Exemplar 3:
Overestimate and Underestimate

Objective: To determine whether estimated values are overestimated or underestimated.

Key Stage: 3

Learning Unit: Numerical Estimation

Materials Required: Worksheets

Prerequisite Knowledge: Rounding off numbers

Description of the Activity:

1. The teacher distributes the worksheet to students and asks them to work in pairs on Questions 1 to 9 in the worksheet.

2. The teacher should remind students that calculating tools must not be used for estimation activities. Exact answers are not required.

3. Having completed Questions 1 to 9, students are required to summarize the estimated values in Question 10.

4. Students are then invited to present their answers and explain their overestimations or underestimations.

5. The teacher should point out that the answer to the same problem may be overestimated or underestimated due to different estimation strategies. Sometimes, it is not even possible to point out whether the estimated value is an overestimation or underestimation.

6. The teacher can ask students to suggest daily life situations where they have to use estimations.

Worksheet : Overestimate and Underestimate

Without using calculators, estimate the answers for the questions below. Before working out the estimates, consider whether the figures used in the process of estimation should be rounded up or rounded down.

1. If May wants to buy 23 apples and each apple costs $1.7, estimate how much she has to pay.

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2. A crane has a maximum loading of 500 kg.　How many boxes, each weighing 19 kg, can this crane lift?

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3. A piece of ribbon, 4 m long is cut into pieces, each 0.36 m long.　How many pieces can be obtained?

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4. Emily wants to buy a book. Its marked price is $140. The book is on sale now and the selling price is 37% off the marked price. Estimate how much Emily needs to pay.

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5. In a district election poll, candidate X received 3 172 votes, candidate Y received 9 327 votes and candidate Z received 11 082 votes. Can you estimate the total number of votes in this election?

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6. The maximum loading of the lift is 600 kg. A number of children, whose weights range from 37 kg to 39 kg, are waiting for the lift. If all of them go into the lift, it will be overloaded unless two of them do not take this lift. Estimate the total number of children.

7. The floor area of a living room in an apartment is 10 m × 9 m. If John wants to cover this floor with tiles of size 0.8 m × 0.8 m, estimate the number of tiles he needs.

8. A painter calculates that 14.9 litres of paint are required to paint a room. How many tins of paint should he buy if each tin contains 2.75 litres of paint?

9. A company wants to transport 20 cars of the same model. If the length of each car is 4.72 m long and the cargo compartment of a car-transport vehicle is 24 m long, how many car-transport vehicles are required to carry all these cars?

10. Which of the answers obtained above are overestimated or underestimated ? Write the question numbers in the table below:

	Overestimated	Underestimated	Not able to determine
Question Numbers			

Notes for Teachers:

1. The answer to the same problem may be overestimated or underestimated due to different estimation strategies. The teacher should accept different answers, which are obtained from reasonable estimation strategies.

2. It is important to remind students that a skillful estimation requires flexibility in choosing estimation strategies. Decisions must be made about how close to the exact answer one should estimate, whether one should overestimate or underestimate the result and what strategy one should use. Students should recognize that

    a) *An estimate can be obtained through various processes*

    > Example: To calculate 6.39 + 3.75 + 5.98
    >   Process 1:   The total value for the integral part: 6 + 3 + 5 = 14
    >   The approximated sum of the decimal part is 2.
    >   The sum is about 16.
    >   Process 2:   Round the numbers to the nearest integers
    >   6+ 4 + 6 = 16
    >   The sum is about 16.

    b) *An acceptable estimate can be more than one*

    > Example: To calculate the money for 31 pencils at $2.7 each.
    >   Estimate 1: $32 \times 2.5 = 32 \times 10 \times 1/4 = 80$
    >   The cost is around $80.
    >   Estimate 2: $30 \times 3 = 90$
    >   The cost is around $90.

    c) *The value of an estimate depends on context*
        The appropriateness of whether to overestimate or underestimate depends on the situation.

    > Example: The capacity of a lift must be an underestimate of what the lift can actually hold.
    > Example: A person should overestimate the money he/she brings to go shopping.

3. Through this activity, students can share their views on the reasons of choosing an overestimated or underestimated value.

4. The teacher can further introduce more daily-life activities for students to practice their strategies and their decision in making an underestimate or overestimate. Some examples are:
    i)   to estimate the cost shared by six people on a lunch bill $562;
    ii) to estimate the selling price of a basketball at 10% discount from the marked price $98;
    iii) to estimate the bill when purchase a bag of rice at $45.4, beef at $28.5, vegetable at $10 and fish at $32 from the supermarket;
    iv) whether it is enough to pay $280 for 38 pens at $6.7 each;
    v)   is it possible to purchase 7 pens at $15.6 each with $100?

Notes for Assessment

Assessing students' ability in estimation is not an easy task. The criteria for good estimates are often ambiguous. Some suggestions in measuring students' estimation skills are:

a) Timing
    The time allocated to make estimates should be carefully controlled. The amount of time depends on three important factors:
    i)   types of operation : division usually takes longer time than addition

    ii) the format : "$47\overline{)8526}$" takes less time than "8526 is divided by 47"

    iii) the complexity of the situation : "216×859" takes more time to estimate than "21×85"

    In order to ensure students to estimate and not to compute, it is better to give <u>less time than enough</u>. Overhead projectors and personal computers are effective means to present the items one at a time while maintaining control over the amount of time for each item. If test booklets are used, it is essential to control the amount of time available to complete the problems on each page.

b) Formats for answers

    i)   open-ended: it provides no clues about the answer, but teachers should establish an acceptable range of values for students' estimates.

> Example: To estimate the sum $98 + 295 + 485$
>   An estimate greater than 900 seems unacceptable.

    ii)   intervals: a pseudo number line is provided and some predetermined values are placed along the continuum. Students are required to place an estimate by putting a dot on the number line.

> Example: If you deliver 95 newspapers a day, about how many papers will you deliver in a year?
>
> 20000   25000   30000   35000   40000
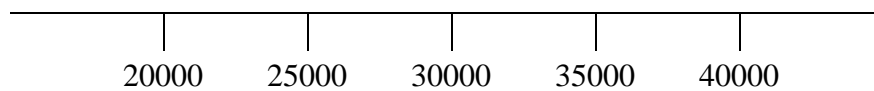
    iii)   multiple-choice

> Example: $22 \div 73 \approx$
>   A. 0
>   B. 0.3
>   C. 0.5
>   D. 1
>   E. 3

    iv)   order of magnitude

> Example: How many days have you lived since your birth? (Circle the nearest answer)
>   A. 300
>   B. 3000
>   C. 30 000
>   D. 300 000

c) Numbers in estimation items

The numbers used in the test questions should be complex enough to encourage students to make estimates.

Example: Question 1   Question 2

4 358   402
+ 2 037   + 305

Students are more willing to estimate the answer for the first question than that for the second question.