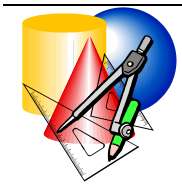


Exemplar 23:

Constructing Squares

Objective: To apply different properties of a square to construct squares using Information Technology

Key Stage: 3

Learning Unit: Quadrilaterals

Materials Required: Dynamic Geometry Software such as *Geometer's Sketchpad* (later referred as *Sketchpad*)

Prerequisite Knowledge:

(1) Properties of special quadrilaterals such as parallelograms, rectangles, squares, rhombuses, etc.
(2) Basic concepts of transformation such as translation, reflection and rotation.
(3) Basic construction skills in using dynamic geometry software.

Description of the Activity:

1. The teacher introduces the activity to students and revises some basic construction skills in using dynamic geometric software such as constructing parallel lines, perpendicular lines, a circle, etc.

2. The teacher discusses with students the two methods below to construct a square (Fig. 1 and Fig. 2).

Method 1:

(i) Construct a segment *AB* and mark its mid-point *M*.

(ii) Construct a circle with *M* as the centre and *AM* as the radius.

(iii) Construct a line *L* through *M* and perpendicular to AB.

(iv) Mark the intersections of *L* and the circle as *C* and *D*.

(v) Join the four points *A*, *D*, *B* and *C* on the circle to get a quadrilateral.

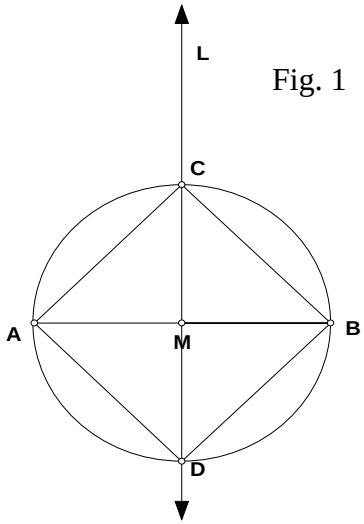


Fig. 1

Method 2:

(i) Construct a line segment *AB*.

(ii) Construct two perpendicular lines L_1 and L_2 through its end points *A* and *B*.

(iii) Mark a point *P* on L_1.

(iv) Construct an angle bisector of angle *PAB*. This angle bisector will intersect L_2 at point *C*.

(v) Construct a line through *C* and perpendicular to L_2. This line intersects L_1 at point *D*. A square *ABCD* is then obtained.

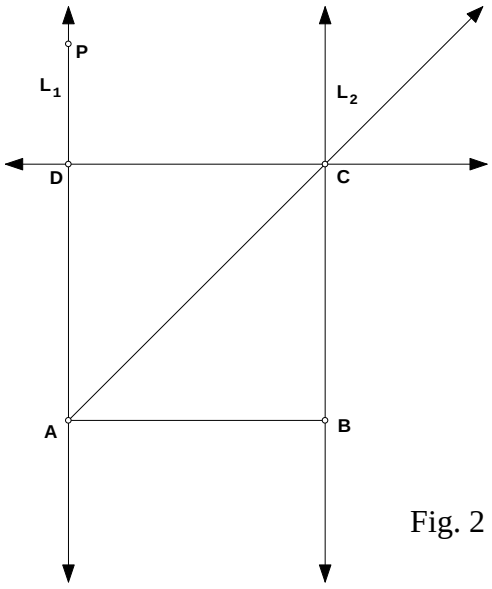


Fig. 2

3. The teacher then asks students whether the figure constructed is a square. Students are required to justify their assertion with reasons.

4. Students are guided to observe that not all the properties of a square are needed in the construction and recall the idea of minimal conditions.

5. Students are asked to construct their own squares using the properties of a square. For less able students, the teacher may give the hints below (Fig. 3 and Fig. 4):

Method 3:

(i) Construct a line segment *AB*.
(ii) Construct a circle centred at *A* with radius *AB*.
(iii) Construct a line through *A* and perpendicular to *AB*.
(iv) Mark the intersection of the circle and this line as point *C*.
(v) Construct a line through *C* and parallel to *AB*.
(vi) Construct a line through *B* and parallel to *AC*.
(vii) The point of intersection of the lines drawn in steps (v) and (vi) is labeled as *D*. Then *ABDC* is a square.

