



Exemplar 22:

Properties of Various Types of Parallelograms

Objective: To explore the properties of various types of special parallelograms such as rhombuses, rectangles and squares.

Key Stage: 3

Learning Unit: Quadrilaterals

Materials Required: Dynamic geometry software such as *Geometer's Sketchpad* (later referred as *Sketchpad*) and the file *Parllegm.gsp*

- Prerequisite Knowledge:**
- (1) Basic knowledge on the definitions of parallelogram and special quadrilaterals.
 - (2) The meaning of parallel lines and angles.
 - (3) The concept of congruent triangles.

Description of the Activity:

1. The teacher asks students to recall the definitions of parallelogram, rhombus, rectangle and square.
2. The following *Sketchpad* file, *Parllegm.gsp*, is demonstrated (Fig.1). The

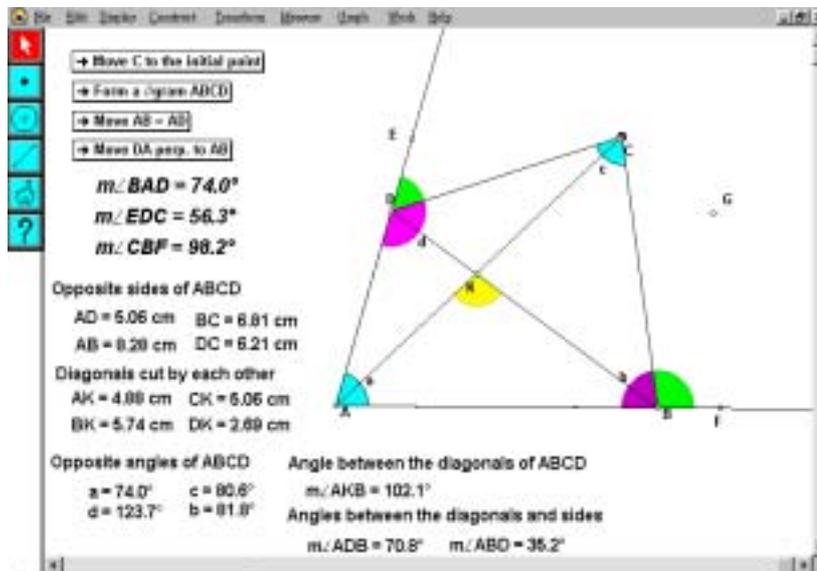


Fig. 1

teacher briefly explains the different measures shown in the diagram.

3. The teacher drags the point C so that the quadrilateral $ABCD$ looks like a parallelogram (Fig. 2). Students are then asked to observe changes in different measures. Which measures are useful for checking whether $ABCD$ is a parallelogram are discussed. The teacher then guides students to conclude that from the definition of parallelogram, the measures of $\angle BAD$, $\angle EDC$ and $\angle CBF$ are sufficient to check the conclusion.

