Exemplar 18:
Limitations of Using 2-D Figures to Represent 3-D Solids

Objectives: (1) To identify 3-D figures from their 2-D representations
(2) To recognize the limitations of the 2-D representations of 3-D solids

Key Stage: 3

Learning Unit: More about 3-D solids

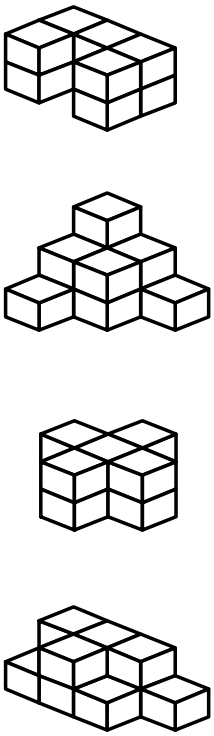
Materials Required: Pencil and paper, Unit cubes

Prerequisite Knowledge: Understanding the terms: face, edge, vertex and net of a solid.

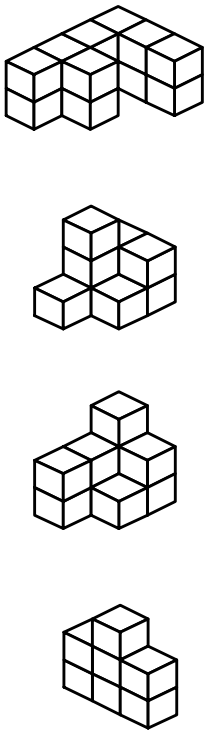
Description of the Activity:

1. Students are divided into groups of two. The teacher distributes each group a number of blocks and 2 sets of 2-D pictures as following (Fig. 1(a) and Fig 1(b)):

Set of Fig. 1a



Set of Fig. 1b



2. As a start, the teacher guides students to answer the following questions when looking at the pictures:
   (a) How many horizontal layers does the solid have?
   (b) How many blocks are in each layer? How are the blocks arranged?
   (c) Is each layer identical?
   (d) How many cubes are needed to construct the solids?

3. One student in each group is asked to describe the solids based on the representations of the solids in the set 1a using the above questions as hints. His/her group-mate needs to construct the solids. After then, other students in the group will describe the solids in the set 1b to help his/her group-mate to construct the solids.

4. Students of each group are asked to display their solids constructed. Their answers and strategies in constructing the solids should be discussed in the class. It is anticipated that different solids may be constructed for the same representation. Hence, students are then guided to find out that sometimes *only one* diagram of a solid is *unable* to give enough information to build the solid.

5. Students are recalled the ways to represent 2-D representations learnt in "Introduction to Geometry" Learning Unit. Besides, the teacher may introduce the other ways with the diagram on the left (Fig. 2a) to show the model on the right (Fig. 2b). The numbers in Fig. 2a indicate how *high the stack of cubes* shall be. The picture of the solid shown is drawn.

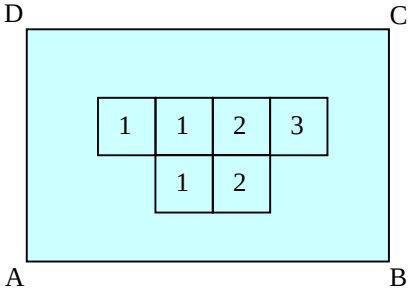


Fig. 2a

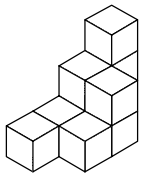


Fig. 2b

The teacher asks students to imagine the following solids and to describe them. (See Fig. 3a to Fig. 3d)

Fig.3a

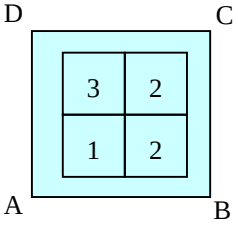


Fig.3b

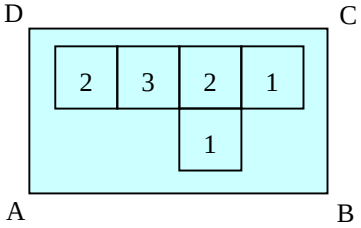


Fig.3c

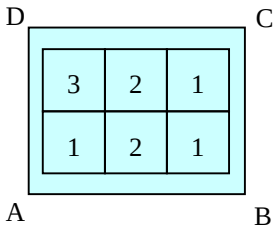
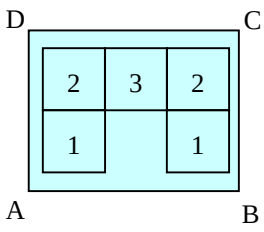


Fig. 3d



6. Discussion is made on the limitations of this representation. The teacher then briefly explains a more popular way of representation — providing different views of the solids such as views from the top, front, right-side/left-side (Fig. 4a and 4b). The teacher briefly explains the representation.

