B. Learning Objectives and Notes on Teaching (KS3)

Unit	Learning objectives	Notes on Teaching	Time ratio
Measures in 2-Dimensional (2-D) and 3-Dimensional (3-D) Figures			
Estimation in Measurement	● recognize the approximate nature of measurement and choose an appropriate measuring tool and technique for a particular purpose ● choose an appropriate unit and the degree of accuracy for a particular purpose ● develop estimation strategies in measurement ● handle and reduce errors in measurement ● estimate, measure and calculate lengths, areas, capacities, volumes, weights, rates, etc.	As students have been introduced direct measurements for real objects and indirect measurements for simple figures at primary levels (refer Annex I of the *Syllabus*), the focus at this learning stage includes ◆ more sophisticated skills in direct measurements; and ◆ indirect measurements such as using formulas and estimation strategies in measurement. Regarding direct measurements, teachers should guide students to see that all measurements are approximations. Appropriate techniques, tools and units should be chosen for a particular purpose such as measuring the weight of a gold ring and that of a piece of steak. Teachers should also guide students to discuss ways to reduce errors or to increase the accuracy of a measurement. For example: ◆ measuring the volume of 1000 drops of water to deduce the volume of a drop of water; ◆ using small grids to estimate the area of an irregular shape. Students should be aware of the distortion on the visual inspection of the measures of objects. Regarding indirect measurements, teachers may use various activities to guide students recognizing the importance of estimation in measurement. Activities should include measuring objects with the help of a few measuring instruments or without any measuring instruments at all. It is a mental process, though there are often visual and manipulative aspects. Teachers can introduce estimation strategies such as ◆ the benchmark strategies, for example, by applying a standard (benchmark/referent) such as the height of the door to estimate that of a room; and ◆ decomposition-recomposition strategies, such as by decomposing the corridor into a number of classrooms of known lengths. The unit “Simple Idea of Areas and Volumes” will focus on formulas related to circles and prisms while the unit “More about Areas and Volumes” on formulas relating to pyramids and spheres. Appropriate activities such as stacking and dissecting figures should be provided before introducing the relevant formulas. Teachers should also enable students to apply the experiences of figures and solids in the unit “Introduction to Geometry” in deriving the formulas for 2-D and 3-D figures.	6
Simple Idea of Areas and Volumes	● find areas of simple polygons ● explore the formula for the area of a circle ● calculate circumferences and areas of circles ● understand and use the formulas for surface areas and volumes of cubes, cuboids, prisms and cylinders ● appreciate the application of formulas, besides measurement, in finding measures and be aware of the accumulated errors arisend		15

Unit	Learning objectives	Notes on Teaching	Time ratio
More about Areas and Volumes	● calculate arc lengths and areas of sectors ● understand and use the formulas for volumes of pyramids, circular cones and spheres ● understand and use the formulas for surface areas of right circular cones and spheres ● understand and use the relationships between sides, surface areas and volumes of similar figures ● distinguish between formulas for length, area, volume by considering dimensions	Below are some exemplary activities: ◆ dissecting a circle into even number of identical small sectors and rearranging these sectors to form a figure that approximates a parallelogram to deduce the formula for the area of a circle; ◆ applying the concept of ratio and the formula for circumference to find arc lengths; ◆ unfolding prisms and pyramids into nets to find the surface areas of the solids; ◆ dividing a cube by six congruent pyramids of square bases to deduce the volume of a pyramid; ◆ unfolding a right circular cone to form a sector and deducing the formula for the curved surface area; ◆ taking the limit of a sequence of pyramids to find the volume of a cone, etc. Teachers should guide students to see that some formulas are obtained from approximations such as the formula for the area of circle. Besides using real models, teachers can illustrate the process of folding/unfolding and combining/dissecting a figure by the animation produced by the application of computer software. Applications of the formulas to solve daily-life examples such as the volume of water in a swimming pool and water-flow in a cylindrical pipe should be discussed. Teachers should also guide students to see intuitively the possible effects of errors on calculations involving measurements. For example: It does not make sense to give the area of a rectangular playground from the measured length 39.5m and width 24.7m. as 975.65m^2 because the given information has only 3 sig. figures. Students should be aware that the calculated area should be between 39.45×24.65m^2 and 39.55×24.75 m^2, that is, between 972.4425 m^2 and 978.8625 m^2 As students come across many formulas for areas and volumes of figures by the end of this learning module, teachers should guide students to compare and distinguish the dimensions of lengths, areas and volumes, such as Lengths in linear measurements such as$4a$, $(a+b)\times 2$, $2\pi r$, etc. Areas in quadratic measurements such as a^2, $a\times b$, πr^2, $4\pi r^2$, πrl, etc. Volumes in cubic measurements such as a^3, $a\times b\times h$, πr^2h, $(4/3)\ \pi r^3$, $(1/3)\pi r^2$h, etc. Teachers may then guide students to see the relations between the ratio of the volumes and that of the surface areas of similar solids to the ratio of their corresponding linear measurements. For more able students, teachers can ask students to use spreadsheet to explore the maximum area of figures for a given perimeter; to design a container with maximum capacity by cutting a square from each corner of a sheet of A4-paper, etc as enrichment activities.	18