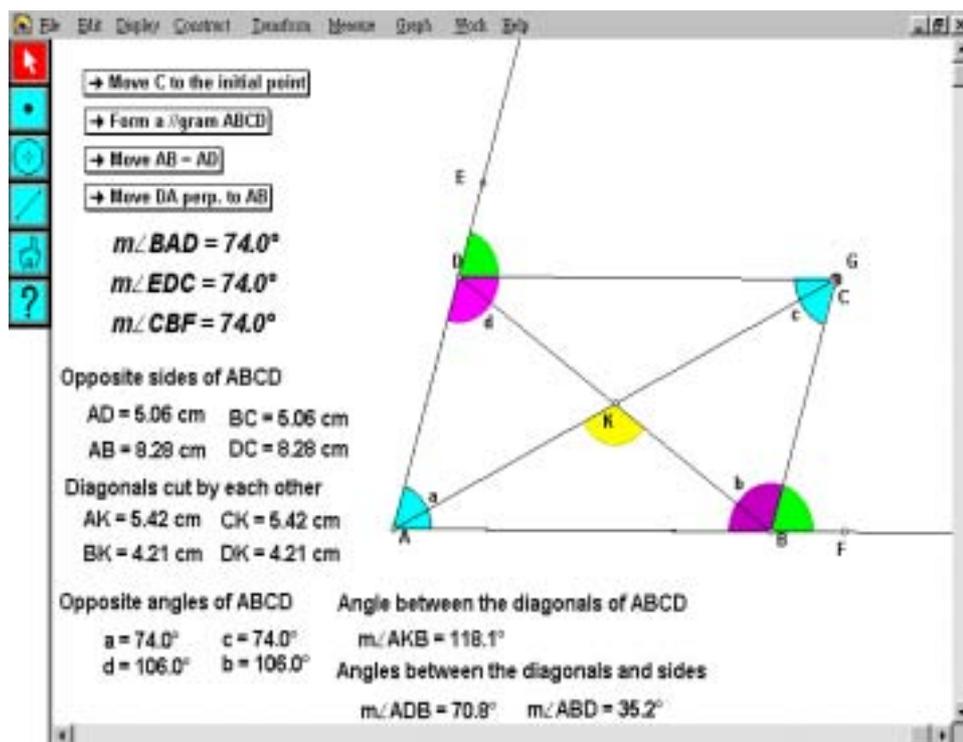


Fig. 2

4. The teacher discusses with students the changes of the following measures when C is dragged to make $ABCD$ to become a parallelogram:
 - (a) Opposite sides of $ABCD$
 - (b) Diagonals
 - (c) Opposite angles
 - (d) Angles between the diagonals
 - (e) Angles between the diagonals and the sides

Students, with the guidance of teachers, should conclude that “The opposite sides, and the opposite angles are respectively equal and diagonals bisect each other when $ABCD$ is a parallelogram”.

5. The points A , B or D are dragged to form various shapes of parallelograms. Students are asked whether the above findings are still true. The teacher guides students to summarize that “For any parallelogram, the opposite sides and the

opposite angles are always equal and the diagonals always bisect each other.”

6. The teacher distributes the *Sketchpad* file, Parllegm.gsp, and the Worksheet to students. Students are asked to use the file to form a rhombus using their methods and to explore the properties of some special types of parallelograms, such as rhombuses. After then, the teacher asks students to explain their methods of forming a rhombus and summarizes the minimal steps in forming a rhombus, that is in the order of pressing the buttons, "**→Move AB=AD**", "**→Form a //gram ABCD**" from the given *Sketchpad* file.
7. Students are asked to observe the special properties of the rhombus in addition to the properties of parallelogram (Fig. 3). They are requested to write their findings on the Worksheet provided and also to drag the points *A*, *B* or *D* to check whether the conjecture holds for any rhombus. The teacher then guides students to summarize that “For any rhombus, it has all properties of a parallelogram and also the diagonals are perpendicular to each other.”

