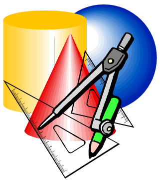

EXEMPLAR 25:

Transformation of Points in Coordinate Plane

Objective: To describe intuitively the effects of transformation (such as translation, reflection with respect to lines parallel to x-axis and rotation about the origin through multiples of 90°) on points in coordinate planes

Key Stage: 3

Learning Unit: Introduction to Coordinates

Materials Required: Dynamic geometry software such as *Cabri Geometry II* and *Cabri* files Tra02.fig, Tra03.fig, Ref02.fig and Rot02.fig

Prerequisite Knowledge:
(1) Meaning of transformation including translation, reflection and rotation
(2) Locating the coordinates of a point in coordinate plane.

Description of the Activity:

1. The teacher gives a brief revision on the meaning of transformation including translation, reflection and rotation of 2-D shapes.

2. The teacher distributes Worksheet 1: "Translation of Points in Coordinate Plane" to students. Students need to use the *Cabri* file Tra02.fig and Tra03.fig to explore the effect of translation of points in the coordinate plane and write down their findings in the worksheets. The idea of vector should be explained prior to students' exploration.

3. The teacher discusses the answers for Worksheet 1 with students and concludes that $(x, y) \rightarrow (x + \mathrm{a}, y + \mathrm{b})$ represents the translation by a vector from O to (a, b) no matter the points lie in the grid or not.

4. The teacher distributes Worksheet 2 "Reflection of Points in Coordinate Plane" and Worksheet 3 "Rotation of Points in Coordinate Plane" to students. Students

have to make use of the *Cabri* files Ref02.fig and Rot02.fig to explore the effect of rotation and reflection of points in the coordinate plane and write down their findings in the worksheets.

5. The teacher discusses with students the answers for the worksheets.

6. For Worksheet 2, the teacher may conclude that the x-coordinate of the point remains unchanged if the line of reflection is always parallel to the x-axis.

7. For Worksheet 3, the teacher may conclude that $(x , y) \to (y , -x)$ represents a rotation of the point through 90°. The teacher can guide students to discover that rotating through 180° is the same as rotating 90° twice.

   That is, $(x , y) \xrightarrow{90^\circ} (y , -x) \xrightarrow{90^\circ} (-x , -y)$ is the same as $(x , y) \xrightarrow{180^\circ} (-x , -y)$.

Worksheet 1: Translation of Points in Coordinate Plane

1. Open the Cabri file Tra02.fig. You can find a vector starting from the origin *O* to a point *P* (see Fig.1).

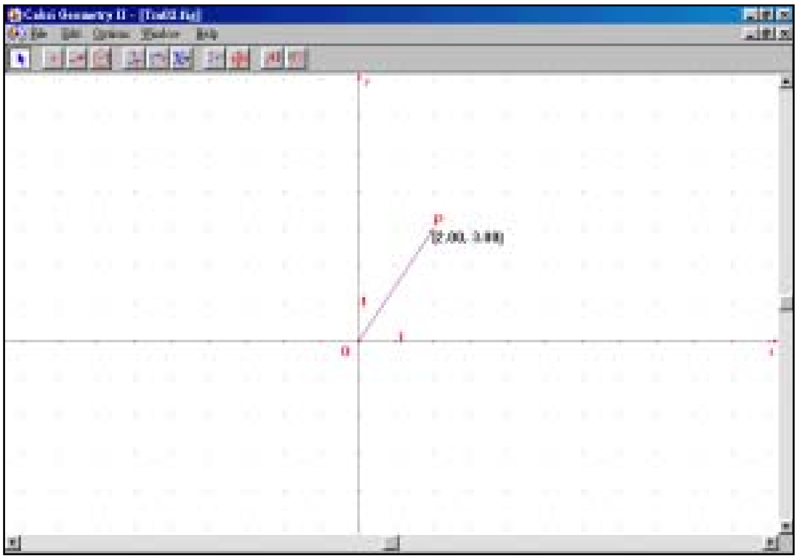

Fig.1

2. Select **Point** from the **Points** toolbox. Move the cursor to the grid point in the plane and click once to create a point. Label it as *A*.

