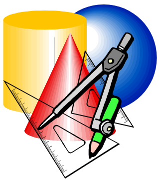

EXEMPLAR 27:

Sine Ratio

Objective: To understand the sine ratio for angles between 0° to 90°

Key Stage: 3

Learning Unit: Trigonometric Ratios and Using Trigonometry

Materials Required: Dynamic geometry software such as *Geometer's Sketchpad* (later referred as *Sketchpad*), spreadsheet such as *Excel* and the file trigo1.gsp

Prerequisite Knowledge:
(a) Basic concepts of similar triangles
(b) Definition of adjacent side, opposite side and hypotenuse for an acute angle of a right-angled triangle

Description of the Activity:

1. The teacher recalls the meaning of adjacent side, opposite side and hypotenuse of a right-angled triangle.

2. The teacher distributes the worksheet to students. Students are asked to complete the worksheet by using the *Sketchpad* file trigo1.gsp. In doing the worksheet, students need to investigate the relationship between the opposite side and the hypotenuse for an angle of a right-angled triangle.

3. Students make their own conjectures on the relationship after the observation. Afterwards, they should follow the instruction in the worksheet to analyze the relationship of the corresponding data by using *Excel*.

4. The teacher asks students to discuss the reasons of their conjectures.

5. The teacher may ask some students to present their conjectures and reasons to the whole class. Students can be guided to use the idea of similarity to prove the

invariance nature of the relation.

6. The teacher may then introduce the definition of sine ratio and guide students to conclude that:
   (a) the sine ratio is independent of the sizes of the triangles formed;
   (b) the ratio is constant for a fixed angle;
   (c) different angles will give different sine ratios.

Worksheet:Investigation of the relationship between the adjacent side and the opposite side for an angle of a right angled triangle

Instruction:

1. Open the *Sketchpad* file trigo1.gsp.

   You will find an angle $\angle ABC$ and its measurement on the top of the window. The middle part of the window shows a right-angled triangle with a right angle at R whereas the lower part of the window shows its measurement as shown in Fig. 1. (Please measure the size of $\angle PRQ$ and verify that ΔPQR is actually a right-angled triangle.)

