers, computer display

Prerequisite Knowledge: Interior angles of regular polygons and simple ideas on transformations.

Description of the Activity:

1. The teacher demonstrates to students the following web sites for the exploration of various tessellated figures and introduces the idea of tessellation:
   http://forum.swarthmore.edu/alejandre/students.tess.html
   (Good tessellated figures made by students in other countries can be found in this web page.)
   http://www18.big.or.jp/~mnaka/home.index.html
   (Interesting tessellated figures with animation effects can be found.)

2. Students are grouped in two or three. Each group is given a sheet of paper with one or more pre-drawn polygons including different types of triangles, quadrilaterals and regular polygons. They are asked to tessellate the plane by using only one type of polygons each time and complete the Worksheet.

3. Completed tessellations are put on the blackboard so that students can discuss and share their work with their classmates.

4. Discussion is conducted on what shapes can tessellate a plane. The following discussion can be done with the use of an overhead projector:
   (a) Try various types of triangles to tessellate a plane on the transparency and discuss whether the conjecture "All triangles can tessellate a plane" is correct.
   (b) Try with various types of quadrilaterals (some may not be convex). Discussion can be made on whether the conjecture "all quadrilaterals can tessellate a plane" is correct.
   (c) Try with other types of polygons and challenge students whether the conjecture "All polygons can tessellate a plane" is correct.
   (d) Try with various types of <u>regular polygons</u> and discuss the conjecture that "Only regular polygons with interior angles 60°, 90° and 120° can tessellate a plane".

5. Through demonstrating students' work on the blackboard or on transparencies with the help of an overhead projector, discussion with students on the conjectures can be carried out.

6. Students are guided to come to the conclusion that all triangles and all quadrilaterals can tessellate because their sums of interior angles are 180° and 360° respectively.

7. For regular polygons, the teacher should help students to get the result that only equilateral triangles, squares, and regular hexagons can tessellate. They should arrive at the conclusion that only regular polygons with interior angles 60°, 90° or 120° can tessellate a plane.

Worksheet: Polygons Tessellating the Plane

1. Try with the following given polygons to tessellate the plane. Fill in Table 1.

Data Table for Polygons		
Polygon	Number of Sides (n)	Tessellate a Plane? (Yes/No)
Equilateral Triangle		
Isosceles Triangle		
Scalene Triangle		
Parallelogram		
Rhombus		
Kite		
Square		
A general quadrilateral with no special characteristics		

Table 1

2. Try with the following regular polygons to tessellate the plane. Fill in Table 2.

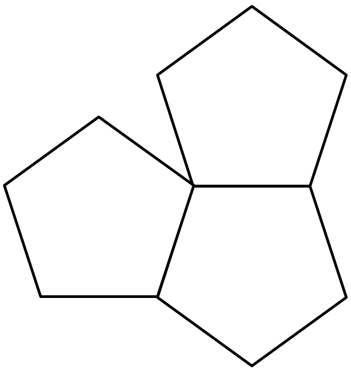
Data Table for Regular Polygons			
Regular Polygon	Number of Sides (n)	Each Interior Angle (180°× (n-2) ÷ n)	Tessellate a Plane? (Yes/No)
Equilateral Triangle			
Square			
Regular Pentagon			
Regular Hexagon			
Regular Heptagon			
Regular Octagon			

Table 2

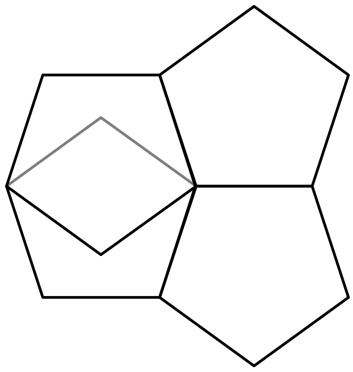
Notes for Teachers:

1. Tessellation comes from the Latin word *tessella*, which was the small, square stone or tile used in ancient Roman mosaics. *Tiling* and *mosaics* are common synonyms for tessellation. A plane tessellation is a pattern made up of one or more shapes, completely covering a surface without any gaps or overlaps. 2-D shapes and 3-D shapes will tessellate. A plane can be tessellated with one or more shapes. The teacher may get the pre-drawn polygons from Appendix B.