Unit	Learning objectives	Notes on Teaching	Time ratio
Learning Geometry through an Intuitive Approach			
Introduction to Geometry	● recognize the common terms and notations in geometry such as line segments, angles, regular polygons, cubes and regular polyhedra (Platonic solids) etc. ● identify types of angles and polygons ● construct 3-D solids and explore their properties, such as Euler's formula ● sketch the 2-D representation of simple solids ● sketch the cross-sections of the solids ● overview tools of geometry and explore ways of using them to construct polygons, circles, parallel and perpendicular lines	In this module, students are expected to have rich experiences in manipulating objects such as paper-folding, constructing models, transformation of figures, etc. to build up an intuitive idea of the properties of various 2-D and 3-D figures. This will be a foundation for the deductive reasoning in studying geometric properties in the later stage. Below are some exemplary activities: **For the unit "Introduction to Geometry"** ◆ cutting a triangle to explore the sum of interior angles of the triangle; ◆ cutting solids made of play-doh to observe cross-sections of the solids; ◆ exploring solids and its 2-D representations with the help of software packages; ◆ exploring the properties on the edges, vertexes and faces of solids like Euler's formula, duality of regular polyhedra through manipulating the real-objects, etc. **For the unit "Transformation and Symmetry"** ◆ folding paper to observe the number of axes of reflectional symmetry of a regular pentagon; ◆ exploring the number of the axes of reflectional symmetry and the number of 2-fold, 4-fold or n-fold points of rotation on figures generated in software packages such as POWERPOINT; ◆ observing the effect of transformations on a geometric figure by dragging the objects in the computer, etc. **For the unit "Angles Related with Lines and Rectilinear Figures"** ◆ cutting a heptagon into triangles to explore the sum of exterior angles; ◆ designing a tessellated pattern by transforming a rectangle; ◆ using the software LOGO to construct squares, rectangles, or regular hexagons, etc. **For the unit "More about 3-D Figures"** ◆ drawing planes of reflectional symmetry with different colours on a cube for counting the number of planes of reflection; ◆ matching nets with solids by playing games in some software packages; ◆ manipulating the wire-model of a pyramid to observe the angle between the edge and the horizontal plane, etc. The unit "Introduction to Geometry" is considered both as an overview of geometry learnt in the primary school (Details can be referred to the Annex I of the *Syllabus*) and an introduction to plane geometry, 3-D geometry and tools of geometry in the secondary school.	10
Transformation and Symmetry	● recognize reflectional and rotational symmetries in 2-dimensional (2-D) shapes ● recognize the effect on 2-D shapes after the transformation including reflection, rotation, translation, dilation/contraction etc. ● appreciate the symmetrical shapes around and transformations on shapes used in daily-life		6
Congruence and Similarity	● recognize the properties for congruent and similar triangles ● extend the ideas of transformation and symmetry to explore the conditions for congruent and similar triangles ● recognize the minimal conditions in fixing a triangle ● identify whether 2 triangles are congruent/similar with simple reasons ● <u>explore and justify the methods to construct angle bisectors, perpendicular bisectors and special angles by compasses and straight edges</u> ● <u>appreciate the construction of lines and angles with minimal tools at hand</u>		14

Note: The objectives <u>underlined</u> are considered as non-foundation part of the syllabus.