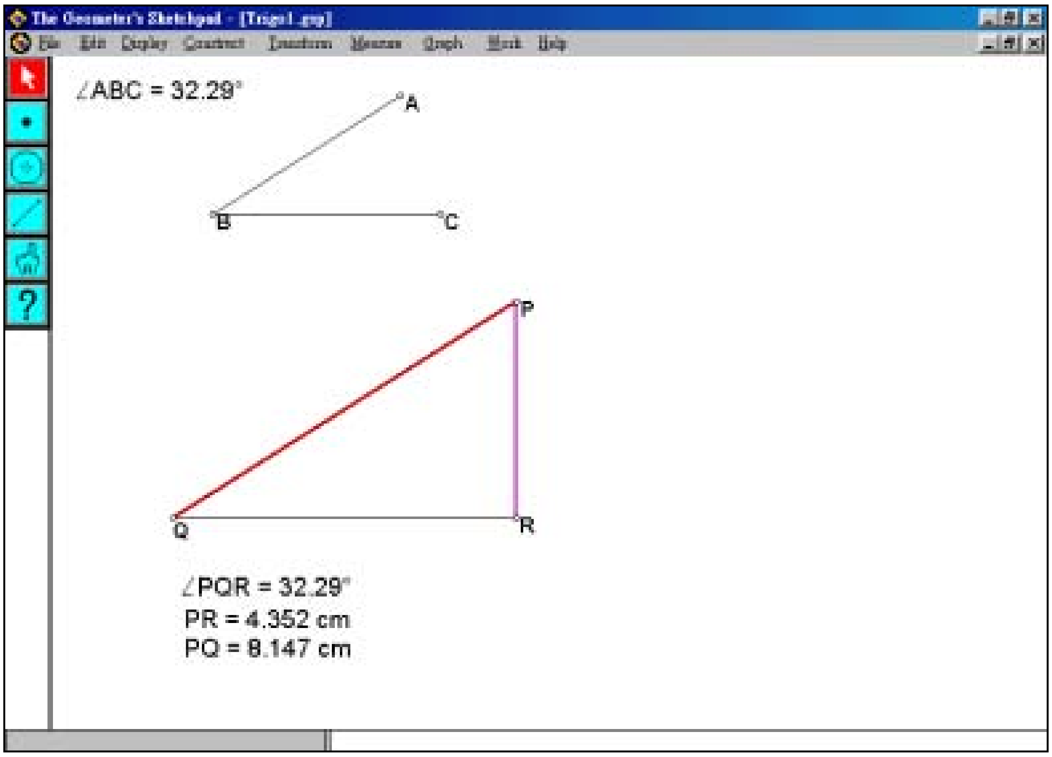

Fig. 1

2. Drag the end points of $\angle ABC$ to observe any change that happens in ΔPQR. Write down your finding(s) below.

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3. Drag the vertex *R* of ΔPQR. You will find that the lengths of *PR* and *PQ* change simultaneously. Try to use another angle *PQR* by dragging an end point of $\angle ABC$. Then drag the vertex *R* again. Can you guess a relationship between the lengths of *PR* and *PQ* for a fixed $\angle PQR$? Write down your conjecture below.

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4. Measure $\angle PQR$, its opposite side *PR* and its adjacent side *PQ* and collect the data by using the following steps.
   (a) Hold down the **Shift** key to highlight the measures of $\angle PQR$, *PR* and *PQ* in the lower part of the window simultaneously.
   (b) Select **Measure** | **Tabulate** in the pull-down menu to create a table.
   (c) Highlight the table formed. Choose **Measure** | **Flip Direction** in the pull-down menu to flip the table.
   (d) Drag the vertex *R* to another position without changing the size of $\angle PQR$. Click to select the table and press Ctrl E to add a new record to the table.
   (e) Repeat the same procedure to add more records (suggested to make up to about 7 records) for this fixed angle to the table as shown in Fig. 2.