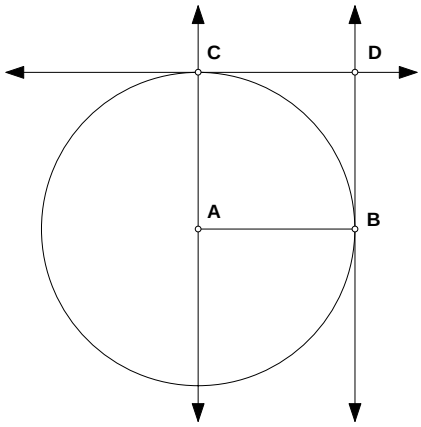


Fig. 3

Method 4:

(i) Construct a line segment *AB*.
(ii) Rotate the segment by 90º in the anti-clockwise direction about point *A*.
(iii) Mark the end point of the resulting segment as *C*.
(iv) Rotate *AC* by 90º in the anti-clockwise direction about point *C*.
(v) Mark the end point of the resulting segment as *D*.
(vi) Then *ABDC* is a square.

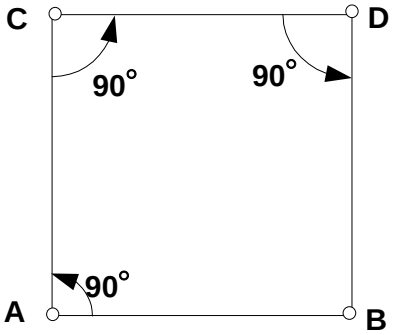


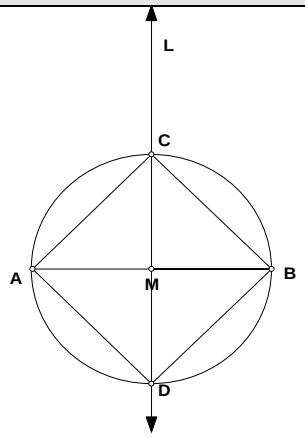
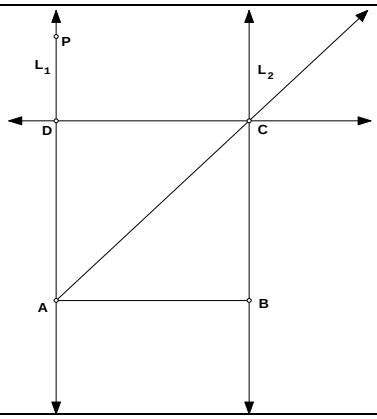
Fig. 4

6. Students are invited to demonstrate the steps in their constructions and give the geometrical reasons in constructing the square. Other students may comment on the methods of construction and discuss which one is most efficient (i.e. which one involves the least number of steps).

7. The teacher then discusses and concludes with students the properties used in each construction of the square. Students are requested to write down the properties adopted in the constructions in the worksheet as homework.

Worksheet: Construction of a Square

Record the properties used in each method to construct a square.

Method	Properties of squares	Diagram
1		
2		
3		
4		

Notes for Teachers:

1. This problem is a very interesting open-ended problem though the problem is embedded in the computer environment. Students may use different methods to construct a simple figure — a square. The computer also provides the environment for students to free play with the properties of square. More time should be allowed for students to explore with the software. If time is not sufficient, the teacher can let students try the constructions at home or using the computer facilities in school after lessons, and discuss in the lessons.

2. For those who are unable to handle the software, the teacher can give the CD-ROM developed by The Hong Kong University of Science and Technology "如何應用 *WinGeom, WinPlot, Sketchpad* 於教學上" which was distributed to schools in July 2000. This CD-ROM provides demonstration in using the software.

3. Teachers can use the enclosed *Sketchpad* Scripts Square1.gss, Square2.gss, Square3.gss and Square4.gss to demonstrate methods 1 to 4 respectively. Alternatively, the teacher can refer to Annex for the construction procedures.