Unit	Learning objectives	Notes on Teaching	Time Ratio
Angles Related with Lines and Rectilinear Figures	• recognize different types of angles • explore and use the angle properties associated with intersecting lines and parallel lines • explore and use the properties of lines and angles of triangles • explore and use the formulas for the angle sum of the interior angles and exterior angles of polygons • explore regular polygons that tessellate • appreciate the past attempts in constructing some special regular polygons with minimal tools at hand • construct some special regular polygons using straight edges and compasses	In plane geometry, a simple introduction including angle sum of triangles is expected in the unit "Introduction to Geometry". All students at primary levels should have studied line symmetry and some students may have learnt rotational symmetry as an enrichment topic. So, in the unit "Transformation and Symmetry", teachers can briefly relate the idea of line symmetry and extend the term to reflectional symmetry. Symmetrical shapes and the applications of transformation found in real-life situations such as Chinese characters, English letters, plants and logos can be used as motivation activities. More focus will be placed on the properties of reflectional symmetry, rotational symmetry and the effect of transformations on polygons. Based on the exploration of the effects of transformations on 2-D figures, teachers can guide students to study the meaning of congruence and similarity. The application of transformations in the tessellation of polygons and tessellated figures should be discussed in the unit "Angles related with Lines and Rectilinear Figures". In this unit, students are expected to treat the properties of angles intuitively and no formal proofs are required. Spatial sense is one of the abilities advocated in the curriculum. This includes building and manipulating mental representations of 2-D and 3-D objects and perceiving the same object from different perspectives. Activities to construct cubes (or combination of cubes), cuboids, cylinders and regular polyhedra in the unit "Introduction to Geometry" can support students' development in spatial visualization. In this unit, students are expected to sketch its 2-D representation from a given solid. However, the treatment should not be as formal as that required in the technical drawing of the subject "Design and Technology". In the unit "More about 3-D Figures", students are expected to recognize the same object when viewing from different angles of 2-D representations. Students are expected to build up an intuitive idea of the relations between lines and relations between planes. All these explorations should be treated qualitatively and these experiences will be the foundation for solving 3-D problems in the "More about Trigonometry" in KS4. In this unit, the objects chosen can be real-objects or models displayed in the computer. For tools of geometry, students have learnt to use protractors to measure angles at primary levels. At this stage, other tools such as compasses, set squares, straight edges, dynamic geometry software packages and some functions provided by common applications of software should be introduced. Activities using compasses and straight edges to construct angle bisectors, perpendicular bisectors and congruent triangles, etc are regarded as platforms for students to apply the conditions for congruent triangles. Through these activities, students can also be guided to appreciate past attempts in constructing figures with minimal tools at hand. Modern tools for construction in the software packages can also be introduced. Students are expected to justify verbally the steps they use. Writing geometric proofs are expected later in the unit "Simple Introduction to Deductive Geometry".	18
More about 3-D Figures	• extend the idea of symmetry in 2-D figures to recognize and appreciate the reflectional and rotational symmetries in cubes and tetrahedron • explore and identify the net of a given solid • imagine and sketch the 3-D objects from given 2-D representations from various views • recognize the limitation of 2-D representations in identifying the solid • explore the properties of simple 3-D object, such as identifying ◆ the projection of an edge on one plane ◆ the angle between a line and a plane ◆ the angle between 2 planes		8