3. Select **Translation** from the **Transformation** toolbox. Click the point *A* and the vector respectively to translate the point *A* by the given vector. Label the translated point as *A*′. Select **Equation and Coordinate** from the **Measure** toolbox. Click the points *A* and *A*′ to measure their coordinates (see Fig.2).

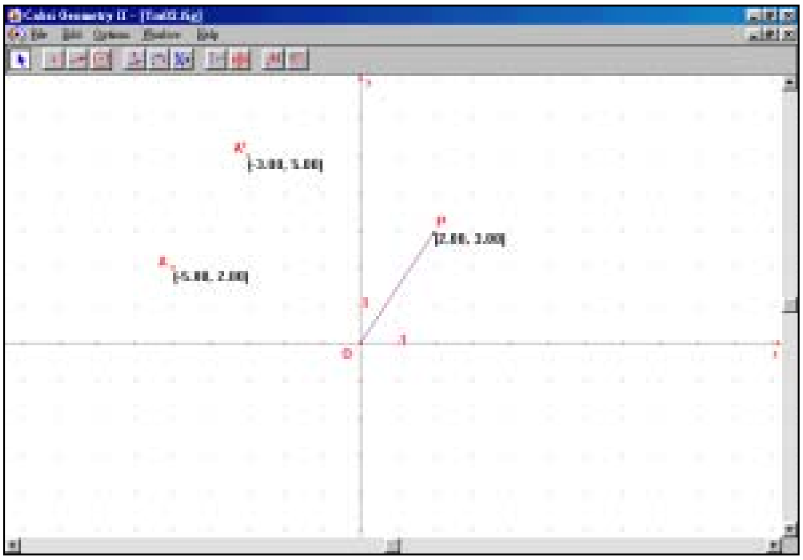

Fig.2

4. Now drag the point *A* to observe the changes in the coordinates of points *A* and *A′*. Without changing the vector, record a set of coordinates of *A* and its translated point *A′* in Table 1. Then change the magnitude of the vector by dragging the end point of the vector. Record other sets of data and fill in the conclusion in the same table.

Vector	Coordinates of the point *A*	Coordinates of the point *A*'
From *O* to (,)	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
From *O* to (,)	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
From *O* to (,)	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
Conclusion		
Vector	Coordinates of the point *A*	Coordinates of the point *A*'
From O to (a , b)	(x , y)	(,)

Table 1

5. Open the *Cabri* file Tra03.fig. You will find that the point *P* may not be lying in the grid. Does your conclusion in Table 1 still hold for points not lying in the grid?

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(You may repeat steps 2 and 3 above to do your own investigation in answering the above question.)

Worksheet 2: Reflection of Points in Coordinate Plane

1. Open the *Cabri* file Ref02.fig. You can find a line *L* which is parallel to *x*-axis. *P* is a point lying on the line *L* (see Fig.1).

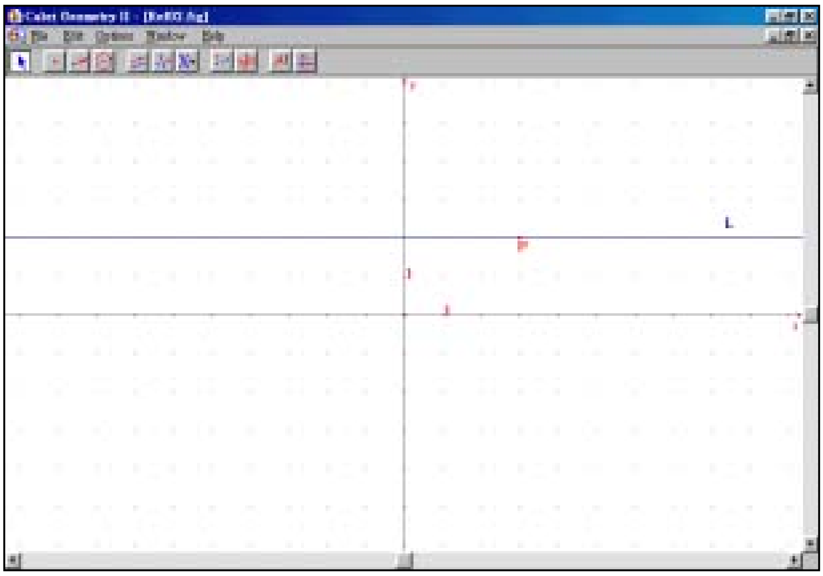

Fig.1

2. Select **Point** from the **Points** toolbox. Move the cursor to the grid point in the plane and click once to create a point. Label it as *A*.