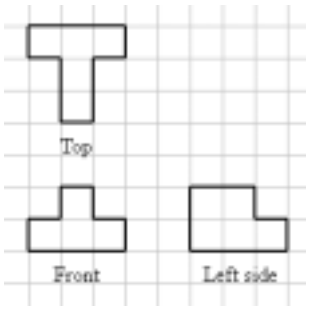


Fig. 4a

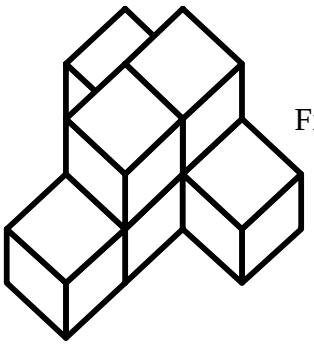


Fig. 4b

7. The teacher gives the following diagrams to students and asks them to describe or draw the 2-D representation of the solids in groups. They are encouraged to imagine the solids in the first instance and then construct the corresponding solids to check their mental models later on (Fig. 5a to Fig. 5c). Students should finish the drawing on the worksheet.

Fig.5a

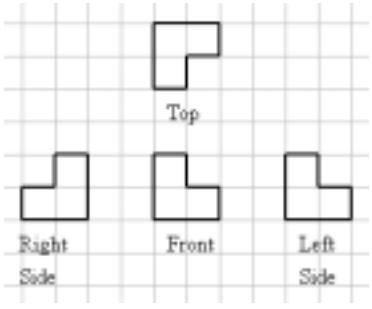


Fig. 5b

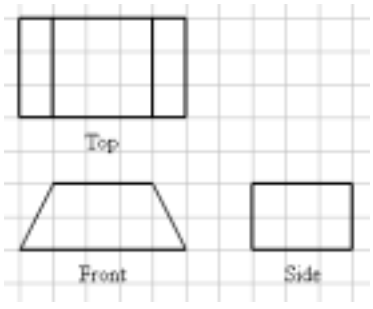
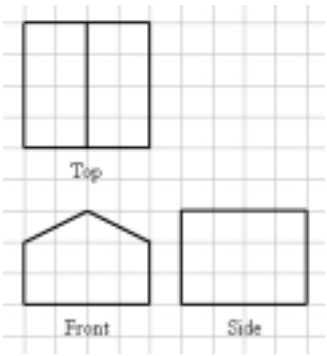


Fig. 5c

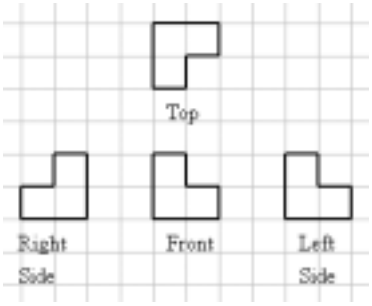


8. The teacher then discusses with students the solids of each diagram. The teacher summarizes the problems of representing solids in 2-D diagrams and the importance of a good 2-D representation.

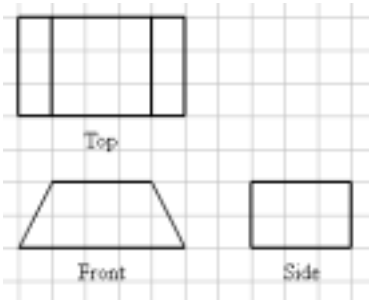
Worksheet:

According to the information provided below, construct the solids and draw a better 2-D representation reflecting the 3-D impression.

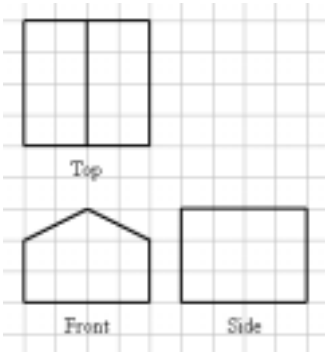
a)



b)



c)



Notes for Teachers:

1. It is expected that this activity can help students develop their spatial concept. All students are encouraged to make predictions first and then check their predictions by building models. Students are also encouraged to reflect on those mental models and refine the models. Students should not merely make 3-D models by trial and error. Their attention should be drawn on the visual imagination rather than on physical activity.

2. The teachers can encourage more able students to explore further by working with the following problem:

   When a 3-D solid is viewed from top, it looks like a circle. When it is viewed from the front, it looks like a square. When it is viewed from the side, it looks like a triangle. Do these three views determine the 3-D shape?

   The solution of the problem can be found in the web-site www.ies.co.jp/math/java/geo/solomon/solomon.html.