Unit	Learning objectives	Notes on Teaching	Time Ratio
Learning Geometry through a Deductive Approach			
Simple Introduction to Deductive Geometry	● develop a deductive approach to study geometric properties through studying the story of Euclid and his books - *Elements* ● develop an intuitive idea of deductive reasoning by presenting proofs of geometric problems relating with angles and lines ● understand and use the conditions for congruent and similar triangles to perform simple proofs ● identify lines in a triangle such as medians, perpendicular bisectors etc. ● explore and recognize the relations between the lines of triangles such as the triangle inequality, concurrence of intersecting points of medians etc. ● explore and justify the methods of constructing centres of a triangle such as in-centre, circumcentre, orthocentre, centroids etc.	In this learning module, teachers should guide students to recognize the deductive reasoning in studying properties of simple geometric figures based on the exploratory experiences in the module 'Learning Geometry through an Intuitive Approach'. Simple activities for students to recognize the limitations of inductive/intuitive (or experimental) approach can be used as an introduction. In introducing deductive reasoning, teachers should guide students to identify what is given and what is to be proved in a problem. Discussions in the flow of thinking in proving the statement prior to writing formally can then be made in class. It is important that students provide reasons to the statements in their proofs. The format in writing proofs should not be confined to any one type such as "2-column" form. In the unit "Simple Introduction to Deductive Geometry", students are expected to write simple geometric proofs related to properties learnt in the module 'Learning Geometry through an Intuitive Approach'. These include proofs related to parallel lines, congruence and similarity. The Story of Euclid can be treated as a motivation activity for students to appreciate past efforts. For the properties related to lines, angles and some important centres of triangles, teachers can guide students to explore their properties with the help of dynamic geometric software. After then, the construction activities using compasses and a straight edge should be tried and students are expected to justify their constructions in verbal forms. Teachers may ask more able students to present their justifications in written forms. One of the profound accomplishments in the history of mathematics is the Pythagoras' Theorem. There are over three hundred proofs of Pythagoras' Theorem. Teachers can illustrate some of the proofs including the *Gou-gu*[1] in *Yuan-dao*[2] of the commentaries of the ancient Chinese mathematical book, *Zhou Bi Suan Jiang*[3]. Teachers can ask students to find and compare different proofs of the theorem as project work. Discussions on the authorship of the theorem – whether Pythagoras' Theorem or *Gou-gu* Theorem can be made. Its application in solving problems reducible to right-angled triangle should be mentioned and practised. This unit can be introduced together with the unit "Rational and Irrational Numbers" in the **Number and Algebra** Dimension. For more able students, teachers may also introduce the story of the first crisis of mathematics –the existence of irrational numbers threatening the assumption of the number system believed by the Pythagorean. The reason in introducing this story is to provide platforms for students to discuss the dynamic element in developing mathematical knowledge.	27
Pythagoras' Theorem	● recognize and appreciate different proofs of Pythagoras' Theorem including those in Ancient China ● recognize the existence of irrational numbers and surds ● use Pythagoras' Theorem and its converse to solve problems ● appreciate the dynamic element of mathematics knowledge through studying the story of the first crisis of mathematics		8

1 The equivalent Chinese characters for *Gou-gu* are 勾股.

2 The equivalent Chinese characters for *Yuan-Dao* are 弦圖.

3 The equivalent Chinese characters for *Zhou Bi Suan Jing* are 周髀算經, which is believed as the oldest Chinese mathematics text known.

Unit	Learning objectives	Notes on Teaching	Time Ratio
Quadrilaterals	• extend the idea of deductive reasoning in handling geometric problems involving quadrilaterals • deduce the properties of various types of quadrilaterals but with focus on parallelograms and special quadrilaterals • perform simple proofs related with parallelograms • understand and use the mid-point and intercept theorems to find unknowns	Special quadrilaterals such as the parallelogram and trapezium are included in the unit "Quadrilaterals". Students can apply the idea of parallelism and the properties of triangles to study properties related to parallelograms and special parallelograms such as rectangles, squares and rhombuses. Dynamic geometry software can be used to illustrate the properties of these parallelograms. Teachers can demonstrate different methods to draw parallelograms with the help of software. Teachers can then drag the constructions to show that no matter how the shape is changed, it is a parallelogram and ask students what conditions are fixed when the constructions are dragged. Alternatively, teachers can ask students to investigate the minimum number of steps to draw the parallelogram. Simple computations that related to these figures are required. Teachers should note that writing simple proofs related to these quadrilaterals is considered as non-foundation part.	15
Learning Geometry through an Analytic Approach			
Introduction to Coordinates	• understand and use the rectangular and polar coordinate systems to describe positions of points in planes • able to locate a point in a plane by means of an ordered pair in rectangular coordinate system • describe intuitively the effects of transformation such as translation, reflection with respect to lines parallel to x-axis, y-axis and rotation about the origin through multiples of 90° on points in coordinate planes • calculate areas of figures that can be cut into or formed by common 2-D rectilinear figures	In primary schools, students have learnt using the 8-point compass bearing to describe directions. In the unit "Introduction to Coordinates", students are expected to describe the location of a point by coordinates. Games such as battleship, seating plan can be designed for motivation. Stories of René Descartes can be introduced to explain the origination of the Cartesian coordinate plane. Overlaying rectangular or polar grids over maps or aerial photographs of regions can show the comparison on the rectangular and polar coordinate systems. To enhance students' understanding in rectangular coordinates, such as how coordinates will be affected by translating points 2 units along the x-axis, activities for students to move points around the grid should be allowed. These activities can be conducted with the help of software, graph boards or graph paper. In the unit "Coordinate Geometry of Straight Lines", students are expected to study the rectangular coordinate system formally. Besides describing the location of points, teachers should guide students to understand the ways to describe the distance between 2 points and the slope of the line joining 2 points in the coordinate system. If students have no prior knowledge in trigonometric ratios, teachers should only introduce the slope of a line as the ratio of the opposite and adjacent sides. Activities to demonstrate the conditions for parallel lines and perpendicular lines should adopt an intuitive approach instead of deducing them from the tangent ratio. The relation between tangent and the slope of a line must be introduced when students study the unit "Trigonometric Ratios and Using Trigonometry". Discussion on the conditions of parallel lines or perpendicular lines can also be made using the idea of similarity. An intuitive idea for comparing different approaches, namely experimental, deductive or analytic approaches, in studying geometric problems can be introduced through activities in proving relevant results of rectilinear figures. This idea will be further elaborated when students get mature and come across more varieties of geometric figures in KS4.	9
Coordinate Geometry of Straight Lines	• understand and use formulas of distance and slope • use ratio to find the coordinates of the internal point of division and mid point • understand the conditions for parallel lines and perpendicular lines • appreciate the analytic approach to prove results relating to rectilinear figures besides deductive approach • choose and use appropriate methods to prove results relating to rectilinear figures		12