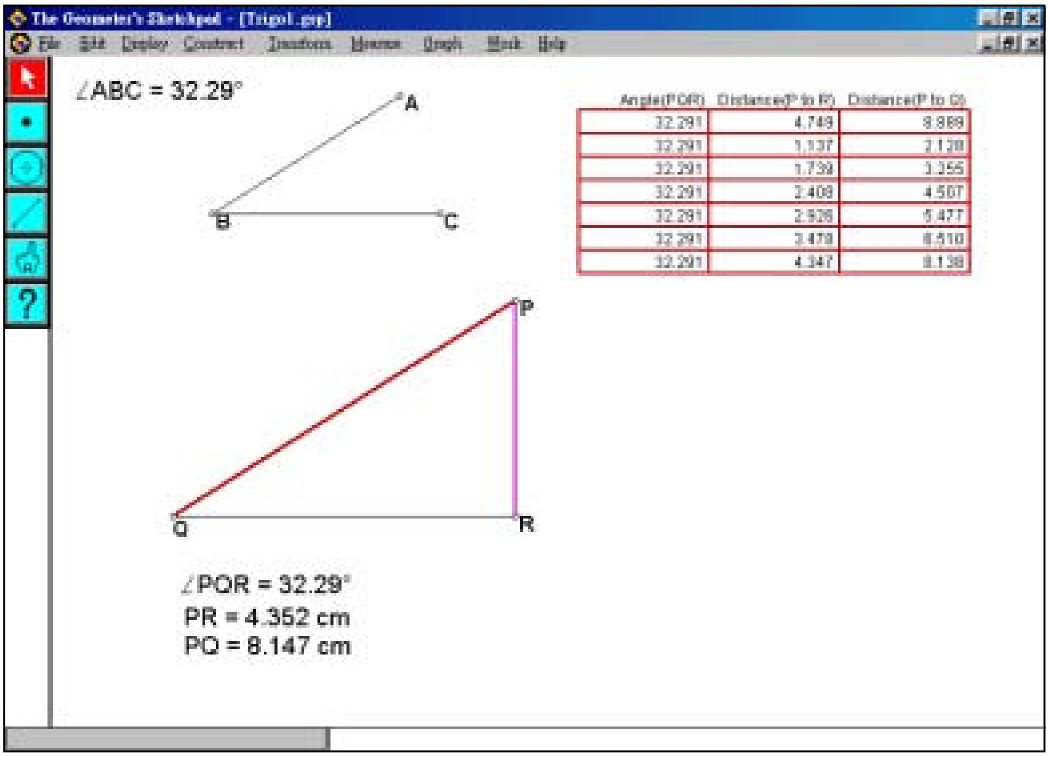
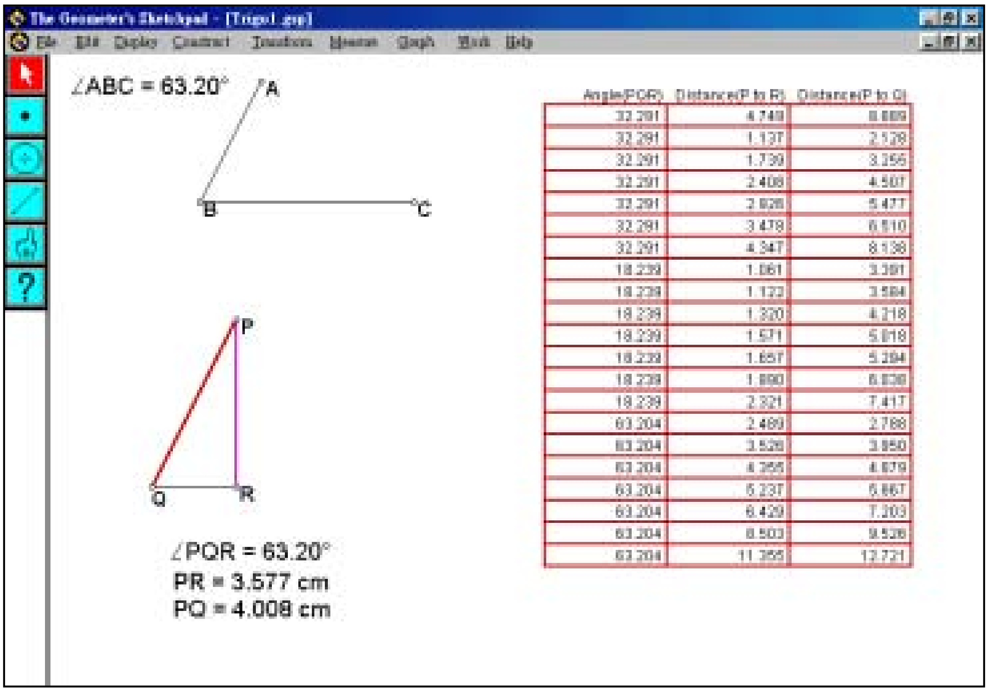


Fig. 2

(f) Drag ∠*ABC* to change the size of ∠*PQR*. Repeat steps (d) and (e) to record other sets of data (each set contains about 7 records for the same fixed angle *PQR*, see Fig. 3).