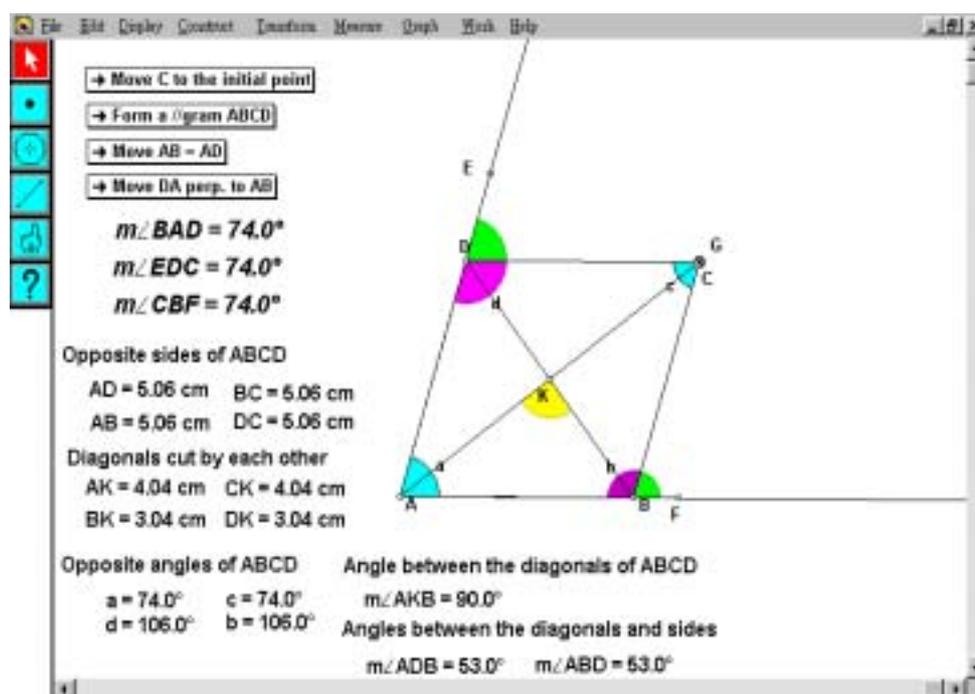


Fig. 3

8. Students explore the properties of other special parallelograms including squares (Fig. 4) and rectangles (Fig. 5) and to write their findings on the Worksheet. These findings include:
 - (a) The construction steps in the *Sketchpad* file, i.e., buttons in sequence.
 - (b) The changes of the measures.
 - (c) The common properties of parallelograms and the properties of special

parallelograms in addition to general parallelograms.

- (d) Students are requested to form different shapes of squares/rectangles by changing the positions of vertices A, B, C or D and countercheck whether the findings in (c) still hold.