3. Select **Reflection** from the **Transformation** toolbox. Click the point *A* and the line *L* respectively to reflect the point *A* by the given line *L*. Label the reflected point as *A′*. Select **Equation and Coordinate** from the **Measure** toolbox. Click the points *A* and *A′* to measure their coordinates (see Fig.2).

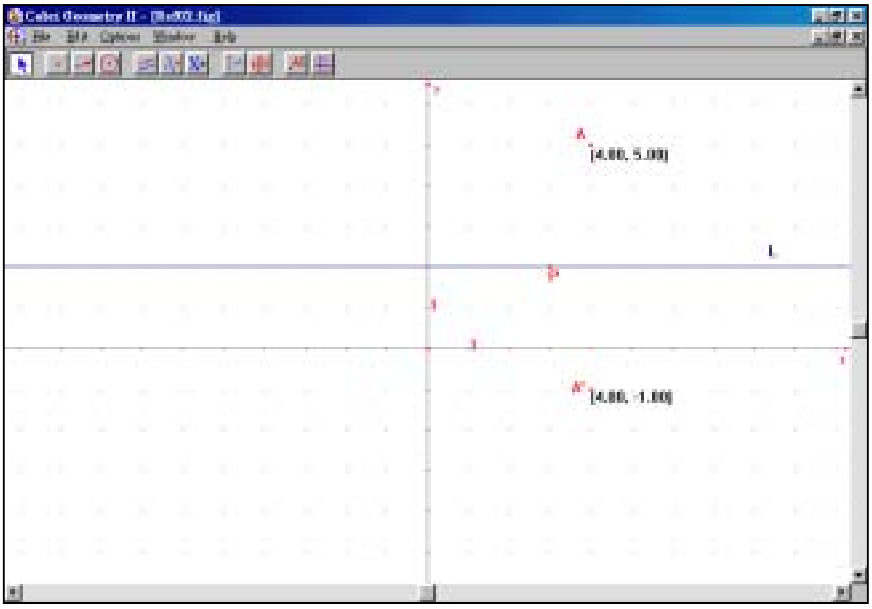

Fig.2

4. Drag the point *P* so that *L* becomes the *x*-axis. Drag the point *A* to observe the changes in the coordinates of points *A* and *A*'. Record a set of coordinates of *A* and its reflected point *A*′ in Table 1 and fill in the conclusion. Then drag the point *P* to (1, 2) so that *L* is 2 units above the *x*-axis. Record another set of data and fill in the conclusion in Table 2. Repeat the above for a new position of *P* as (1, -3) so that *L* is 3 units below the *x*-axis.

Axis of reflection (The line *L*)	Coordinates of the point *A*	Coordinates of the point *A*'
the *x*-axis	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
Conclusion		
the *x*-axis	(*x* , *y*)	(,)

Table 1

Axis of reflection (The line *L*)	Coordinates of the point *A*	Coordinates of the point *A*'
2 units above the *x*-axis	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
Conclusion		
2 units above the *x*-axis	(*x* , *y*)	(,)

Table 2

Axis of reflection (The line L)	Coordinates of the point A	Coordinates of the point A'
3 units below the x-axis	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
Conclusion		
3 units below the x-axis	(x , y)	(,)

