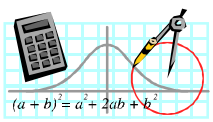


Exemplar 7: Formulating Algebraic Formula

Objective : To formulate algebraic formulas to solve problems

Key Stage : 3

Learning Unit : Formulating Problems with Algebraic Language

Materials Required : Rulers

Prerequisite Knowledge :
(1) Using letters to represent numbers
(2) Solving simple linear equations in one unknown.

Description of the Activity :

1. The teacher puts an equilateral triangle on the blackboard (Figure 1) and explains that the perimeter of the equilateral triangle is 3 units if its side is 1 unit.

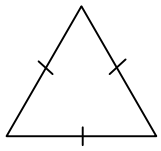


Figure 1

2. The teacher puts another equilateral triangle of identical size alongside with the first triangle so that the two triangles touch each other on one side only (Figure 2).

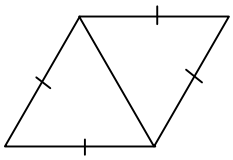


Figure 2

3. The teacher distributes Worksheet 1 to students and asks them to work independently on Questions 1 and 2.

4. For Questions 3 to 6, the teacher helps students form groups. Discussion is encouraged among group members in doing these questions. The teacher may ask students the following questions in order to guide them to solve the problem:
   (a) Can you obtain the solution by observation?
   (b) What strategy will you use to solve the problem?
   (c) Is there any alternative in solving the problem?
   (d) Which method is the best in solving the problem?

5. The teacher may advise students to set up a table to record the number of triangles and the perimeter of the figure to explore their relations if students have difficulties in deriving the formula.

6. After obtaining a formula for the problem, students can verify it by using different values of n.

7. After completing Worksheet 1, students can do Worksheet 2 on their own to consolidate the concept of the method learnt in Worksheet 1.

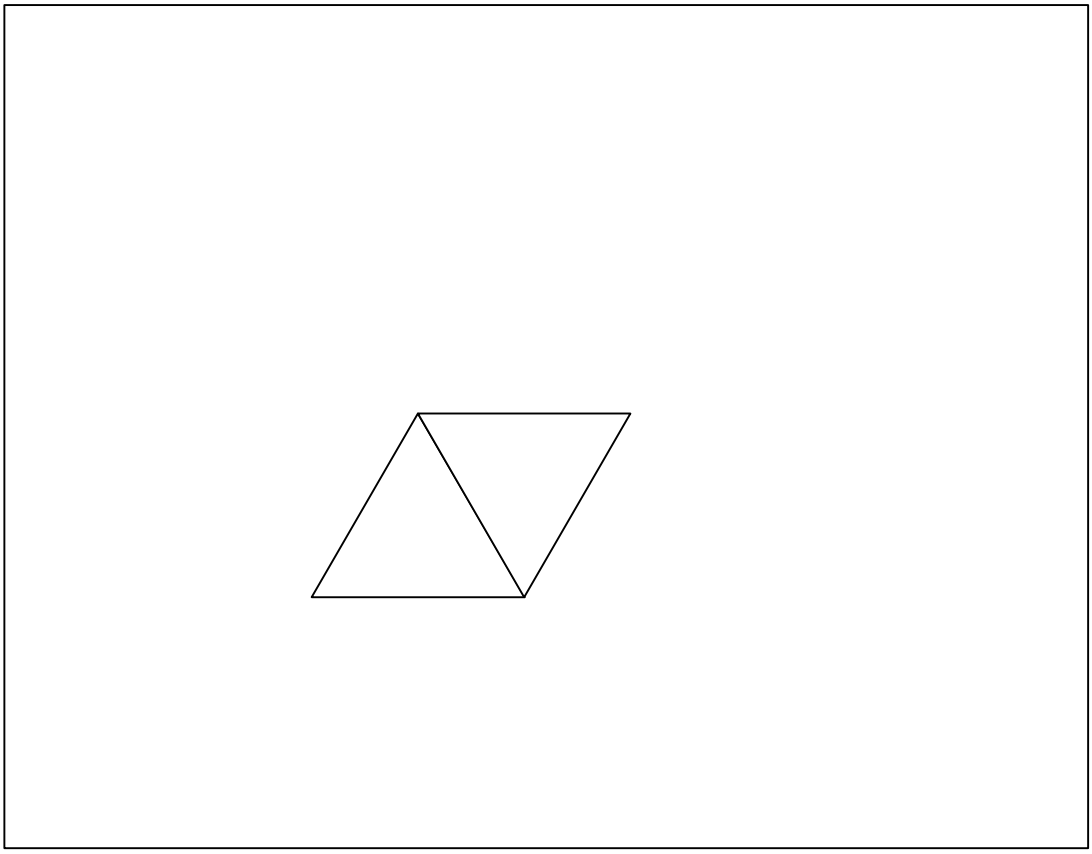
8. The teacher may ask students to answer and explain the following question after they have completed the worksheets:
   What will happen to the relation between the number of triangles and the perimeter of the figure in Worksheet 1 (or hexagon in Worksheet 2) if the new triangle (or hexagon) may be allowed to attach to two of the sides of the figure?