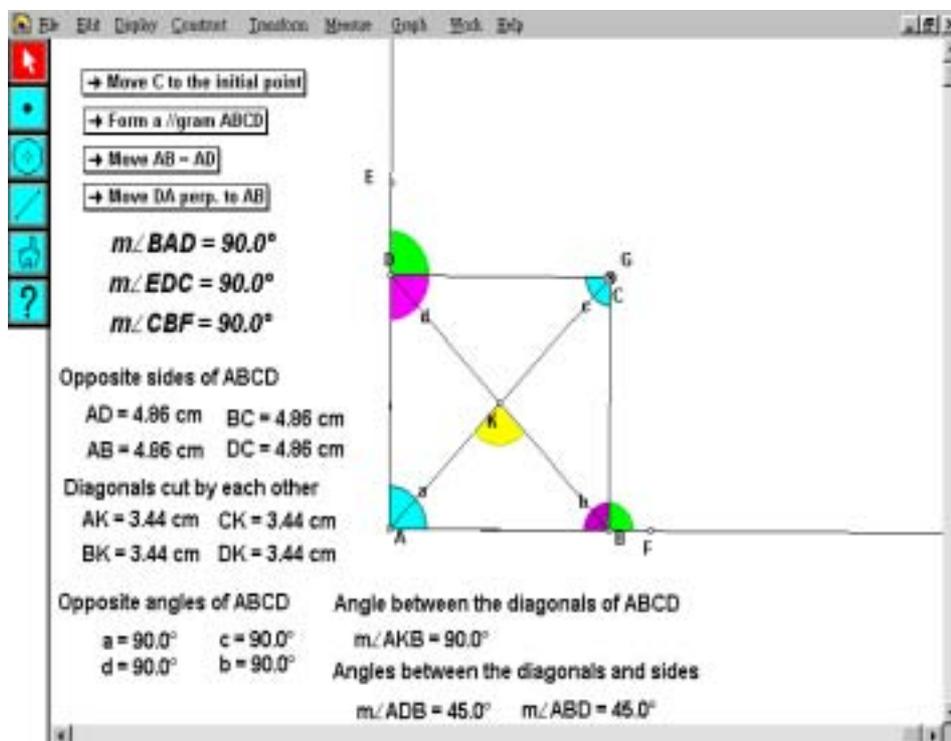


Fig. 4

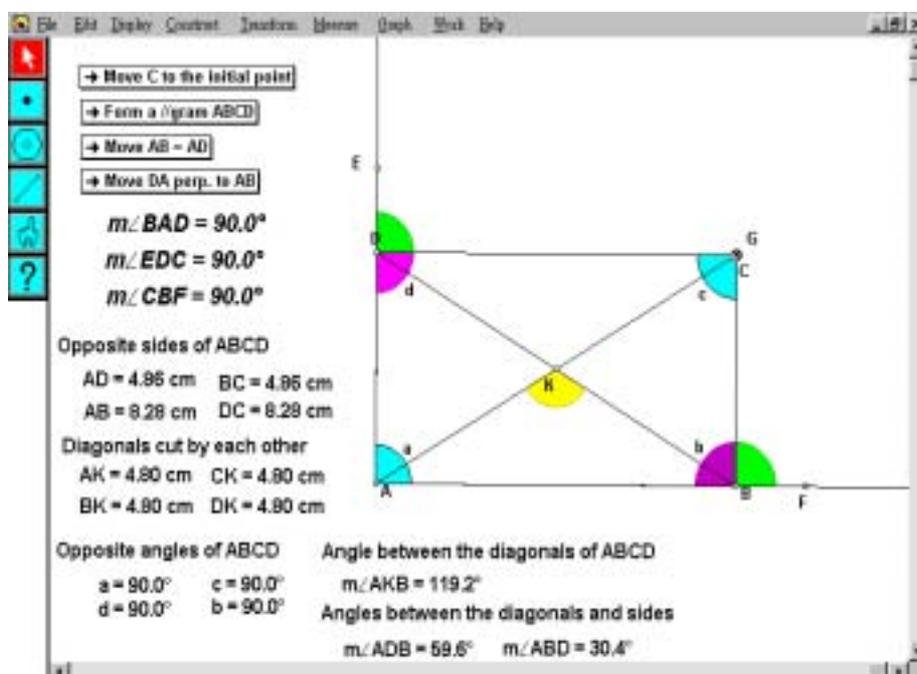


Fig. 5

9. Students are invited to present their findings. The teacher then guides students to summarize the properties of squares and rectangles.

Worksheet: Properties of Special Parallelograms

Put a tick in the appropriate boxes to match the following parallelograms with their corresponding properties.

Properties	Parallelogram	Rectangle	Square	Rhombus
1. Four right angles				
2. Opposite angles equal				
3. Consecutive angles supplementary				
4. Four sides equal				
5. Two distinct pairs of consecutive sides equal				
6. Opposite sides equal				
7. Opposite sides parallel				
8. One pair of sides parallel				
9. Diagonals equal				
10. Diagonals perpendicular				
11. Diagonals bisect each other				

Construction steps for rectangle: _____

Construction steps for square: _____

Construction steps for rhombus: _____

Notes for Teachers:

1. The teacher may construct the *Sketchpad* file prior to the lesson or use the provided application file *Parllegm.gsp*. But for classes who are familiar with the *Sketchpad* program, the teacher may ask students to construct their own application files. The icons "**→Form a //gram $ABCD$** ", "**→Move $AB=AD$** " and "**→Move DA perp. to AB** " are designed to help students who find it difficult to drag the mouse appropriately. For students who are more competent in using computers, the teacher may ask them to drag the point C rather than providing these icons.
2. This activity may also be used for students' self-exploration. Nevertheless, students should be given appropriate guidance in controlling the computers and sufficient time for finding the properties of special parallelograms.
3. The buttons "**→Move C to the initial point**", "**→Form a //gram $ABCD$** ", "**→Move $AB=AD$** " and "**→Move DA perp. to AB** " are Move Buttons which help students to move one or more points to specified destinations.

Here are the meanings of the 4 Move Buttons:

- "**→Move C to the initial point**": move the vertices A , B , C and D to their initial positions.
- "**→Form a //gram $ABCD$** ": move the vertex C such that $AB \parallel CD$ and $AD \parallel BC$.
- "**→Move $AB=AD$** ": move the vertex B such that $AB=AD$.
- "**→Move DA perp. to AB** ": move the vertex D such that $AD \perp AB$.

Moreover, it should be noted that the sequence in pressing buttons will affect the shapes formed. For example, if the "**→Form a //gram $ABCD$** " button is pressed prior to the button "**→Move $AB=AD$** ", the diagram will not be the same as that constructed in the reverse sequence. The teacher should remind students that this fault is mainly due to the limitation of the file. In other construction methods, there should be no difference in the images constructed.

4. For this Activity, students are asked to observe that the properties are not only true for a particular parallelogram but for all types of parallelograms. Therefore, dragging the points A , B or D is a very important step in the activities. For more able students, the teacher can also skip the guided discovery as mentioned and let them explore by their own.

5. Due to the limitation of the software, it is sometimes difficult to drag the point C to make the measures of $\angle BAD$, $\angle EDC$ and $\angle CBF$ equal. It is, thus, important to remind students that the activities only provide intuitive experience to visualize the conjecture and to guide them to recognize the limitation of this approach. The teacher can then guide students to see the importance of deductive approach in the study of the properties.