Exemplar 1:
Addition and Subtraction of Directed Numbers

Objective: To understand the concepts of addition and subtraction of directed numbers

Key Stage: 3

Learning Unit: Directed Numbers and the Number Line

Materials Required: Black and white magnetic buttons

Prerequisite Knowledge: Concepts of directed numbers and the addition and subtraction of positive integers

Description of the Activity:

1. The teacher prepares 10 black and 10 white magnetic buttons for this activity.

2. The teacher asks students to decide which colour represents positive and which colour represents negative. Without loss of generality, black is used to represent positive and white is used to represent negative in the following description.

3. Each button represents one unit. Therefore, ⬤ represents "+1" and ◯ represents "–1".

4. The teacher explains the following points on the addition and subtraction of directed numbers using magnetic buttons.
    (a) one black button and one white button together make a "zero". That is, " 1" plus "–1" equals 0;
    (b) the **addition** of directed numbers means **adding** corresponding numbers of black and/or white buttons;
    (c) the **subtraction** of directed numbers means **taking away** corresponding numbers of black and/or white buttons.

5. The teacher uses the examples suggested in Table 1 or any other similar examples to demonstrate the addition and subtraction of directed numbers. The following questions can be raised from time to time during the illustration:

Addition
i)   How many buttons are left? What colour are they?

ii)   Which directed number is represented by the buttons left behind?

iii)   For 2 numbers of the same sign, will addition of the two numbers produce a sum with a sign different from the original numbers?

iv)   For 2 numbers of opposite signs in *Cases 3* and *7*, how many "zeros" are produced? What is the sign of the result? Which number in the original 2 numbers will affect the sign of the result? What is the general feature of this number ("–4" in *Case 3* or "+4" in *Case 7*) in comparing to another number?

Subtraction
i)   We have some black buttons, but we need to take away more black buttons than we have, what should we do? How can we do it?

ii)   How many "zeros" should we add ? Explain.

iii)   How many buttons are left after the required number of buttons are taken away? What colour are they?

iv)   Which directed number do those buttons left represent?

v)   For 2 numbers of the same sign, will the result have the opposite sign of the original numbers after subtraction? Does subtraction always change the sign of the numbers (by referring to *Cases 4* and *8*)?

vi)   For 2 numbers of opposite signs, which one of the original numbers will affect the sign of the result after subtraction?

vii)   What is the general feature of the "dominated" number ("+4" in *Case 4* or "–4" in *Case 8*) in comparing to another number?

6.   After completing the demonstration of all cases in Table 1, the teacher then distributes the worksheet to students to have hands-on experience of using the buttons to do addition and subtraction. In Part A, questions are structured from easy to difficult while in Part B, questions are of mixed levels of difficulties.

7.   In concluding the activity, the teacher may ask students the following questions for discussion.
i)   What is the sign of the result when adding two numbers of the same sign? Is it true for subtraction?

ii)   How is the sign of the result determined when two numbers of opposite signs are added?

Activity on Addition and Subtraction

When dealing with addition, pair the black and white buttons to make zero(s) and count how many are left.

When dealing with subtraction, just take away the buttons if there is a sufficient number of buttons of the same colour. Otherwise, add enough "zeros" i.e. pair(s) of black and white buttons, to make sufficient number of buttons to be taken away.
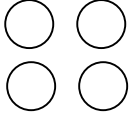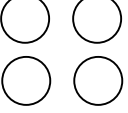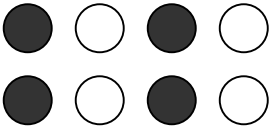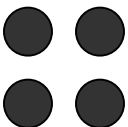
Table 1
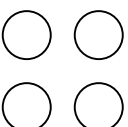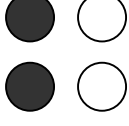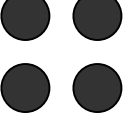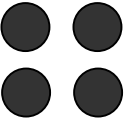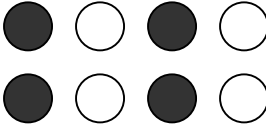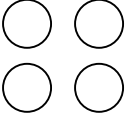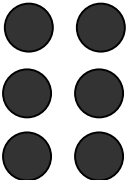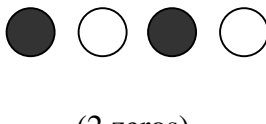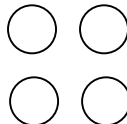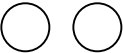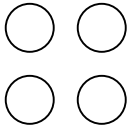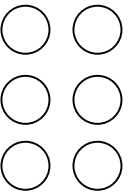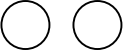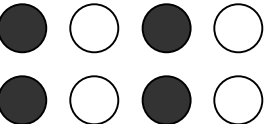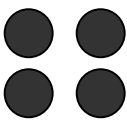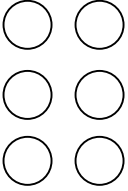
Case	Expression	Start	Buttons added	Buttons taken away	Buttons left
1	0 + (−4) = −4	No buttons	○ ○ ○ ○		○ ○ ○ ○
2	0 − (+4) = −4	No buttons	● ○ ● ○ ● ○ ● ○ (4 zeros)	● ● ● ●	○ ○ ○ ○
3	(+2) + (−4)= −2	● ●	○ ○ ○ ○		○ ○
4	(+2) − (+4) = −2	● ●	● ○ ● ○ (2 zeros)	● ● ● ●	○ ○
5	(+2) +(+ 4)= + 6	● ●	● ● ● ●		● ● ● ●