Worksheet 1 : Formulating Algebraic Formula

Problem :

All the triangles used in this worksheet are equilateral and identical. The length of each side of the triangles is 1 unit. In this learning activity, you will add triangles to the following figure. For every triangle added to the figure, it can only attach to one of the sides of the figure.

1. Put a third triangle alongside with any one of these two triangles. Draw it in the following figure:



2. What is the perimeter of the resulting figure?

   _____

   _____

3. Without drawing anything in the figure, can you guess the perimeter of the figure if a 4^{th} triangle is added? What is the perimeter?

   _____

   _____

4. Using a similar argument, what is the perimeter of the figure after the 5^{th} triangle is added ?

_____

_____

5. Write down the perimeter of the figure containing
   (i) 6 triangles;
   (ii) 7 triangles; and
   (iii) 8 triangles.

_____

_____

_____

6. Let n be the number of triangles and p units be the perimeter of the figure. Can you derive a formula relating n and p?

_____

_____

7. What is the perimeter of the figure with 100 triangles?

_____

_____

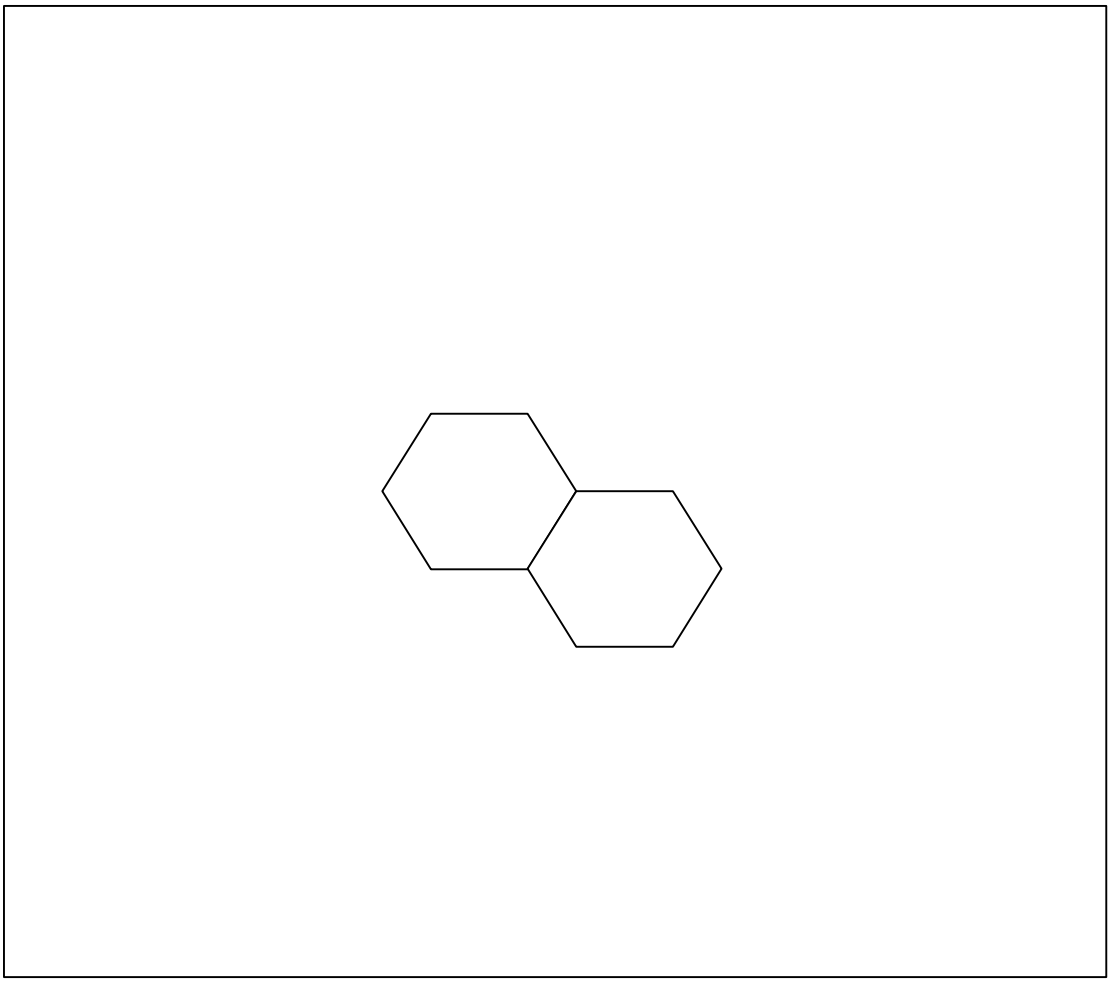
_____

_____

Worksheet 2 : Formulating Algebraic Formula

Problem :

All the shapes used in this worksheet are regular hexagons of identical size with the length of each side equal to 1. To start the activity, you are given two regular hexagons of identical size attached side by side. You have to add new hexagons to the figure. Every new regular hexagon added can only be attached to one of the sides of the figure.



1. What is the perimeter of the above figure?

   _____

2. Draw a 3rd regular hexagon in the above figure such that this hexagon will only touch the figure on one side. What is the perimeter of the new figure ?

   _____

3. Similarly, write down the perimeter of the figure with 5 regular hexagons.

4. Write down the perimeter of the figure containing
   (i) 6 hexagons;
   (ii) 7 hexagons; and
   (iii) 8 hexagons.

5. Let *n* be the number of hexagons and *p* units be the perimeter of the figure. Can you derive a formula relating n and p?

6. Based on the methods used above, can you guess the formula for the perimeter of the figure formed in a similar way with *n* regular octagons?

Notes for Teachers :

1. In this exemplar, students are expected to apply mathematical ideas and skills to unfamiliar questions, evaluate and justify the solution and investigate the problem in a heuristic learning process so that their problem solving skill could be developed. The possible strategies (not exhaustive) used in solving the questions in the worksheet include:

   - Observing
   - Counting
   - Trial and error
   - Tabulating the feasible solution
   - Setting up equations

2. The answer to Question 2 of Worksheet 1 is 4.
   In this question, students may solve it by simply counting the number of sides or observing the pattern of increments. The teacher can ask students to compare the two different methods. They may find that they can obtain the answer easily from both methods.

3. In Questions 3, 4 and 5 of Worksheet 1, students may use the counting method or the tabulation method. When comparing these two methods, they may find that the second is better. The table is shown below:

Number of triangle (n)	Perimeter (p)
1	3
2	4
3	5
4	6
5	7
6	8
7	9
8	10

4. Question 6 of Worksheet 1 is a more challenging problem since there is an unknown n in the question. Students need to make some guesses on the values of the unknown. The focus is on how guesses can be proceeded in the right direction. The teacher can encourage students to set up a table similar to the one shown in the

last point to explore the relation between n and p. Students can also use some more values of n to find p if they have difficulties in deriving the formula. If necessary, an additional column showing the pattern of the numbers could be included. An example is shown below:

Number of triangle (n)	Perimeter (p)	Pattern
1	3	3 = 1 + 2
2	4	4 = 2 + 2
3	5	5 = 3 + 2
4	6	6 = 4 + 2
5	7	7 = 5 + 2
6	8	8 = 6 + 2
7	9	9 = 7 + 2
8	10	10 = 8 + 2
n	n + 2	$p = n + 2$

5. Answer to Question 7 of Worksheet 1:
   The perimeter is 102 units when there are 100 triangles in the figure.

6. Answers to questions in Worksheet 2:
   Q1. 10
   Q2. 14
   Q3. 22
   Q4. (i) 26
   (ii) 30
   (iii) 34

7. A table similar to the one in Point 4 could be set up to find the formula.

Number of hexagon (n)	Perimeter (p)	Pattern
1	6	$6 = 4 \times 1 + 2$
2	10	$10 = 4 \times 2 + 2$
3	14	$14 = 4 \times 3 + 2$
4	18	$18 = 4 \times 4 + 2$
5	22	$22 = 4 \times 5 + 2$
6	26	$26 = 4 \times 6 + 2$
7	30	$30 = 4 \times 7 + 2$
8	34	$34 = 4 \times 8 + 2$
n	$4n + 2$	$p = 4n + 2$

8. Answer to Question 6 : 8n + 2

9. For Point 8 in the Description of the Activity, students would realize that the numbers n and p do not form a relation. Adding an additional shape to one side of the existing figure increases the perimeter by a fixed number of units. If the additional shape is attached to two of the sides of the existing figure, the pattern of adding a fixed number will be no longer valid.