Unit	Learning objectives	Notes on Teaching	Time Ratio
Trigonometry			
Trigonometric Ratios and Using Trigonometry	• understand the sine, cosine and tangent ratios for angles between 0° to 90° • explore the properties and relations of trigonometric ratios • explore the exact value of trigonometric ratios on special angles 30°, 45°, 60° • rationalize denominators such as $\sqrt{2}$ • apply trigonometric ratios to find measures of 2-D figures • introduce the ideas of bearing, gradient, angle of elevation, angle of depression and to solve related 2-dimensional problems	"Trigonometry" is concerned with the measurement of angles. It is new to students. From past experiences, it is suggested to introduce the unit after students have a thorough understanding of the properties of geometric figures and mensuration. The focus of the unit in this learning stage is on the application of trigonometric ratios whereas discussions on the properties of trigonometric functions are expected in KS4. Teachers are recommended to use simple practical problems to introduce the importance of trigonometric ratios, for example, to estimate the height of a building. Teachers may consider using the lengths of 3 sides of a right-angled triangle to define the trigonometric ratios for angles between 0° to 90° or using the coordinates of a point on the unit-circle to define trigonometric ratios. Activities through calculators, or measuring distances can be carried out in the class to explore how the values of trigonometric ratios change as the angle varies. After introducing the trigonometric ratios, teachers may consider to apply the ratios to solve practical problems first and later to explore the trigonometric relations and values for trigonometric ratios of special angles. Students are expected to solve problems involving right-angled triangles and simple practical problems involving gradient, bearing, angle of elevation and angle of depression. It should be noted that students may come across bearings, gradients in Geography. Thus, teachers should be aware of the terminology used in other subjects such as reduced bearing. If students have learnt the unit "Coordinate Geometry of Straight Lines" prior to this learning unit, teachers in this learning unit should define the slope of the line using the tangent ratio. Discussions on the properties of tangent ratio to introduce the conditions for parallel lines and perpendicular lines should also be made. It is recommended through exploratory activities for students to observe the patterns of properties and relations of trigonometric ratios. The relations include: $\sin(90°-\theta) = \cos\theta$, $\cos(90°-\theta) = \sin\theta$ $\tan(90°-\theta) = \frac{1}{\tan\theta}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$ Calculators and graphing software can be used to present the tables of values and the graphs of the ratios simultaneously to facilitate students to explore and to make conjectures on their relations. The proofs of trigonometric identities should be minimized at this stage. Some students may not be keen in finding the exact value as the approximated values can be obtained easily from calculators. Finding the exact values of trigonometric ratios of special angles may then be considered as an application of trigonometric ratios and the properties of triangles. Teachers can arrange students in groups to find the exact values for these special angles as a challenge for them. The rationalization of the denominators in the form of irrational numbers can be highlighted, as students will come across different representations of the same ratio in various situations.	26