Table 1 (continued)

Case	Expression	Start	Buttons added	Buttons taken away	Buttons left
6	(+2) – (–4) = +6	● ●	● ○ ● ○ ● ○ ● ○ (4 zeros)	○ ○ ○ ○	● ● ● ● ● ●
7	(–2) + (+4) = +2	○ ○	● ● ● ●		● ●
8	(–2) – (–4) = +2	○ ○	● ○ ● ○ (2 zeros)	○ ○ ○ ○	● ●
9	(–2) + (–4) = –6	○ ○	○ ○ ○ ○		○ ○ ○ ○ ○ ○
10	(–2) – (+4) = –6	○ ○	● ○ ● ○ ● ○ ● ○ (4 zeros)	● ● ● ●	○ ○ ○ ○ ○ ○

Worksheet

Use buttons of two different colours to find the answers.

Part A

1. $0 + (-5) = $ _____

2. $0 - (-5) = $ _____

3. $(+3) + (-5) = $ _____

4. $(+3) - (+5) = $ _____

5. $(+4) + (+6) = $ _____

6. $(+4) - (-6) = $ _____

7. $(-1) + (+7) = $ _____

8. $(-1) - (-7) = $ _____

9. $(-1) + (-5) = $ _____

10. $(-1) - (+5) = $ _____

Part B

1. $(+6) + (+3) = $ _____

2. $(+6) - (+5) = $ _____

3. $(-7) + (+4) = $ _____

4. $(-4) + (-5) = $ _____

5. $(-5) - (+4) = $ _____

6. $(-3) - (-1) = $ _____

7. $(-8) - (-3) = $ _____

8. $(+5) - (-5) = $ _____

9. $(-3) - (-3) = $ _____

10. $(-2) - (-7) = $ _____

Notes for Teachers:

1. This learning activity is designed as an introductory lesson to the topic on the addition and subtraction of directed numbers. It can be used as a hands-on activity for students to discover the concepts of the addition and subtraction of directed numbers.

2. Answers to the worksheet:

   Part A

1.	−5	2.	+5	3.	−2	4.	−2	5.	+10
6.	+10	7.	+6	8.	+6	9.	−6	10.	−6

   Part B

1.	+9	2.	+1	3.	−3	4.	−9	5.	−9
6.	−2	7.	−5	8.	+10	9.	0	10.	+5

3. Answers to the questions for discussion in Point 7 of the Description of the Activity:

   i) Adding 2 numbers of the same sign will produce a sum with the same sign as the two numbers;

   ii) the result of adding 2 numbers of different signs will have the same sign of the number which has a greater magnitude (the "dominated" number) and the magnitude of the result will be the same as the difference of the magnitudes of the 2 numbers (as in *Case 3*).

4. After the activity, the teacher guides students to arrive at the following conclusions.

   For any two positive numbers a and b,

Addition	*Subtraction*
$(+a)+(+b) = a+b$	$(+a)-(+b) = a-b$
$(+a)+(-b) = a-b$	$(+a)-(-b) = a+b$
$(-a)+(+b) = -a+b$	$(-a)-(+b) = -a-b$
$(-a)+(-b) = -a-b$	$(-a)-(-b) = -a+b$