Exemplar 11 :
Number Patterns — Fibonacci Sequence (2)

Objectives : (1) To investigate the pattern of numbers in Fibonacci sequence
(2) To use algebraic symbols to represent the pattern

Key Stage : 3

Learning Unit : Formulating Problems with Algebraic Language

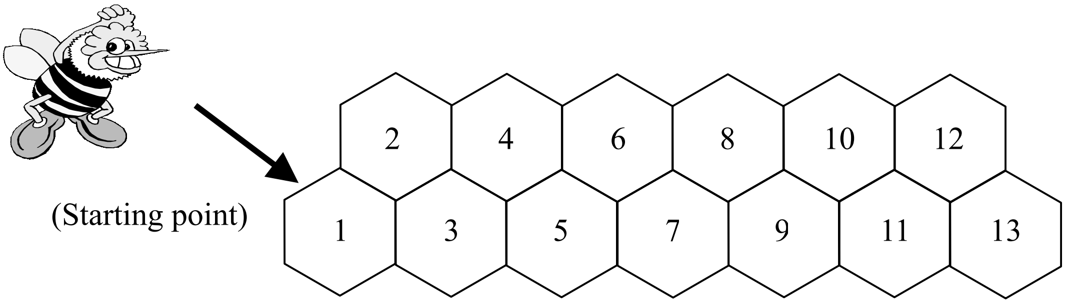
Materials Required : Overhead projector, transparencies, markers and worksheets

Prerequisite Knowledge : (1) The four basic arithmetic operations of integers
(2) The functional notation

Description of the activity :

1. The teacher distributes the worksheet to students.
2. With the aid of transparencies, the teacher describes the problem in the worksheet to students:
3. The teacher asks students to complete the worksheet.
4. Students are invited to explain how they obtain their answers orally.
5. The teacher comments on the answers.

Worksheet : Number Pattern



In the figure, a bee starts at a point shown in the above diagram and only moves to the right to a cell with a bigger number. Count the number of paths that brings the bee from the **starting point** to the cell numbering *n* where *n* = 1, 2, 3,……

n	**T(*n*)** **(No. of paths from the starting point to the cell numbering *n*)**
1	
2	
3	
4	
5	
6	
7	
8	

1. What is the relationship among T(1), T(2) and T(3)? Explain the relationship by referring to the movement of the bee.

2. What is the relationship among T(2), T(3) and T(4)? Explain the relationship by referring to the movement of the bee.

3. What is the relationship among any three consecutive terms of T(n)? Use T($n-2$), T($n-1$) and T(n) to denote three consecutive terms of the sequence. Explain the relationship by referring to the movement of the bee.

4. Can you describe the pattern of the number of paths by the relationship in Question 3 only? If not, what other conditions must be added to describe the whole pattern?

5. Write down all the conditions that are sufficient to describe the pattern of the number of paths from the starting point to the cell numbering n.

Notes for Teachers :

1. Before doing the worksheet, the teacher could revise with students on how to use the functional notation, T(n), to denote the n^{th} term in a sequence.

2. The number of paths from the **starting point** to the cell numbering n, T(n), is given by the following conditions:
   (a) T(1) = 1;
   (b) T(2) = 2; and
   (c) $T(n) = T(n-2) + T(n-1)$ for $n > 2$ and n is a natural number.

3. Students may not find it easy to explain why condition (c) holds in general. Guidance from the teacher should be offered.
   (a) The teacher may guide students to consider the following questions:
      (i) Which cell does the bee reach before entering the cell numbering n?
      Answer: Cell numbering $n - 2$ or cell numbering $n - 1$.
      (ii) How many paths are there from the cell numbering $n - 1$ to the cell numbering n and from the cell numbering $n - 2$ to the cell numbering n?
      Answer: There is only one path for either case.
      (iii) How many paths are there from the starting point to the cell numbering n via the cell numbering $n - 2$ and via the cell numbering $n - 1$?
      Answer: $T(n - 2)$ for the first part and $T(n - 1)$ for the second part
   (b) The teacher then helps students to derive/obtain that the number of paths for the bee going from the starting point to the cell numbering n is the sum of the number of paths from the starting point to the cell numbering $n - 2$ and the number of paths from the starting point to the cell numbering $n - 1$.

4. Students should be told that any sequence satisfying the conditions given in (2) are in Fibonacci sequence.

5. T(n) may also be defined by

$$T(n) = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right], \quad \text{for } n \geq 1.$$