Table 3

Worksheet 3: Rotation of Points in Coordinate Plane

1. Open the *Cabri* file Rot02.fig. You will find a point *A* joining to the origin *O* (see Fig.1).

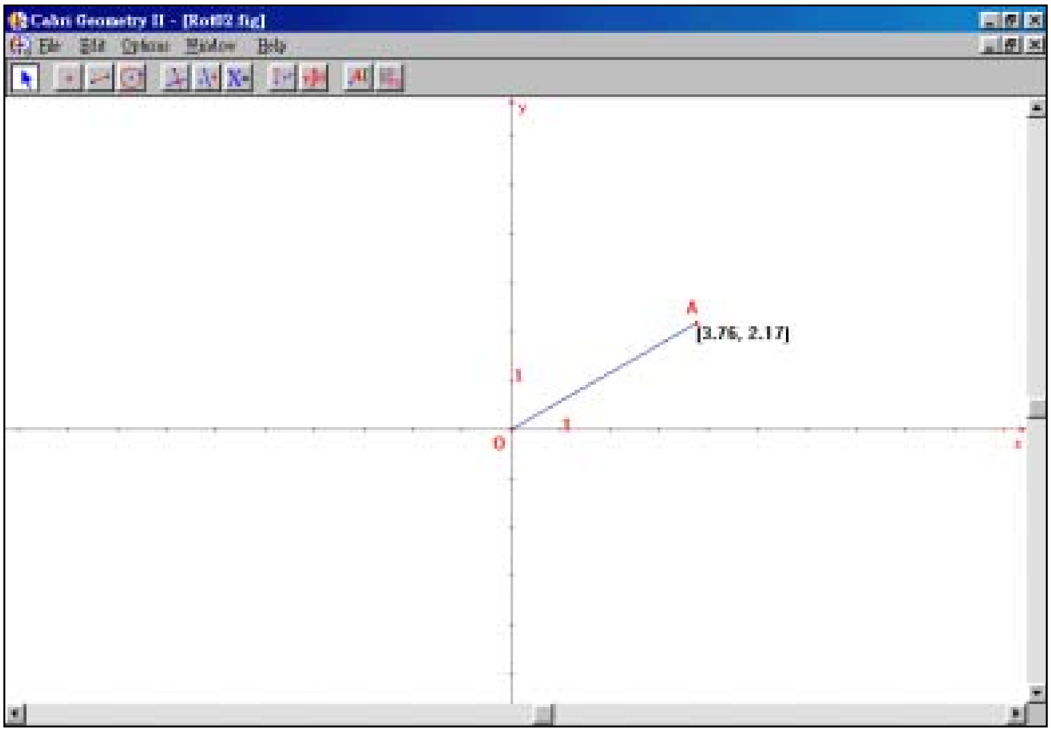

Fig.1

2. Select **Numerical Edit** from the **Display** toolbox.

3. Click to place an edit box anywhere in the drawing window for creating an interactive number.

4. Type the numerical value 90 in the box. Press **Ctrl U** to select **Degree**.

5. Select **Rotation** from the **Transformation** toolbox. Click the point *A*, the origin *O* and the numerical value 90° to rotate the point *A* by 90°. Label the rotated point as *A*'. Select **Equation and Coordinate** from the **Measure** toolbox. Click the points *A* and *A*' to indicate their coordinates.

6. Select the **Segment** from the **Lines** toolbox. Draw the line segment *OA*'. Select **Mark Angle** from the **Display** toolbox. Select *A*, *O* and *A*' sequentially to mark the right angle *AOA*' (see Fig. 2 on next page).