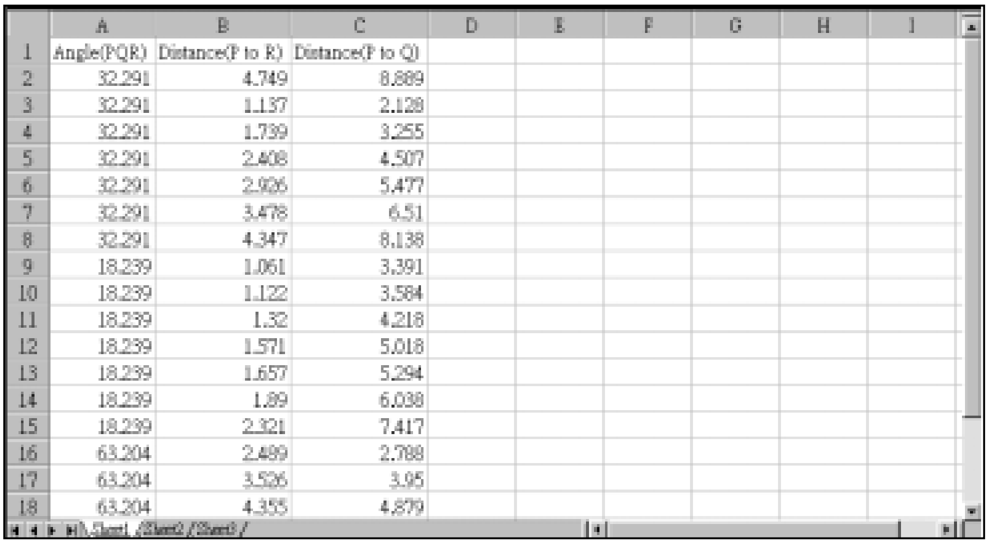
Angle(PQR)	Distance(P to R)	Distance(P to Q)
32.291	4.749	8.889
32.291	1.137	2.128
32.291	1.739	3.255
32.291	2.408	4.507
32.291	2.926	5.477
32.291	3.478	6.510
32.291	4.347	8.138
18.239	1.061	3.391
18.239	1.122	3.584
18.239	1.320	4.218
18.239	1.571	5.018
18.239	1.657	5.294
18.239	1.890	6.038
18.239	2.321	7.417
63.204	2.489	2.788
63.204	3.526	3.950
63.204	4.355	4.879
63.204	5.237	5.867
63.204	6.429	7.203
63.204	8.503	9.526
63.204	11.355	12.721

Fig. 3

5. To copy the data into an *Excel* file in order to analyze the conjecture:

(a) Highlight the table and press **Ctrl C**.

(b) Open a new *Excel* file. Click the cell A1 and press **Ctrl V** to paste the data into the *Excel* file as shown in Fig. 4.



	A	B	C	D	E	F	G	H	I
1	Angle(PQR)	Distance(P to R)	Distance(P to Q)						
2	32.291	4.749	8.889						
3	32.291	1.137	2.128						
4	32.291	1.739	3.255						
5	32.291	2.408	4.507						
6	32.291	2.926	5.477						
7	32.291	3.478	6.51						
8	32.291	4.347	8.138						
9	18.239	1.061	3.391						
10	18.239	1.122	3.584						
11	18.239	1.32	4.218						
12	18.239	1.571	5.018						
13	18.239	1.657	5.294						
14	18.239	1.89	6.038						
15	18.239	2.321	7.417						
16	63.204	2.489	2.788						
17	63.204	3.526	3.95						
18	63.204	4.355	4.879						

Fig. 4

6. Verify your conjecture in step 3 above by using the *Excel* file you constructed as shown in Fig. 4.
   Is your conjecture correct? ______________
   If not, find out a relationship between *PR* and *PQ* and write it down below.
   (Hint: You may consider $PR + PQ$, $PR - PQ$, $PR \times PQ$ and PR/PQ)

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7. Discuss with other students for a proof to your conjecture. Write down below.

Notes for Teachers:

1. This exemplar aims at introducing the definition of the sine ratio of an acute angle to students. For the sine ratio, students should be able to define it in a right-angled triangle and understand that it is the ratio of the opposite side to the hypotenuse of this angle. Besides, this ratio is independent of the sizes of the triangles formed, as all the corresponding triangles are similar to each other.

2. In question 2 of the worksheet, students may find that $\angle PQR$ is always equal to $\angle ABC$. In question 3, students may find that if PR increases, PQ will also increase. On the other hand, if PR decreases, PQ will also decrease. Instead of asking students to consider $\frac{PR}{PQ}$, the teacher may ask students to consider the four basic operations for PR and PQ. Students can experience that a new mathematical concept or relationship is usually formed from a lot of data analysis. The inquiring skill as well as reasoning skill can be enhanced through this exemplar.

3. The teacher should note that there might be round up error in computing $\frac{PR}{PQ}$ and etc. So students should use sufficient number of data to test their conjectures.

4. The teacher can also adopt the idea of this exemplar for the investigation of cosine ratio and tangent ratio of an acute angle.