4. Properties of a square used in each suggested construction procedure are listed in the table.

Method	Properties used	Remarks
1	Two diagonals are equal, perpendicular and bisect each other.	These 3 methods are similar to the strategies in using straight edges and compasses in the construction of a square.
2	Two right angles and the diagonals bisect opposite angles.	
3	Opposite sides are parallel and adjacent sides are equal and perpendicular to each other.	
4	Four right angles and four equal sides	The last method illustrates the application of transformation in the construction process by using the reflectional and rotational symmetries of a square.

Annex

Suggested Operation Procedure for the demonstration: (Based on *Sketchpad* 3.0)

Method 1

1. Click the **Segment** tool  and drag to draw a horizontal segment.

2. Click the **Label** tool  and label the endpoints of the segment as *A* and *B*.

3. Select the segment *AB*. Go to the **Construct** menu and choose **Point At Midpoint**.
4. Click the **Label** tool and give a label to the midpoint. Double click the label and rename it as *M*.
5. Click the **Selection Arrow** tool  . Hold down the **Shift** key.
   Select points *A* and *M*. Go to the **Construct** menu and choose **Circle By Centre And Point**.
6. Hold down the **Shift** key. Select point *M* and segment *AB*. Go to the **Construct** menu and choose **Perpendicular Line** to construct a line through *M* and perpendicular to *AB*.
7. Click the **Label** tool and label the line as *L*.
8. Click the **Selection Arrow** tool. Hold down the **Shift** key. Select the circle and line *L*. Go to the **Construct** menu and choose **Point At Intersection**.
9. Click the **Label** tool and label the points of intersection as *C* and *D*.
10. Click the **Selection Arrow** tool. Hold down the **Shift** key. Select the points *A*, *B*, *C* and *D*. Go to the **Construct** menu and choose **Segment**. Then a square *ADBC* is obtained.

Method 2

1. Repeat step 1 and 2 in Method 1.
2. Click the **Selection Arrow** tool. Hold down the **Shift** key. Select point *A* and segment *AB*. Go to the **Construct** menu and choose **Perpendicular Line** to construct a line through A and perpendicular to *AB*.
3. Hold down the **Shift** key. Select point *B* and segment *AB*. Go to the **Construct** menu and choose **Perpendicular Line** to construct a line through *B* and perpendicular to *AB*.
4. Click the **Label** tool and label the perpendicular lines at *A* and *B* as L_1 and L_2 respectively.
5. Click the **Point** tool and mark a point *P* on L_1.

6. Click the **Selection Arrow** tool. Hold down the **Shift** key. Select points *P, A* and *B*. Go to the **Construct** menu and choose **Angle Bisector**.
7. Hold down the **Shift** key. Select the angle bisector and L_2. Go to the **Construct** menu and choose **Point At Intersection**.
8. Label this point as *C*.
9. Click the **Selection Arrow** tool. Hold down the **Shift** key. Select point *C* and L_2. Go to the **Construct** menu and choose **Perpendicular Line**.
10. Mark the point of intersection of the perpendicular line drawn in step 9 and L_1.
11. Label this point as *D*. Then a square *ABCD* is obtained.

Other operation procedure involving in methods 3 to 4:

(I) *Construct a circle centre at point A and with radius AB*

Click the **Selection Arrow** tool. Hold down the **Shift** key. Select points *A* and *B*. Go to the **Construct** menu and choose **Circle By Centre And Point**.

(II) Rotate a line segment AB around the point *A* by *90º*

1. Click the **Selection Arrow** tool. Select point *A*. Go to the **Transform** menu and choose **Mark Centre**. Then point *A* is the centre of rotation.
2. Hold down the **Shift** key. Select segment *AB* and point *B*. Go to the **Transform** menu and choose **Rotate**. Enter **90º** in the box and click **OK**.

(III) Reflect a point A about a line segment BC

1. Click the **Selection Arrow** tool. Select segment *BC*. Go to the **Transform** menu and choose **Mark Mirror**. Then *BC* is the line of reflection.
2. Select point *A*. Go to the **Transform** menu and choose **Reflect**. Then an image of *A* is obtained.