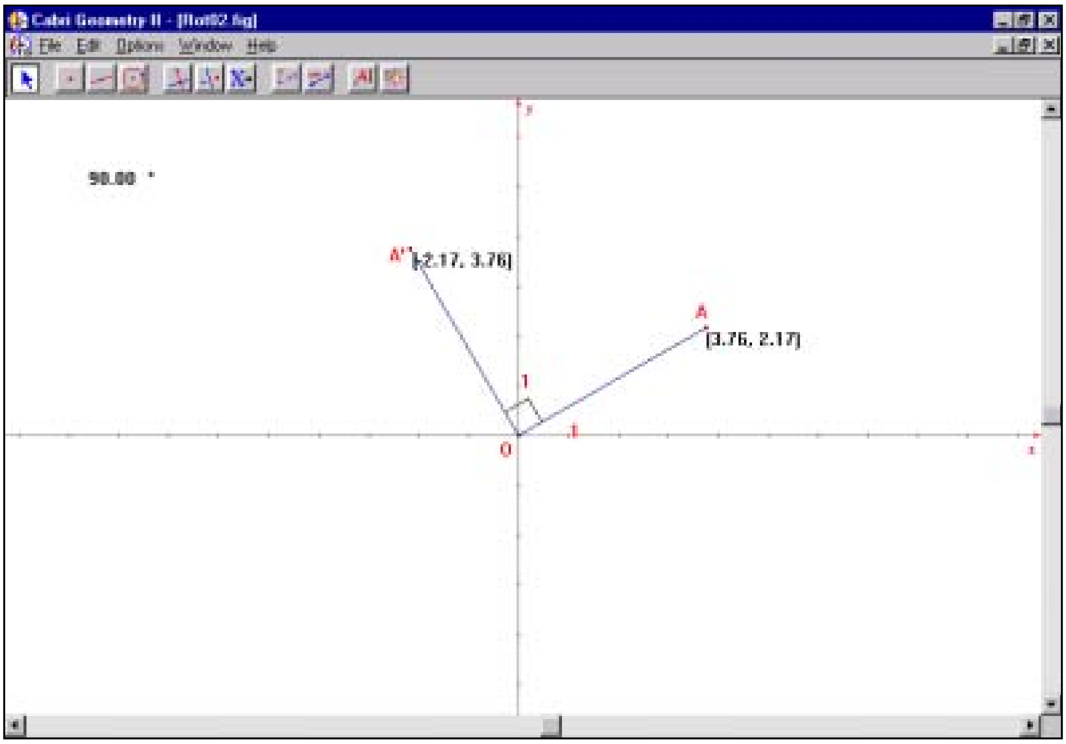

Fig.2

7. Drag the point A to observe the changes in the coordinates of points A and A'. Record a set of coordinates of A and its rotated point A' in Table 1.

Angle of rotation	Coordinates of the point A	Coordinates of the point A'
90°	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
Conclusion		
90°	(x, y)	(,)

Table 1

8. Change the angle of rotation to 180° and 270° subsequently and collect other 2 sets of data. Record them and summarize your conclusions in Tables 2 and 3. You can use the following steps to change the angle of rotation.
   (a) Double click the angle to be rotated. You will find the arrow keys appear on the right hand side of the angle.
   (b) Press the arrow up key or the arrow down key to modify the angle until 180°.
   (c) Drag the point *A* to different positions. Record the coordinates of point *A* and its rotated point *A*' in Table 2.
   (d) Repeat (a) to (c) for the angle of rotation as 270°.

Angle of rotation	Coordinates of the point *A*	Coordinates of the point *A*'
180°	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
Conclusion		
180°	(*x* , *y*)	(,)

Table 2

Angle of rotation	Coordinates of the point *A*	Coordinates of the point *A*'
270°	(,)	(,)
	(,)	(,)
	(,)	(,)
	(,)	(,)
Conclusion		
270°	(*x* , *y*)	(,)

Table 3

Notes for Teachers:

1. The objective of this exemplar is to let students describe **intuitively** the effects of transformation on points in coordinate planes. Students only need to generalize their own investigations from a few data. Geometric proofs are advisable only for more able students.

2. For those students who are keen on using *Cabri*, the teacher can ask them to try the exploration without using the given *Cabri* files.

3. Answer for Worksheet 1:

Vector	Coordinates of the point A	Coordinates of the point A'
From O to (a , b)	(x, y)	$(x + \text{a},\ y + \text{b})$

4. Answers for Worksheet 2:

Axis of reflection (The line L)	Coordinates of the point A	Coordinates of the point A'
the x-axis	(x, y)	$(x, -y)$
2 units above the x-axis	(x, y)	$(x, 4-y)$
3 units below the x-axis	(x, y)	$(x, -6-y)$

5. Answers for Worksheet 3:

Angle of rotation	Coordinates of the point A	Coordinates of the point A'
90°	(x, y)	$(y, -x)$
180°	(x, y)	$(-x, -y)$
270°	(x, y)	$(-y, x)$

6. In Worksheet 2, we only consider lines parallel to the x-axis as the lines of reflection. The teacher may modify the worksheet to let students investigate the effect of reflection of lines parallel to the y-axis. For very brilliant students, the teacher can even change the line of reflection with equation $y = x$ for further exploration. This transformation is called the *inverse transformation* because the

point (x , y) is transformed to the point (y , x).

7. In Worksheet 3, we only consider the cases for rotation through 90°, 180° and 270° only. Teachers may modify the worksheet for the rotation through 360°, –90°, –180°,–270° and also –360°.

8. The teacher may refer to **Appendix C** for Tools in *Cabri Geometry II*.