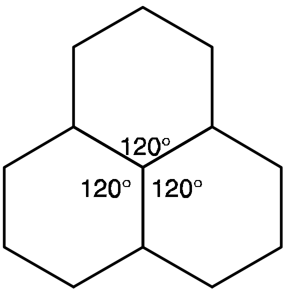
2. Students should have come across examples of tessellation at the primary levels. At the secondary levels, students are encouraged to look deeper into the criteria that make polygons tessellate, especially from the aspect of interior angles of a regular polygon. The general problem of tessellation by regular polygons was first solved by Rev Jones in 1785.



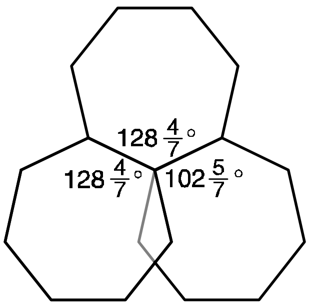
Three regular pentagons



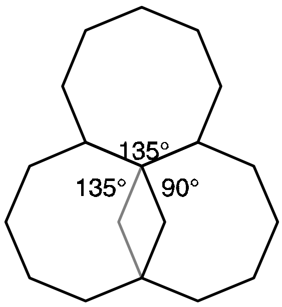
Four regular pentagons



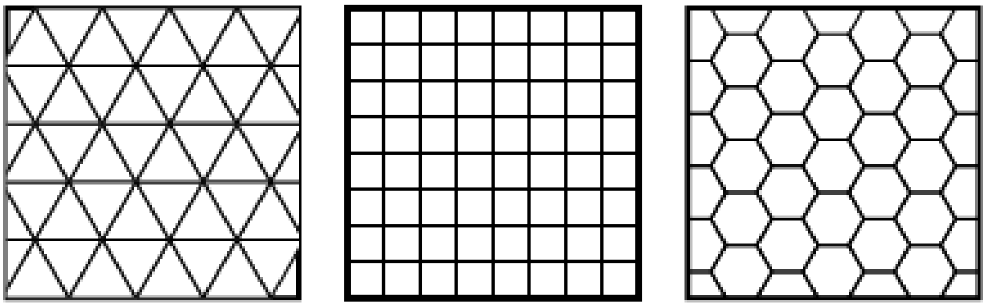
Three regular hexagons



Three regular heptagons



Three regular octagons



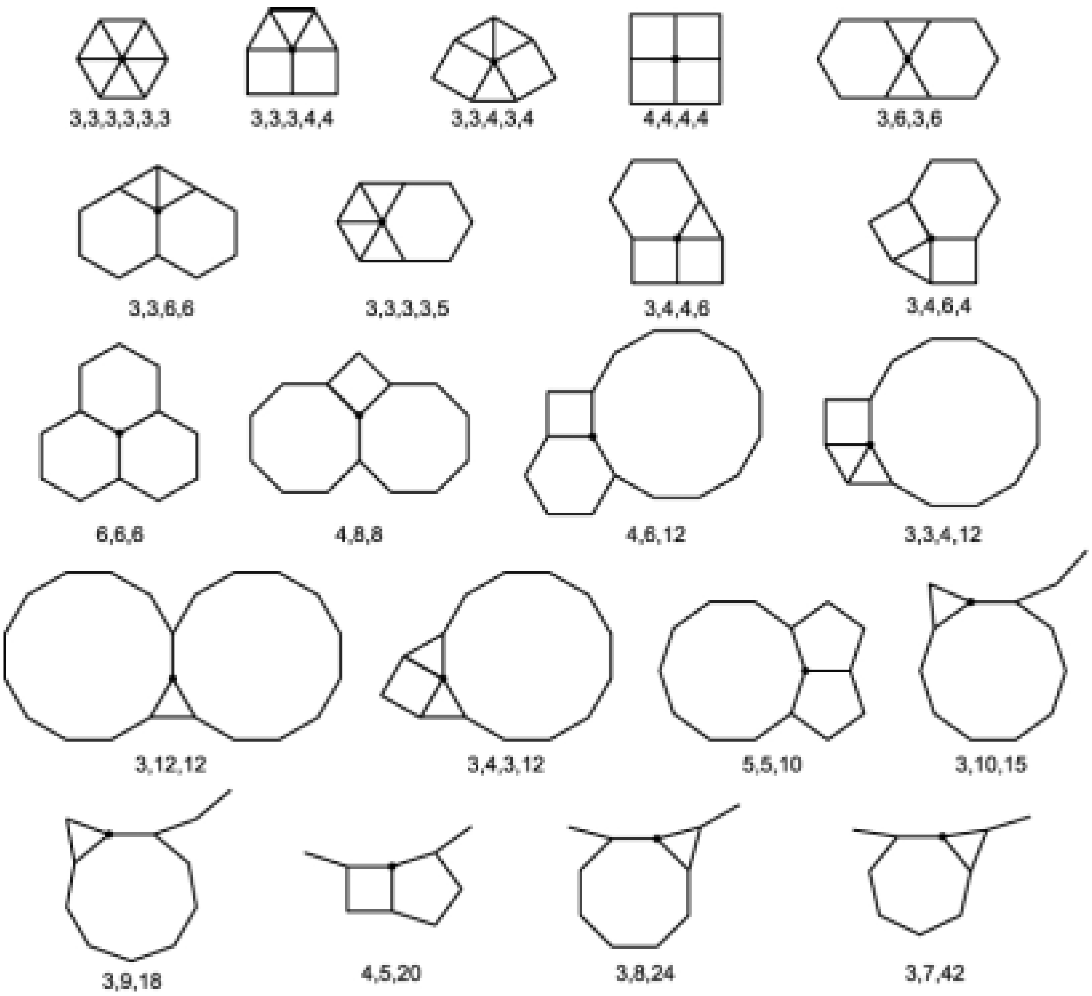
Three regular tessellations

3. The tessellated pictures can be easily organized by scanning photographs into computer and use the software like *PowerPoint* for presentation. During the presentation, students may revise their knowledge of tessellation by answering questions on how the tessellation is done.

4. For the activity to tessellate with polygons, the teacher can ask students to use some computer software packages such the program *Shape* in the CD-ROM **Fun with Learning 98** to explore the tessellation with these shapes.

5. For more able students, the teacher can arrange students in groups to tessellate with more than one type of regular polygons. They are asked to draw as many combinations as possible. The following criteria should be discussed with students:
   (a) There should not be more than six polygons (six angles of equilateral triangles are already 360 degrees);
   (b) There cannot be less than three polygons (angle of a regular polygon is less than 180 degrees);
