Exemplar 10:

Number Patterns — Fibonacci Sequence (1)

Objectives: (1) To investigate the pattern of numbers in Fibonacci sequence
(2) To use algebraic symbols to represent the pattern

Key Stage: 3

Learning Unit: Formulating Problems with Algebraic Language

Materials Required : Worksheets

Prerequisite Knowledge: (1) Ability in observing the patterns of simple number sequences
(2) Using algebraic symbols to represent an expression

Description of the Activity:

9. The teacher distributes the worksheet to students.

10. The teacher describes the problem to students.

11. Students are asked to work in pairs. If possible, students could go outside the classroom and use stairs to help to answer the questions in the worksheet.

12. Students in each group should try to think of the possible ways of going up a flight of stairs with a given number of steps. One student tries the number of ways of going up the stairs while the other records the observations. They can swap their roles for the activity.

5. Students are then invited to describe and explain the relation they found on the pattern on the numbers of ways of climbing the stairs. The teacher gives comments at appropriate times.

6. After checking the answers in the worksheet, the teacher may invite some students to explain why the condition "T (n) = T (n – 1) + T (n – 2) for natural numbers $n > 2$" holds in general in this activity. That is, the number of ways of climbing a flight of

stairs with n steps is the sum of the number of ways of climbing a flight of stairs with $n-1$ steps and the number of ways climbing a flight of stairs with $n-2$ steps.

7. The following questions could be raised for discussion:

   (a) Before reaching the n^{th} step, which step were you on if
      (i) you leap 2 steps to reach the n^{th} step;
      (ii) you just go up 1 step to reach the n^{th} step?

   (b) How many ways are there for you to climb to the n^{th} step if you leap two steps forward in the last move?

   (c) How many ways are there for you to climb to the n^{th} step if you climb one step forward in the last move?

   (d) How many ways are there for you to climb to the n^{th} step if you move one step or two steps forward at any point?
   (The teacher may use the 3rd step as an example to explain the questions in this point)

8. The teacher introduces the term Fibonacci sequence and concludes that the problem of counting stair steps can be solved by computing the terms in Fibonacci sequence which is a recursive sequence with the first two terms both equal to 1 and each successive term obtained by adding together the two previous terms. The sequence is 1, 1, 2, 3, 5, 8, … which can be described by the conditions $T(1) = T(2) = 1$ and $T(n) = T(n-1) + T(n-2)$ for $n > 2$ and n is a natural number.

9. The teacher points out that the number pattern obtained in this exemplar is in Fibonacci sequence.

Worksheet

Problem:

Every morning I have to take the stairs to go to the classroom. My legs are not long enough to leap 3 steps, so I can only go up one step or leap two steps at a time. How many ways can I go up a flight of stairs with n steps?

1. Complete the following table.

Number of steps, *n*	Total number of ways of climbing up the stairs with n steps, T (*n*)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

2. In your own words, describe the pattern of the total number of ways of climbing n steps by referring to the above table.

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3. Derive a formula for T (n) to represent the relation described in Question 2.

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4. Can you generate the sequence T(n) in the table of Question 1 by the formula obtained in Question 3 only? If not, what other conditions must be added to generate the whole sequence?

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5. Write down all the conditions that are sufficient to describe the pattern of the number of ways of going up a flight of stairs with n steps.

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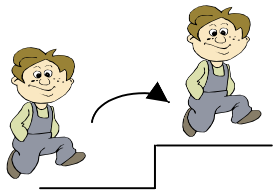
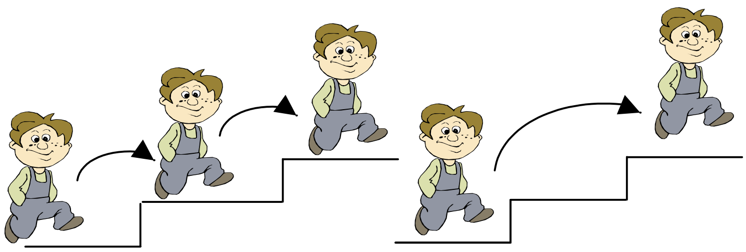
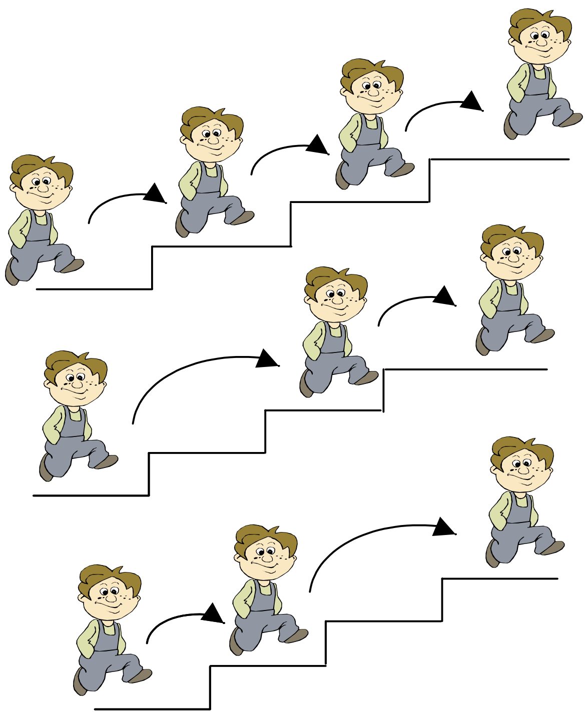
Notes for Teachers:

1. Before doing the worksheet, the teacher may revise with students on how to use T (n) to denote the nth term of a sequence.

2. The teacher may use the following diagram to explain to students the ways that he/she can go up a stairs with 1 step, 2 steps and 3 steps:

No. of steps	Different ways to climb up the stairs	Total no. of ways
1		1
2		2
3		3

3. Answer to Question 1 in the worksheet is

Number of steps, *n*	Total number of ways, T (*n*)
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89

4. If students have difficulties in finding the number of ways, the teacher can suggest students to add an additional column to investigate the relation of each T (*n*) with the previous ones.

Number of steps, *n*	Total number of ways, T (*n*)	Relation of T (*n*) with the previous ones
1	1	1
2	2	2
3	3	3 = 2 + 1
4	5	5 = 3 + 2
5	8	8 = 5 + 3
6	13	13 = 8 + 5
7	21	21 = 13 + 8
8	34	34 = 21 + 13
9	55	55 = 34 + 21
10	89	89 = 55 + 34

5. Answer to Question 2:

   The number of ways of climbing a flight of stairs with n steps is the sum of the number of ways of climbing a flight of stairs with $n - 1$ steps and the number of ways of climbing a flight of stairs with $n - 2$ steps.

6. Answer to Question 3:

   $T(n) = T(n - 1) + T(n - 2)$ where $n > 2$

7. Answer to Question 4:

   $T(1) = 1$

   $T(2) = 2$

8. Answer to Question 5:

   $T(1) = 1$

   $T(2) = 2$

   $T(n) = T(n - 1) + T(n - 2)$ where $n > 2$