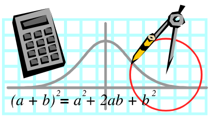

Exemplar 6 :
Exploring Zero Index and Negative Integral Indices

Objectives : (1) To recognise that $a^0=1$ where $a \neq 0$

(2) To recognise that $a^{-n} = \dfrac{1}{a^n}$

Key Stage : 3

Learning Unit : Laws of Integral Indices

Materials Required : Calculators

Prerequisite Knowledge : (1) Use of calculators in finding a^n

(2) Laws of indices involving positive integral indices

Description of the Activity :

1. The teacher revises with students how to use the button $\boxed{x^y}$ of a calculator to find the value of a^n and a^{-n}.

2. The teacher then distributes Worksheet 1 and asks students to suggest the value of a^0 after completing the worksheet.

3. The teacher distributes Worksheet 2 and asks students to suggest the meaning of a^{-n}.

Worksheet 1

1. Use a calculator to find the value of a^0 in each case and complete the table.

a	1	1.5	2	200	–1	–1.5	–111.1
a^0							

2. From the above table, the value of a^0 = ____ .

3. Can you find 0^0 ? What do you get from your calculator?

____

4. For any non-zero real number a, $a^m \times a^n = a^{m+n}$, where m and n are positive integers. Assuming that this law also holds true for numbers with a zero index, find

(i) $a^m \times a^0 = a^{\square + \square} = a^{\square}$

(ii) $a^0 \times a^m$ = __________ = _______

5. From questions 1 and 4, can you suggest the value of a^0 ?

a^0 = _______ for $a \neq$ _______

Worksheet 2

1. Complete the following tables with the help of a calculator.

(i)

$2^{1} = 2$	$2^{-1} = 0.5$	$2^{1} \times 2^{-1} = 2 \times 0.5 = 1$
$2^{2} =$	$2^{-2} =$	$2^{2} \times 2^{-2} =$ ___ $\times$ ___ $=$ ___
$2^{3} =$	$2^{-3} =$	
$2^{4} =$	$2^{-4} =$	
$2^{5} =$	$2^{-5} =$	

From your observation, $2^{n} \times 2^{-n} =$ _________

(ii)

$(-5)^{1} = -5$	$(-5)^{-1} = -0.2$	$(-5)^{1} \times (-5)^{-1} = (-5) \times (-0.2) = 1$
$(-5)^{2} =$	$(-5)^{-2} =$	$(-5)^{2} \times (-5)^{-2} =$ ____ $\times$ _____ $=$ ____
$(-5)^{3} =$	$(-5)^{-3} =$	
$(-5)^{4} =$	$(-5)^{-4} =$	
$(-5)^{5} =$	$(-5)^{-5} =$	

From your observation, $(-5)^{n} \times (-5)^{-n} =$ _________

2. From the result above, what is the value of $a^{n} \times a^{-n}$ for non-zero integral values of a ?

____

____

3. For any non-zero real number a, $a^{m} \times a^{n} = a^{m+n}$ and $a^{m} \div a^{n} = a^{m-n}$, where m and n are positive integers. Assuming that these laws also hold for negative integral indices, find

(i) $a^{n} \times a^{-n} = a^{\square + \square} = a^{\square} =$ _________

(ii) $a^{n} \times \frac{1}{a^{n}} = a^{n} \div a^{n} = a^{\square - \square} = a^{\square} =$ _________

From your observation, $a^{-n} =$ ______ for $a \neq$ _______

Notes for Teachers :

1. The teacher should remind students how to attach a negative sign to a number by using the button $\boxed{+/-}$ of a calculator. Particular attention should be paid to the calculation of values like 5^{-2}, $(-5)^{-2}$, etc.
2. Answers to Worksheets

Worksheet 1

(1)

a	1	1.5	2	200	–1	–1.5	–111.1
a^0	1	1	1	1	1	1	1

(2) $a^0 = 1$

(3) An error message, usually denoted by “E”

(4) (i) $a^{m+0} = a^m$

(ii) $a^{0+m} = a^m$

(5) $a^0 = 1$, for $a \neq 0$

Worksheet 2

(1) (i)

4	0.25	$\underline{4} \times \underline{0.25} = \underline{1}$
8	0.125	$8 \times 0.125 = 1$
16	0.0625	$16 \times 0.0325 = 1$
32	0.03125	$23 \times 0.03125 = 1$

$2^n \times 2^{-n} = 1$

(ii)

25	0.04	$\underline{25} \times \underline{0.04} = \underline{1}$
–125	0.008	$(-125) \times (-0.008) = 1$
625	0.0016	$625 \times (0.0016) = 1$
–3125	0.00032	$(-3125) \times (-0.00032) = 1$

$(-5)^n \times (-5)^{-n} = 1$

(2) $a^n \times a^{-n} = 1$

(3) (i) $a^{n+(-n)} = a^0 = 1$

(ii) $a^{n-n} = a^0 = 1$

$a^{-n} = \dfrac{1}{a^n}$, for $a \neq 0$

3. In finding the value of 0^0, some students may write ‘E’ as the answer. The teacher should point out that it is incorrect to write 0^0 = E. ‘E’ only denotes an error message from a calculator.

4. The teacher may ask the students to make conjectures on the values of a^0 and a^{-n} so as to encourage discussion.

5. A brief review of the laws of indices involving positive integral indices may be appropriate for less able students.