   (c) There cannot be more than three types of polygons (a triangle, a square and a regular pentagon have an angle sum of 258 degrees);
   (d) If there are four polygons used, then two must be of the same type;
   (e) If there are five polygons, then the combinations should either be 2+2+1 or 3+1+1.

   Teachers can introduce the system x, y, z to represent tessellation with regular polygons of x, y, and z sides. For example, 3, 3, 4, 12 represent two triangles, a square together with a dodecagon in tessellation. Students should distinguish 3, 3, 4, 12 from 3, 4, 3, 12 as two different types of tessellation. Teachers can challenge students to find out all the possible types of tessellation with regular polygons. This can be a project and it is not difficult for students to present the solution as there are only 21 types of tessellation by regular polygons.



Reference Materials:

Books:

1. Britton, Jill and Britton, Walter. (1992). *Teaching tessellating art: activities and transparency masters.* Palo Alto, California: Dale Seymour Publication.
2. Burn, Bob. (1987). *The Design of Tessellations*. Cambridge: Cambridge University Press.
3. Seymour, Dale and Britton, Jill. (1989). *Introduction to tessellations*. Palo Alto, California: Dale Seymour Publication.
4. Seymour, Dale. (1989). *Tessellation teaching masters.* Palo Alto, California: Dale Seymour Publication.

CD-ROMs

1. The program *Shape* in the CD-ROM **Fun with Learning 98**.
2. *TesselMania*: A computer software to explore geometry with tessellating shapes etc.

Web Sites:

1. http://library.thinkquest.org/16661/templates/index.html
2. http://library.thinkquest.org/16661/escher.html